We will start quantum chemistry today.

policy

1. we do not aim at being comprehensive
2. we will learn minimum topics to …

perform astronomical observations

1. **statistic degeneracy** — necessary to calculate N_J
2. **selection rule** — which transition to look
3. **notation** — J, K, Ka, Kc, m; understand literature
4. **nuclear spin** — what is ortho and para?

principle

> reasons of many things :
>
> **if we think in that way, it matches reality.**
> **or even have power of prediction.**

1 statistic degeneracy

$g_J = 2J + 1$

$$\frac{N_J}{g_J} = N_0 \; exp\left(-\frac{E_J}{kT}\right)$$

1 statistic degeneracy
2 selection rule
3 notation
4 nuclear spin

- notation 3
- quantum numbers

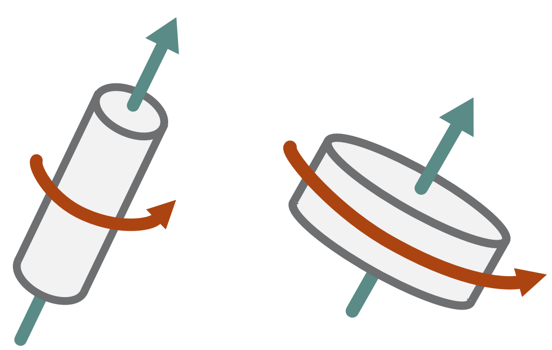
J, K, Ka, Kc

ϕ_r wave function

