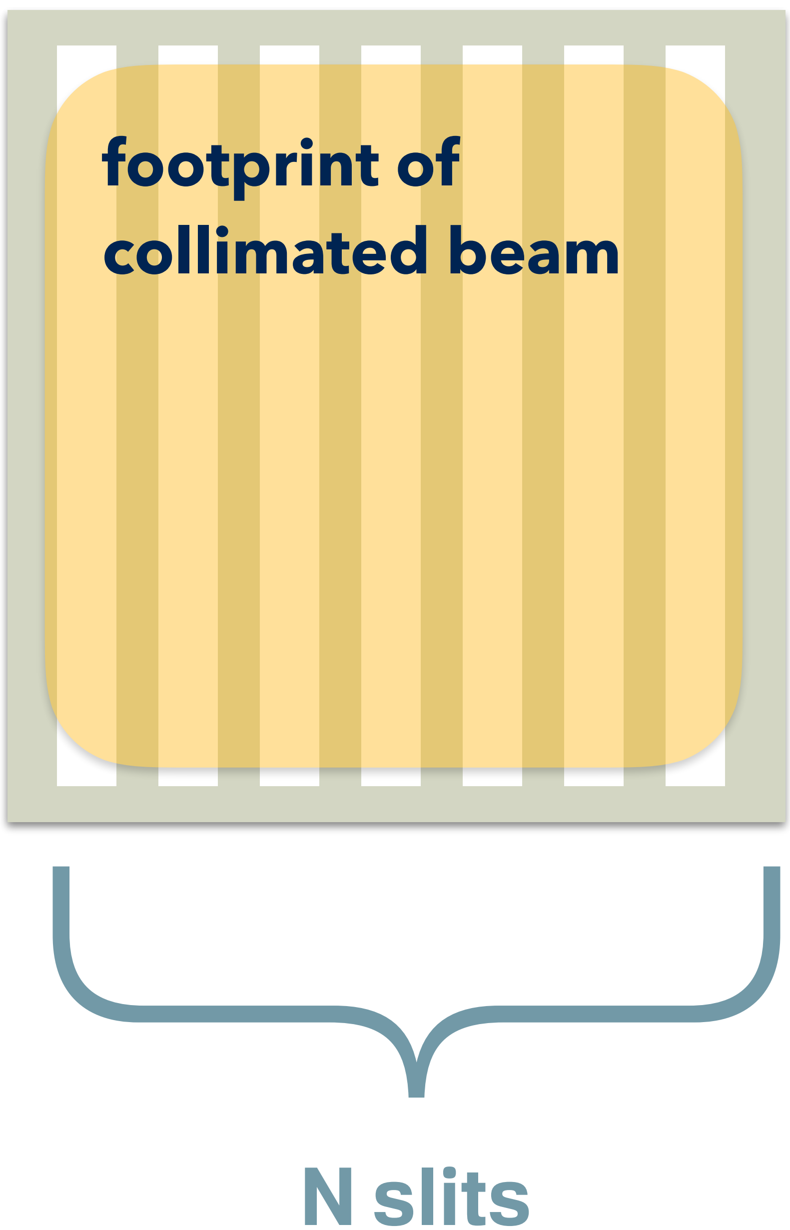
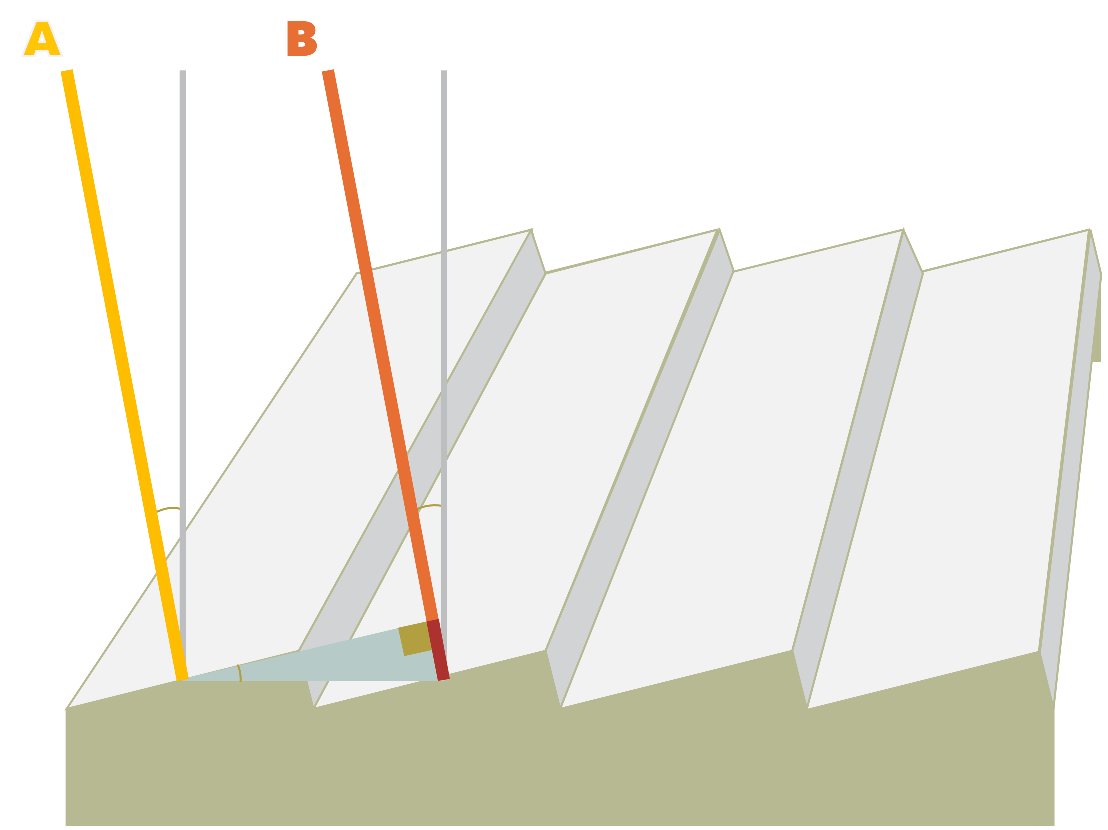
Diffraction grating