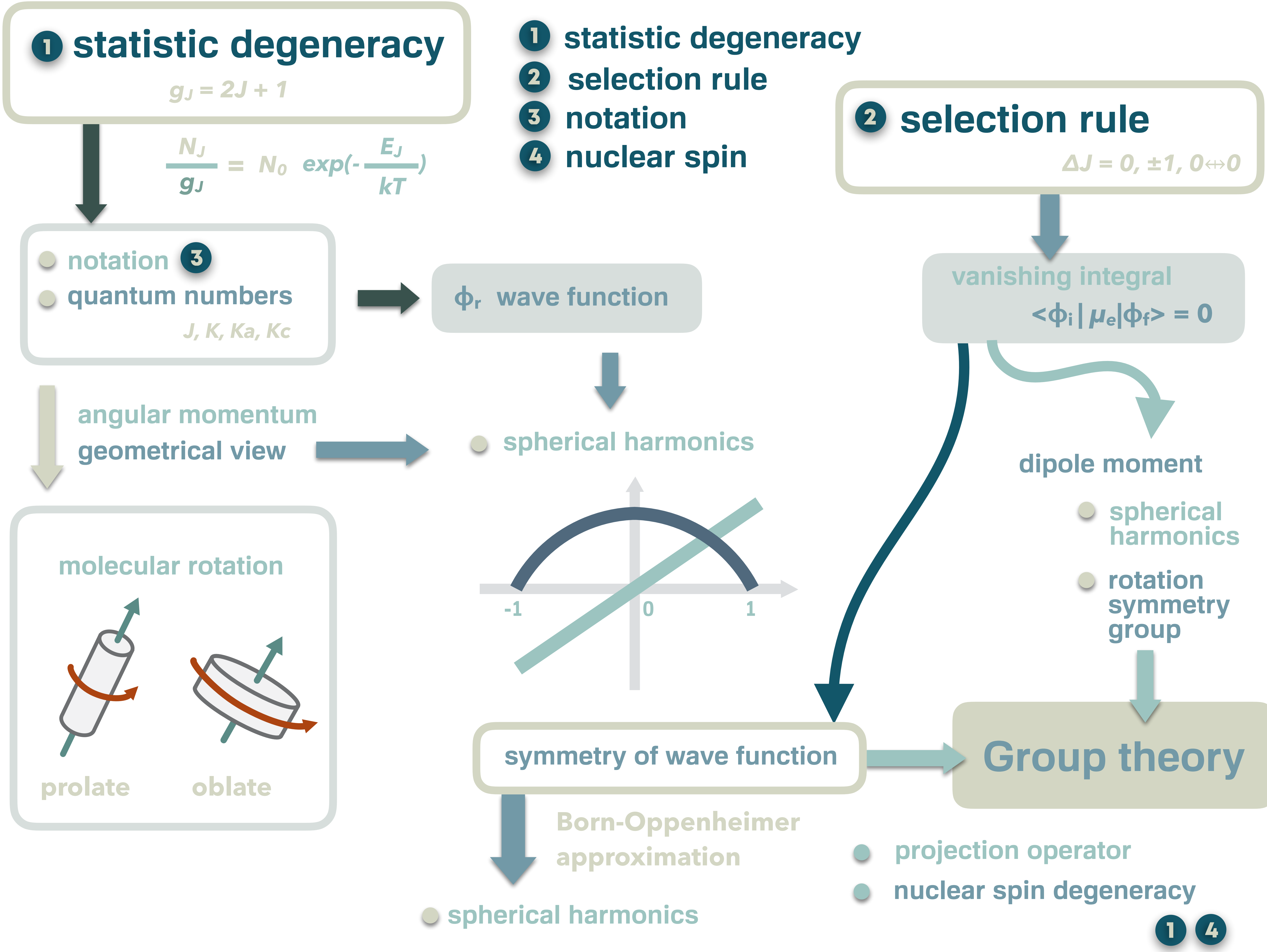
angular momentum
geometrical view

- spherical harmonics

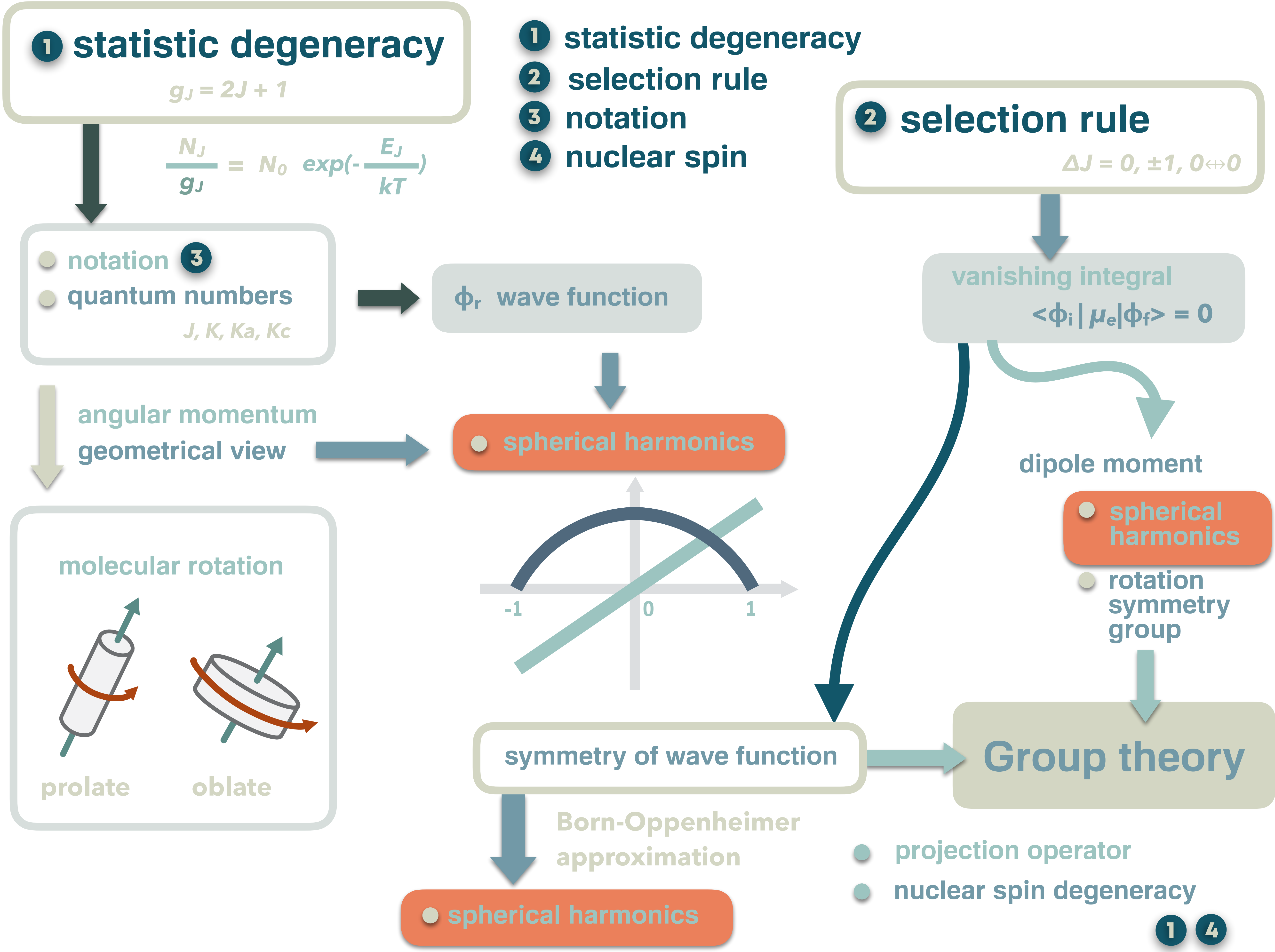
molecular rotation



prolate oblate



1 statistic degeneracy
$g_J = 2J + 1$
$\frac{N_J}{g_J} = N_0 \exp(-\frac{E_J}{kT})$
1 statistic degeneracy
2 selection rule
3 notation
4 nuclear spin
2 selection rule
$\Delta J = 0, \pm 1, 0 \nleftrightarrow 0$
notation 3
quantum numbers
J, K, Ka, Kc
ϕ_r wave function
vanishing integral
$<\phi_i | \mu_e | \phi_f> = 0$
angular momentum
geometrical view
spherical harmonics
dipole moment
spherical harmonics
rotation symmetry group
-1
0
1
molecular rotation
prolate
oblate
symmetry of wave function
Group theory
Born-Oppenheimer approximation
spherical harmonics
projection operator
nuclear spin degeneracy
1 4

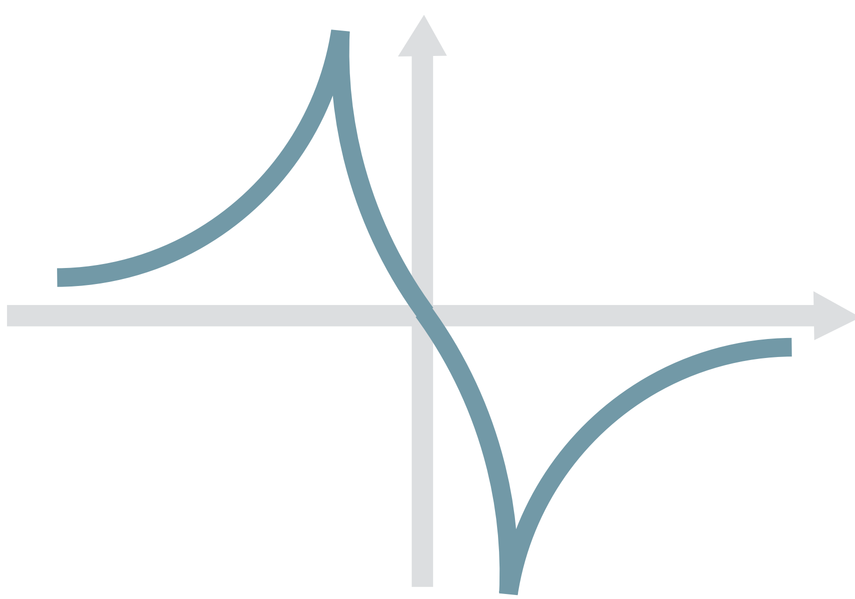


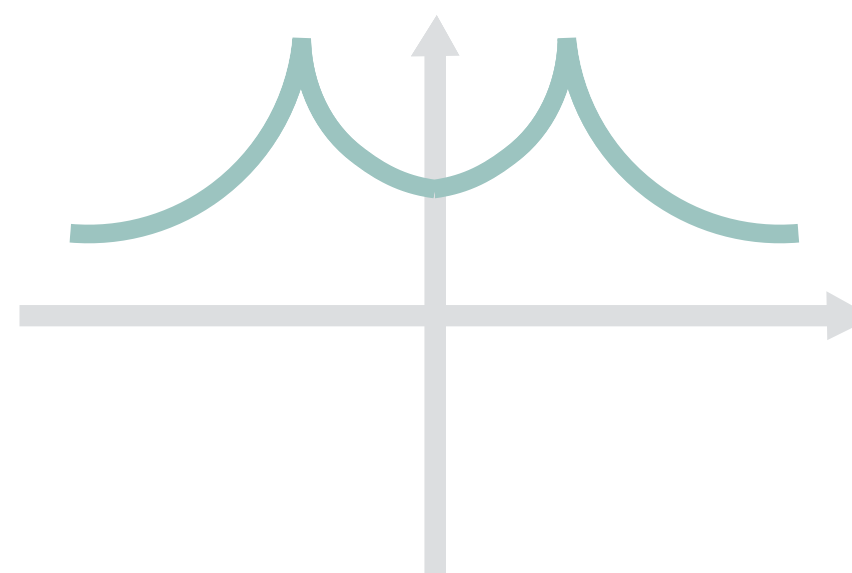
1 statistic degeneracy
$g_J = 2J + 1$
$\frac{N_J}{g_J} = N_0 \exp(-\frac{E_J}{kT})$
1 statistic degeneracy
2 selection rule
3 notation
4 nuclear spin
2 selection rule
$\Delta J = 0, \pm 1, 0 \not\leftrightarrow 0$
notation 3
quantum numbers
J, K, Ka, Kc
ϕ_r wave function
vanishing integral
$\langle \phi_i | \mu_e | \phi_f \rangle = 0$
angular momentum
geometrical view
spherical harmonics
dipole moment
spherical harmonics
rotation symmetry group
molecular rotation
-1
0
1
prolate
oblate
symmetry of wave function
Group theory
Born-Oppenheimer approximation
spherical harmonics
projection operator
nuclear spin degeneracy
1 4

Wave function

what do you think is a wavefunction?

1. solution of Schrödinger equation
2. squared, probability distribution of electron
3. $<J, K, m \,|\, O \,|\, J', k', m' >$ bra.
4. something related to covalent bond





can we explain what is **rotational wavefunction**?

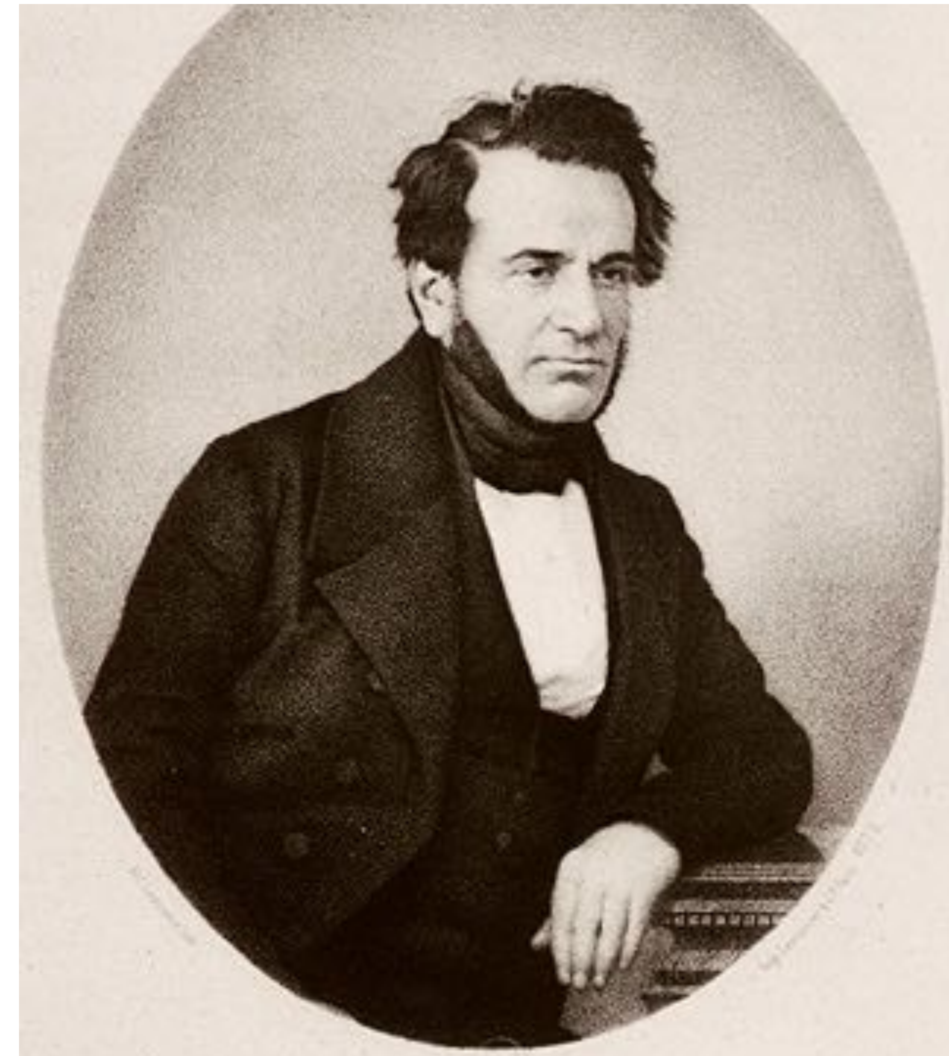
Road to spherical harmonics

1. Hamiltonian in central field $$H = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{Ze^2}{r}$$
2. separate r and θ, ϕ $$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\Lambda^2$$
3. Laplacian in polar coordinate $$\Lambda^2 = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}$$

---

4. Legendre differential equation is part of Laplacian
5. Rodrigues formula is solution Leibniz rule
6. full Laplacian is associated Legendre differential equation
7. Derivative of Rodrigues formula is solution