difference of pathlength

make certain color of light more intense
toward certain direction

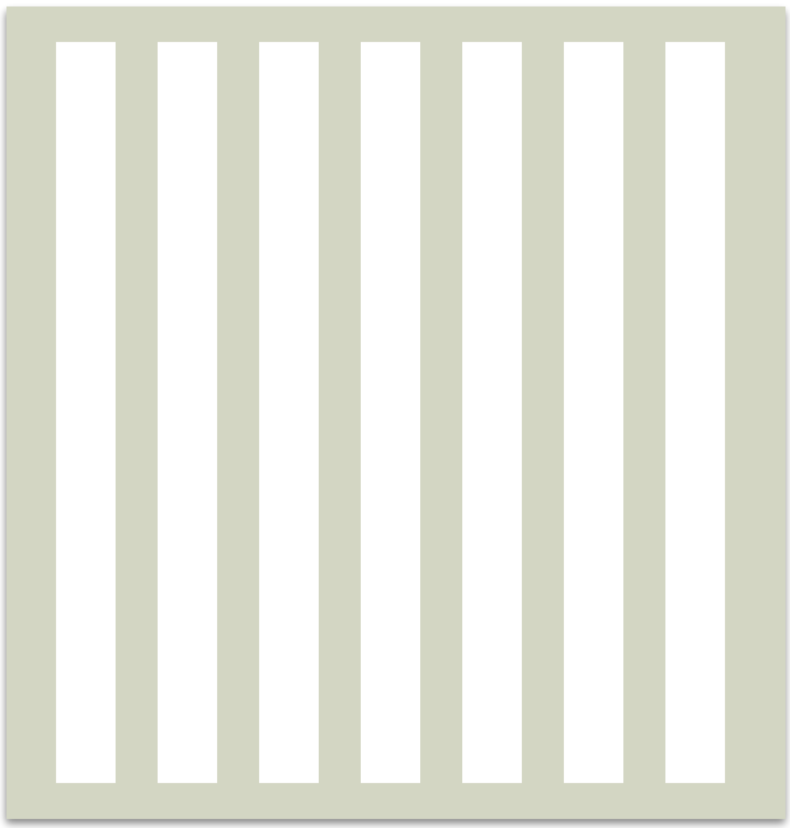
why not use just 2 slits?