→ spherical harmonics



Pierre-Simon Laplace
1749-1827

Adrien-Marie Legendre
1752-1833

Olinde Rodrigues
1795-1851

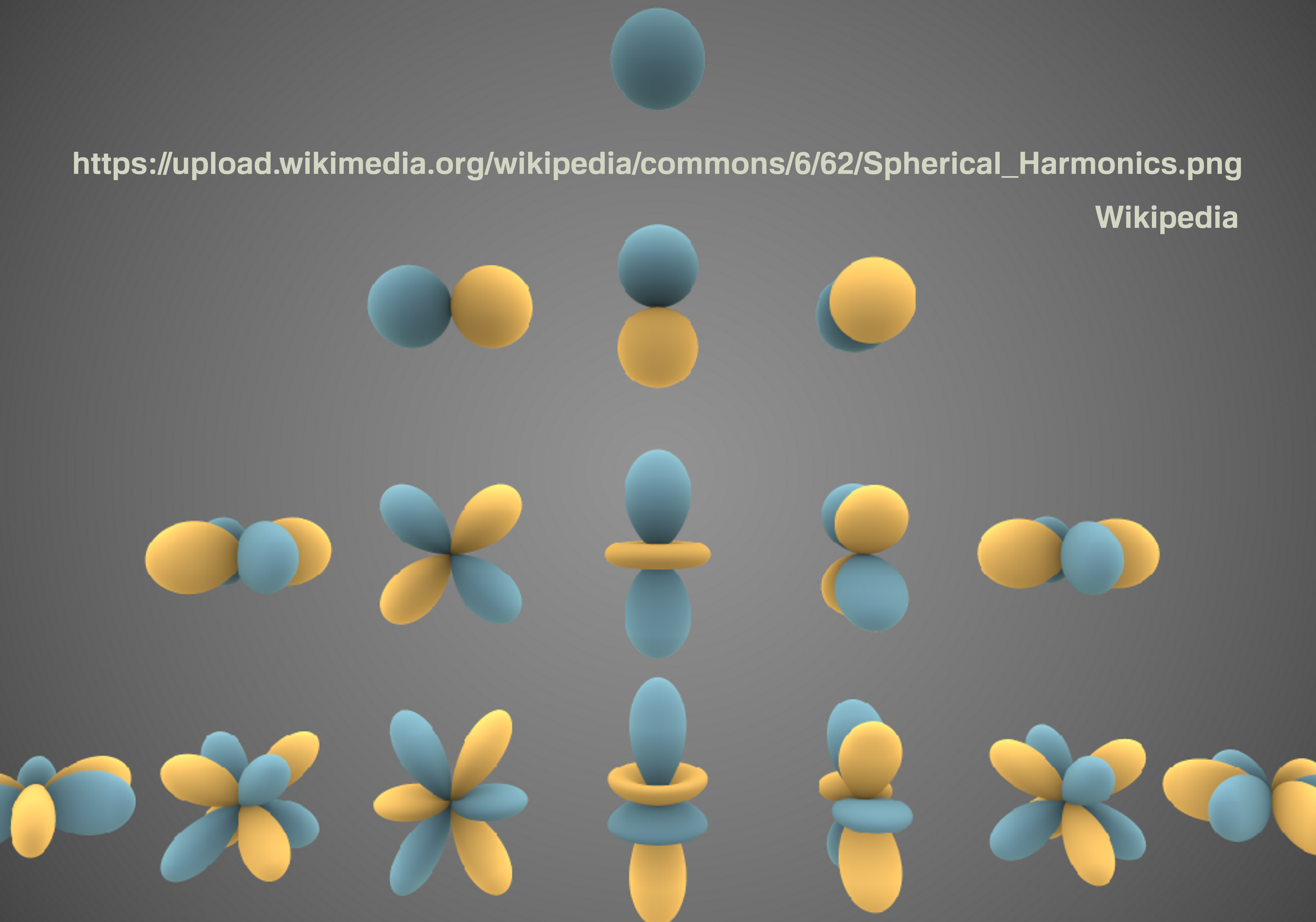
Sir William Rowan Hamilton
1805-1865

sparing one day
of your life
for spherical harmonics
is not too bad

Gottfried Wilhelm Leibniz
1646-1715 born in Leipzig

Benefits of taking time for spherical harmonics

1. it is a wave function
2. angular momentum — J, K, Ka, Kc
3. rotational energy — $E = Bh\,J(J+1)$
4. selection rule — $\Delta J = 0, \pm 1, 0 \nleftrightarrow 0$
5. parity, symmetry — $(-1)^J$
6. vanishing integral



https://upload.wikimedia.org/wikipedia/commons/6/62/Spherical_Harmonics.png

Wikipedia

Hamiltonian in central field

$$H\Psi = E\Psi$$

$$p = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$H = \frac{p^2}{2m} + V(x)$$

$$= \sum_{i}^{n} \frac{p_i^2}{2m_i} + \sum_{i<j}^{n} \frac{Z_i Z_j e^2}{r_{ij}}$$

kinetic potential

$$H\Psi = E\Psi$$

$$\mathbf{x}\Psi = x\Psi$$

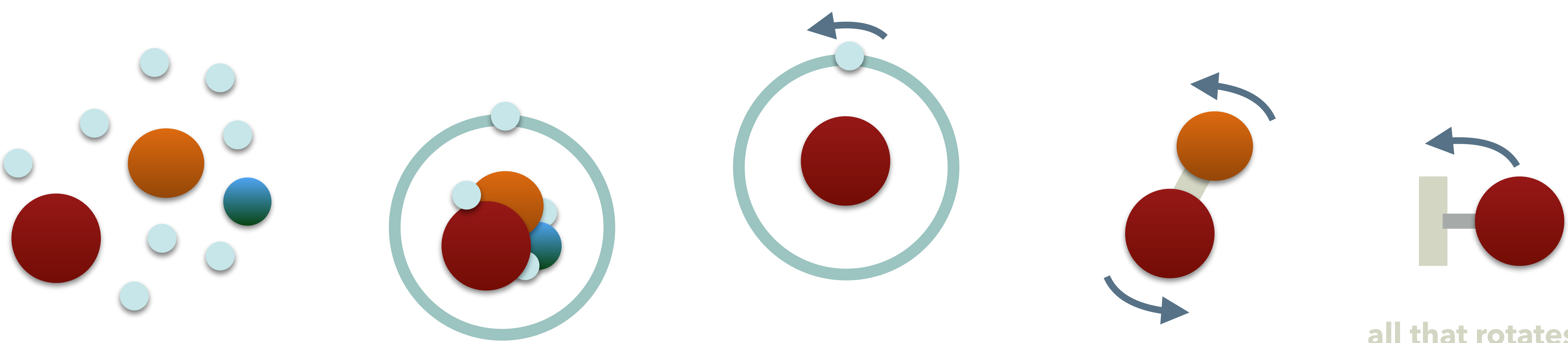
$$\mathbf{L}^2\Psi = l(l+1)\Psi$$

current computers can calculate the formation of structure since the Big Bang,

but it is hard to calculate accurate wave function with a system n > 3

using assumptions to make problem simple

assumptions give us qualitatively important knowledges, like selection rules



$$p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$H = \frac{p^2}{2m} + V(x)$$

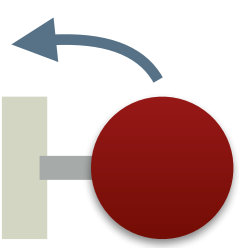
$$p^2 = -\hbar^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$= \sum_i^n \frac{p_i^2}{2m_i} + \sum_{i<j}^n \frac{Z_i Z_j e^2}{r_{ij}}$$

kinetic **potential**

$$H = -\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) - \frac{Ze^2}{r}$$

$$= -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{Ze^2}{r}$$



all that rotates

Laplacian

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\Lambda^2$$

$$\Lambda^2 = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial \phi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}$$

Legendrian

$$H\Psi = E\Psi$$

if $\Psi(r,\theta,\phi) = R(r)Y(\theta,\phi)$

$$HRY = -\frac{\hbar^2}{2\mu}\nabla^2(RY) - \frac{Ze^2}{r}RY$$

$$= -\frac{\hbar^2}{2\mu}\left(\frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\Lambda^2\right)RY - \frac{Ze^2}{r}RY$$

$$= -\frac{\hbar^2}{2\mu}\left(Y\frac{1}{r}\frac{\partial^2}{\partial r^2}rR + R\frac{1}{r^2}\Lambda^2 Y\right) - \frac{Ze^2}{r}RY$$

if $\Lambda^2 Y = c_1 Y$ **equation involves Y only**

$$Y\frac{1}{r}\frac{\partial^2}{\partial r^2}rR + \frac{R}{r^2}c_1 Y - V(r)RY = -\frac{2\mu}{\hbar^2}ERY$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}rR + \frac{R}{r^2}c_1 - V(r)R = -\frac{2\mu}{\hbar^2}ER$$

equation involves R only

We left 2 issues behind

1

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2} r + \frac{1}{r^2}\Lambda^2$$

Laplacian

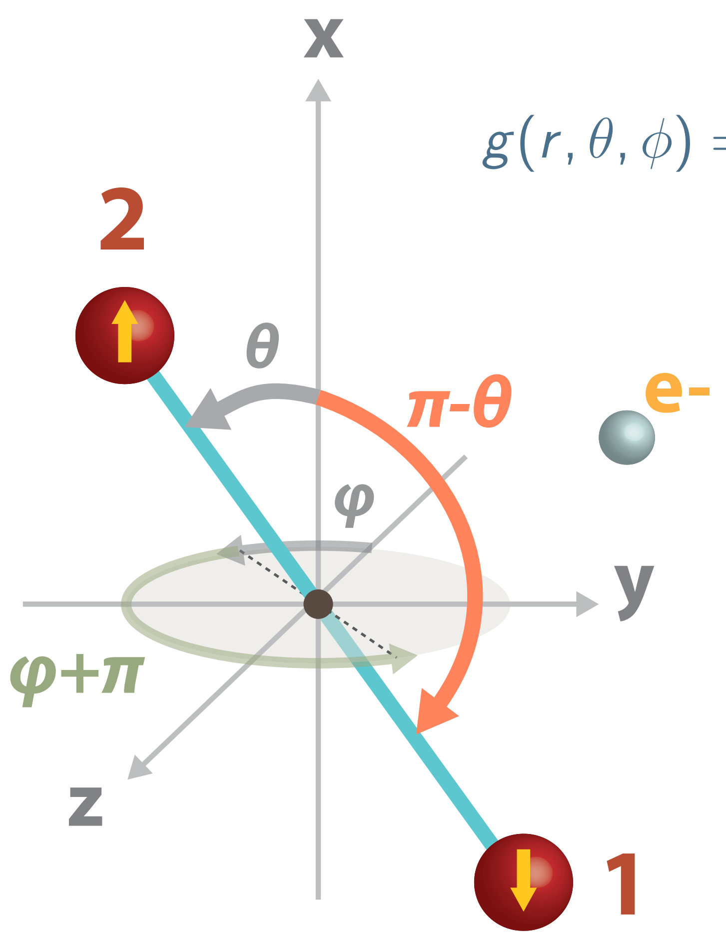
$$\Lambda^2 = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}$$

Legendrian

2

$$\Lambda^2 Y = c_1 Y$$

spherical harmonics



$$g(r,\theta,\phi) = g(r(x,y,z),\theta(x,y,z),\phi(x,y,z)) = f(x,y,z)$$

$$\frac{\partial g}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r}$$

$$g_r = f_x\frac{\partial x}{\partial r} + f_y\frac{\partial y}{\partial r} + f_z\frac{\partial z}{\partial r}$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

our mission here is

to write $f_{xx} + f_{yy} + f_{zz}$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

in g_{rr}

$g_{\phi\phi}$

g_{θ}

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\Lambda^2$$

$$\Lambda^2 = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}$$

$$\frac{\partial x}{\partial\phi} = -r\sin\theta\sin\phi$$

$$\frac{\partial y}{\partial\phi} = r\sin\theta\cos\phi$$

$$\frac{\partial z}{\partial\phi} = 0$$

$$\frac{\partial x}{\partial\theta} = r\cos\theta\cos\phi$$

$$\frac{\partial y}{\partial\theta} = r\cos\theta\sin\phi$$

$$\frac{\partial z}{\partial\theta} = -r\sin\theta$$

$$\frac{\partial x}{\partial r} = \sin\theta\cos\phi$$

$$\frac{\partial y}{\partial r} = \sin\theta\sin\phi$$

$$\frac{\partial z}{\partial r} = \cos\theta$$

without knowing why, let us try

$$g_r = f_x \frac{\partial x}{\partial r} + f_y \frac{\partial y}{\partial r} + f_z \frac{\partial z}{\partial r}$$

$$g_r = f_x \sin\theta\cos\phi + f_y \sin\theta\sin\phi + f_z \cos\theta \quad (1)$$

$$\frac{g_\theta}{r} = f_x \cos\theta\cos\phi + f_y \cos\theta\sin\phi - f_z \sin\theta \quad (2)$$

$$\frac{g_\phi}{r\sin\theta} = -f_x \sin\phi + f_y \cos\phi \quad (3)$$

(1) · sinθ + (2) · cosθ

$$g_r \sin\theta + \frac{g_\theta}{r}\cos\theta$$

$$f_x \sin^2\theta\cos\phi + f_y \sin^2\theta\sin\phi + f_x \cos^2\theta\cos\phi + f_y \cos^2\theta\sin\phi$$

$$= f_x \cos\phi + f_y \sin\phi$$

$$\frac{\partial x}{\partial \phi} = -r\sin\theta\sin\phi$$

$$\frac{\partial y}{\partial \phi} = r\sin\theta\cos\phi$$

$$\frac{\partial z}{\partial \phi} = 0$$

$$\frac{\partial x}{\partial \theta} = r\cos\theta\cos\phi$$

$$\frac{\partial y}{\partial \theta} = r\cos\theta\sin\phi$$

$$\frac{\partial z}{\partial \theta} = -r\sin\theta$$

$$\frac{\partial x}{\partial r} = \sin\theta\cos\phi$$

$$\frac{\partial y}{\partial r} = \sin\theta\sin\phi$$

$$\frac{\partial z}{\partial r} = \cos\theta$$

without knowing why, let us try

$$g_r = f_x \frac{\partial x}{\partial r} + f_y \frac{\partial y}{\partial r} + f_z \frac{\partial z}{\partial r}$$

$$g_r = f_x \sin\theta\cos\phi + f_y \sin\theta\sin\phi + f_z \cos\theta$$ (1)

$$\frac{g_\theta}{r} = f_x \cos\theta\cos\phi + f_y \cos\theta\sin\phi - f_z \sin\theta$$ (2)

$$\frac{g_\phi}{r\sin\theta} = -f_x \sin\phi + f_y \cos\phi$$ (3)

(1) · cosθ - (2) · sinθ

$$g_r \cos\theta - \frac{g_\theta}{r}\sin\theta$$

$$f_x \sin\theta\cos\theta\cos\phi + f_y \sin\theta\cos\theta\sin\phi - f_x \sin\theta\cos\theta\cos\phi - f_y \sin\theta\cos\theta\sin\phi + f_z$$

$$= f_z$$

$$\frac{\partial x}{\partial \phi} = -r\sin\theta\sin\phi$$

$$\frac{\partial y}{\partial \phi} = r\sin\theta\cos\phi$$

$$\frac{\partial z}{\partial \phi} = 0$$

$$\frac{\partial x}{\partial \theta} = r\cos\theta\cos\phi$$

$$\frac{\partial y}{\partial \theta} = r\cos\theta\sin\phi$$

$$\frac{\partial z}{\partial \theta} = -r\sin\theta$$

$$\frac{\partial x}{\partial r} = \sin\theta\cos\phi$$

$$\frac{\partial y}{\partial r} = \sin\theta\sin\phi$$

$$\frac{\partial z}{\partial r} = \cos\theta$$

$$f_z = g_r \cos\theta - \frac{g_\theta}{r} \sin\theta$$ 6

$$f_x \cos\phi + f_y \sin\phi = g_r \sin\theta + \frac{g_\theta}{r} \cos\theta$$

$$-f_x \sin\phi + f_y \cos\phi = \frac{g_\phi}{r \sin\theta}$$

4	5
• cosθ	• sinθ
-	+
• sinθ	• cosθ

$$f_x = g_r \sin\theta \cos\phi + \frac{g_\theta}{r} \cos\theta \cos\phi - \frac{g_\phi \sin\phi}{r \sin\theta}$$ 4

$$f_y = g_r \sin\theta \sin\phi + \frac{g_\theta}{r} \cos\theta \sin\phi + \frac{g_\phi \cos\phi}{r \sin\theta}$$ 5

$$f_x = g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta} \quad (4)$$

$$f_y = g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta} \quad (5)$$

$$f_z = g_r \cos\theta - \frac{g_\theta}{r}\sin\theta \quad (6)$$

$$f_{xx} = (f_x)_x$$

$$g(r,\theta,\phi) = g\left(r(x,y,z), \theta(x,y,z), \phi(x,y,z)\right) = f(x,y,z)$$

$$h(r,\theta,\phi) = h\left(r(x,y,z), \theta(x,y,z), \phi(x,y,z)\right) = f_x(x,y,z)$$

$$f_x = g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}$$

$$(f_x)_x = h_r \sin\theta\cos\phi + \frac{h_\theta}{r}\cos\theta\cos\phi - \frac{h_\phi \sin\phi}{r\sin\theta}$$

$$f_x = g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta} \quad (4)$$

$$f_y = g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta} \quad (5)$$

$$f_z = g_r \cos\theta - \frac{g_\theta}{r}\sin\theta \quad (6)$$

$$f_{xx} = (f_x)_x$$

$$g(r,\theta,\phi) = g\left(r(x,y,z),\theta(x,y,z),\phi(x,y,z)\right) = f(x,y,z)$$

$$h(r,\theta,\phi) = h\left(r(x,y,z),\theta(x,y,z),\phi(x,y,z)\right) = f_x(x,y,z)$$

$$(f_x)_x = h_r \sin\theta\cos\phi + \frac{h_\theta}{r}\cos\theta\cos\phi - \frac{h_\phi \sin\phi}{r\sin\theta}$$

$$\begin{aligned}(f_y)_y = &\left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi\cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi \\ &+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi\cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r} \\ &+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi\cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}\end{aligned}$$

$$\begin{aligned}(f_z)_z = &\left(g_r\cos\theta - \frac{g_\theta\sin\theta}{r}\right)_r \cos\theta \\ &- \left(g_r\cos\theta - \frac{g_\theta\sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}\end{aligned}$$

$$\begin{aligned}(f_x)_x = &\left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi\sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi \\ &+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi\sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r} \\ &- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi\sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}\end{aligned}$$

$$f_x = g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta} \quad (4)$$

$$f_y = g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta} \quad (5)$$

$$f_z = g_r \cos\theta - \frac{g_\theta}{r}\sin\theta \quad (6)$$

$$f_{xx} = (f_x)_x$$

$$g(r,\theta,\phi) = g\left(r(x,y,z),\theta(x,y,z),\phi(x,y,z)\right) = f(x,y,z)$$

$$h(r,\theta,\phi) = h\left(r(x,y,z),\theta(x,y,z),\phi(x,y,z)\right) = f_x(x,y,z)$$

$$(f_x)_x = h_r \sin\theta\cos\phi + \frac{h_\theta}{r}\cos\theta\cos\phi - \frac{h_\phi \sin\phi}{r\sin\theta}$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$

$$+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$

$$- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$

$$+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$

$$- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$f_x = g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta} \quad (4)$$

$$f_y = g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta} \quad (5)$$

$$f_z = g_r \cos\theta - \frac{g_\theta}{r}\sin\theta \quad (6)$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi$$

$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r}$$

$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_r \cos\theta$$

$$- \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta} \right)_r \sin\theta\cos\phi$$
$$+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta} \right)_\theta \frac{\cos\theta\cos\phi}{r}$$
$$- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta} \right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$f_x = g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta} \quad (4)$$

$$f_y = g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta} \quad (5)$$

$$f_z = g_r \cos\theta - \frac{g_\theta}{r}\sin\theta \quad (6)$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta} \right)_r \sin\theta\sin\phi$$
$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta} \right)_\theta \frac{\cos\theta\sin\phi}{r}$$
$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta} \right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r} \right)_r \cos\theta$$
$$- \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r} \right)_\theta \frac{\sin\theta}{r}$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$
$$+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$
$$- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi$$
$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r}$$
$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_r \cos\theta$$
$$- \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}$$

$$f_x = g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta} \quad (4)$$

$$f_y = g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta} \quad (5)$$

$$f_z = g_r \cos\theta - \frac{g_\theta}{r}\sin\theta \quad (6)$$

our mission here is

to find $f_{xx} + f_{yy} + f_{zz}$

how many terms?

g_{rr}	$g_{r\theta} = g_{\theta r}$	g_r
$g_{\theta\theta}$	$g_{r\phi} = g_{\phi r}$	g_θ
$g_{\phi\phi}$	$g_{\theta\phi} = g_{\phi\theta}$	g_ϕ

3 + 3 + 3 = 9 terms

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$
$$+\left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$
$$-\left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi$$
$$+\left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r}$$
$$+\left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_r \cos\theta$$
$$-\left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}$$

$$g_{rr} \quad \left[\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta\right] = 1$$

$$g_{\theta\theta} \quad \left[\frac{\cos^2\theta\cos^2\phi}{r^2} + \frac{\cos^2\theta\sin^2\phi}{r^2} + \frac{\sin^2\theta}{r^2}\right] = \frac{1}{r^2}$$

$$g_{\phi\phi} \quad \left[\frac{\sin^2\phi}{r^2\sin^2\theta} + \frac{\cos^2\phi}{r^2\sin^2\theta}\right] = \frac{1}{r^2\sin^2\theta}$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$
$$+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$
$$- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi$$
$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r}$$
$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_r \cos\theta$$
$$- \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}$$

$$g_{rr} \quad \left[\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta\right] = 1$$

$$g_{\theta\theta} \quad \left[\frac{\cos^2\theta\cos^2\phi}{r^2} + \frac{\cos^2\theta\sin^2\phi}{r^2} + \frac{\sin^2\theta}{r^2}\right] = \frac{1}{r^2}$$

$$g_{\phi\phi} \quad \left[\frac{\sin^2\phi}{r^2\sin^2\theta} + \frac{\cos^2\phi}{r^2\sin^2\theta}\right] = \frac{1}{r^2\sin^2\theta}$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$
$$+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$
$$- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi$$
$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r}$$
$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_r \cos\theta$$
$$- \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}$$

$$g_{rr} \quad \left[\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta\right] = 1$$

$$g_{\theta\theta} \quad \left[\frac{\cos^2\theta\cos^2\phi}{r^2} + \frac{\cos^2\theta\sin^2\phi}{r^2} + \frac{\sin^2\theta}{r^2}\right] = \frac{1}{r^2}$$

$$g_{\phi\phi} \quad \left[\frac{\sin^2\phi}{r^2\sin^2\theta} + \frac{\cos^2\phi}{r^2\sin^2\theta}\right] = \frac{1}{r^2\sin^2\theta}$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$

$$+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$

$$- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi$$

$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r}$$

$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_r \cos\theta$$

$$- \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}$$

$$g_{r\theta} \left[\frac{\sin\theta\cos\theta\cos^2\phi}{r} + \frac{\sin\theta\cos\theta\cos^2\phi}{r} + \frac{\sin\theta\cos\theta\sin^2\phi}{r} + \frac{\sin\theta\cos\theta\sin^2\phi}{r} - \frac{\sin\theta\cos\theta}{r} - \frac{\sin\theta\cos\theta}{r} \right] = 0$$

$$g_{\theta\phi} \left[-\frac{\cos\theta\sin\phi\cos\phi}{r^2\sin\theta} - \frac{\cos\theta\sin\phi\cos\phi}{r^2\sin\theta} + \frac{\cos\theta\sin\phi\cos\phi}{r^2\sin\theta} + \frac{\cos\theta\sin\phi\cos\phi}{r^2\sin\theta} \right] = 0$$

$$g_{r\phi} \left[-\frac{\sin\phi\cos\phi}{r} - \frac{\sin\phi\cos\phi}{r} + \frac{\sin\phi\cos\phi}{r} + \frac{\sin\phi\cos\phi}{r} \right] = 0$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$

$$+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$

$$- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi$$

$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r}$$

$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_r \cos\theta$$

$$- \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}$$

$$g_r \left[\frac{\cos^2\theta\cos^2\phi}{r} + \frac{\sin^2\phi}{r} + \frac{\sin^2\theta}{r} + \frac{\cos^2\theta\sin^2\phi}{r} + \frac{\cos^2\phi}{r} \right] = \frac{2}{r}$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$

$$+ \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$

$$- \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi$$

$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r}$$

$$+ \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_r \cos\theta$$

$$- \left(g_r \cos\theta - \frac{g_\theta \sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}$$

$$g_r \left[\frac{\cos^2\theta\cos^2\phi}{r} + \frac{\sin^2\phi}{r} + \frac{\sin^2\theta}{r} + \frac{\cos^2\theta\sin^2\phi}{r} + \frac{\cos^2\phi}{r} \right] = \frac{2}{r}$$

$$g_\theta \left[-\frac{\sin\theta\cos\theta\cos^2\phi}{r^2} - \frac{\sin\theta\cos\theta\cos^2\phi}{r^2} + \frac{\cos\theta\sin^2\phi}{r^2\sin\theta} - \frac{\sin\theta\cos\theta\sin^2\phi}{r^2} - \frac{\sin\theta\cos\theta\sin^2\phi}{r^2} + \frac{\cos\theta\cos^2\phi}{r^2\sin\theta} + \frac{\sin\theta\cos\theta}{r^2} + \frac{\sin\theta\cos\theta}{r^2} \right] = \frac{\cos\theta}{r^2\sin\theta}$$

$$(f_x)_x = \left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_r \sin\theta\cos\phi$$

$$+\left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\cos\phi}{r}$$

$$-\left(g_r \sin\theta\cos\phi + \frac{g_\theta}{r}\cos\theta\cos\phi - \frac{g_\phi \sin\phi}{r\sin\theta}\right)_\phi \frac{\sin\phi}{r\sin\theta}$$

$$(f_y)_y = \left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_r \sin\theta\sin\phi$$

$$+\left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\theta \frac{\cos\theta\sin\phi}{r}$$

$$+\left(g_r \sin\theta\sin\phi + \frac{g_\theta}{r}\cos\theta\sin\phi + \frac{g_\phi \cos\phi}{r\sin\theta}\right)_\phi \frac{\cos\phi}{r\sin\theta}$$

$$(f_z)_z = \left(g_r\cos\theta - \frac{g_\theta \sin\theta}{r}\right)_r \cos\theta$$

$$-\left(g_r\cos\theta - \frac{g_\theta \sin\theta}{r}\right)_\theta \frac{\sin\theta}{r}$$

$$g_r \left[\frac{\cos^2\theta\cos^2\phi}{r} + \frac{\sin^2\phi}{r} + \frac{\sin^2\theta}{r} + \frac{\cos^2\theta\sin^2\phi}{r} + \frac{\cos^2\phi}{r}\right] = \frac{2}{r}$$

$$g_\theta \left[-\frac{\sin\theta\cos\theta\cos^2\phi}{r^2} - \frac{\sin\theta\cos\theta\cos^2\phi}{r^2} + \frac{\cos\theta\sin^2\phi}{r^2\sin\theta} - \frac{\sin\theta\cos\theta\sin^2\phi}{r^2} - \frac{\sin\theta\cos\theta\sin^2\phi}{r^2} + \frac{\cos\theta\cos^2\phi}{r^2\sin\theta} + \frac{\sin\theta\cos\theta}{r^2} + \frac{\sin\theta\cos\theta}{r^2}\right] = \frac{\cos\theta}{r^2\sin\theta}$$

$$g_\phi \left[\frac{\sin\phi\cos\phi}{r^2} + \frac{\cos^2\theta\sin\phi\cos\phi}{r^2\sin^2\theta} + \frac{\sin\phi\cos\phi}{r^2\sin\theta} - \frac{\sin\phi\cos\phi}{r^2} - \frac{\cos^2\theta\sin\phi\cos\phi}{r^2\sin^2\theta} - \frac{\sin\phi\cos\phi}{r^2\sin\theta}\right] = 0$$

g_{rr} $\left[\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta\right] = 1$

$g_{\theta\theta}$ $\left[\frac{\cos^2\theta\cos^2\phi}{r^2} + \frac{\cos^2\theta\sin^2\phi}{r^2} + \frac{\sin^2\theta}{r^2}\right] = \frac{1}{r^2}$

$g_{\phi\phi}$ $\left[\frac{\sin^2\phi}{r^2\sin^2\theta} + \frac{\cos^2\phi}{r^2\sin^2\theta}\right] = \frac{1}{r^2\sin^2\theta}$

g_r $\frac{2}{r}$

g_θ $\frac{\cos\theta}{r^2\sin\theta}$

g_ϕ 0

$$f_{xx} + f_{yy} + f_{zz}$$

$$= g_{rr} + \frac{2}{r}g_r + \frac{1}{r^2}g_{\theta\theta} + \frac{\cos\theta}{r^2\sin\theta}g_\theta + \frac{1}{r^2\sin^2\theta}g_{\phi\phi}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\Lambda^2$$

1 **find this**

$$\Lambda^2 = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}$$

Road to spherical harmonics

1. Hamiltonian in central field $H = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{Ze^2}{r}$
2. separate r and θ, ϕ $\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\Lambda^2$
3. Laplacian in polar coordinate $\Lambda^2 = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}$

---

4. Legendre differential equation is part of Laplacian
5. Rodrigues formula is solution   Leibniz rule
6. full Laplacian is associated Legendre differential equation
7. Derivative of Rodrigues formula is solution

→ **spherical harmonics**

Exercise today

1. calculate Legendrian

$$g_{rr} \quad \left[\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta\right] = 1$$

$$g_{\theta\theta} \quad \left[\frac{\cos^2\theta\cos^2\phi}{r^2} + \frac{\cos^2\theta\sin^2\phi}{r^2} + \frac{\sin^2\theta}{r^2}\right] = \frac{1}{r^2}$$

$$g_{\phi\phi} \quad \left[\frac{\sin^2\phi}{r^2\sin^2\theta} + \frac{\cos^2\phi}{r^2\sin^2\theta}\right] = \frac{1}{r^2\sin^2\theta}$$

$$g_r \quad \frac{2}{r}$$

$$g_\theta \quad \frac{\cos\theta}{r^2\sin\theta}$$

$$g_\phi \quad 0$$

$$f_{xx} + f_{yy} + f_{zz}$$

$$= g_{rr} + \frac{2}{r}g_r + \frac{1}{r^2}g_{\theta\theta} + \frac{\cos\theta}{r^2\sin\theta}g_\theta + \frac{1}{r^2\sin^2\theta}g_{\phi\phi}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\Lambda^2$$

$$\Lambda^2 = \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}$$

Seminar topics

20 minutes

	subject	keywords
1	NIRSpec on JWST	MSA
2	iSHELL on IRTF	cross-dispersing spectrograph
3	ELT vs JWST	sensitiivty, spatial resolution
4	Adaptive optics	choose from Shack-Hartmann / curvature / pyramid
5	IR detectors	band gap, intrinsic semiconductor
6	Frequency comb	mechanism, objectives
7	Laplacian in polar coordinate	vector analysis
8	2D/3D representation of spherical harmonics	minimum 3 different ways
9	development of quantum mechanics in 1920s	Bohr model, spin