because

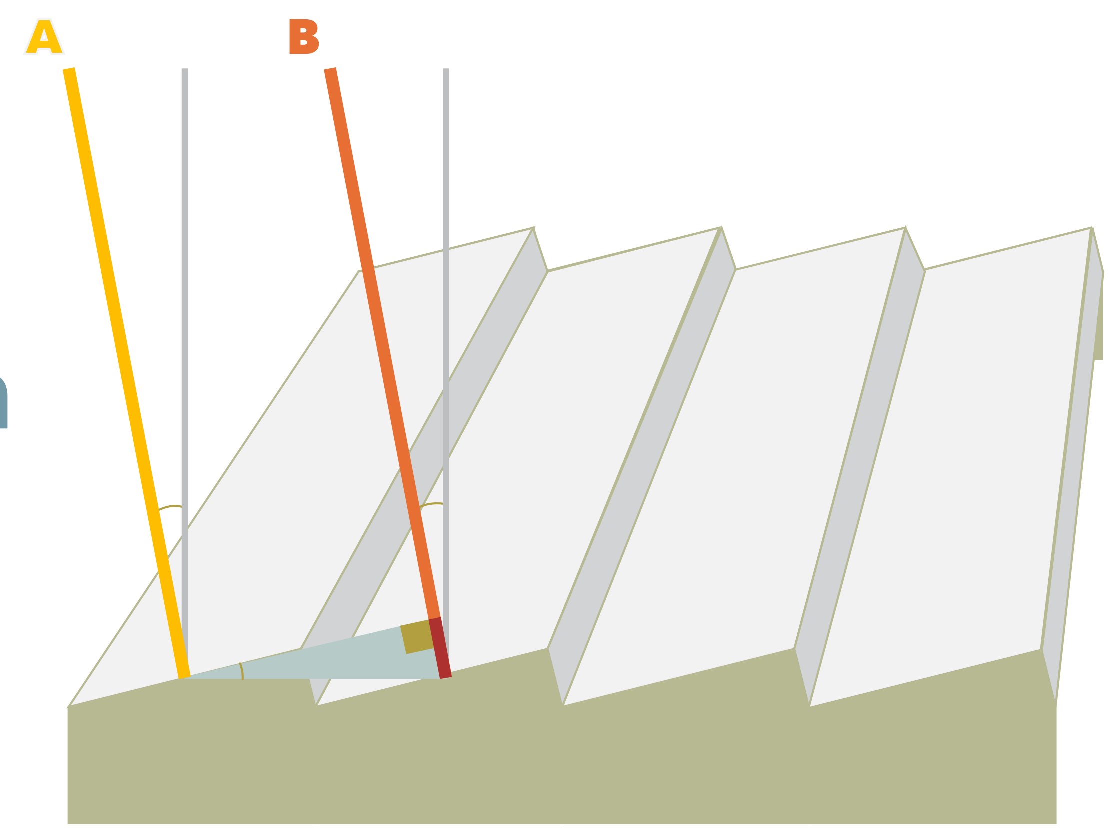
line Intensity goes up as $\propto N^2$

Diffraction grating



transmissive grating with slits

regular openings



1 diffraction by a single slit

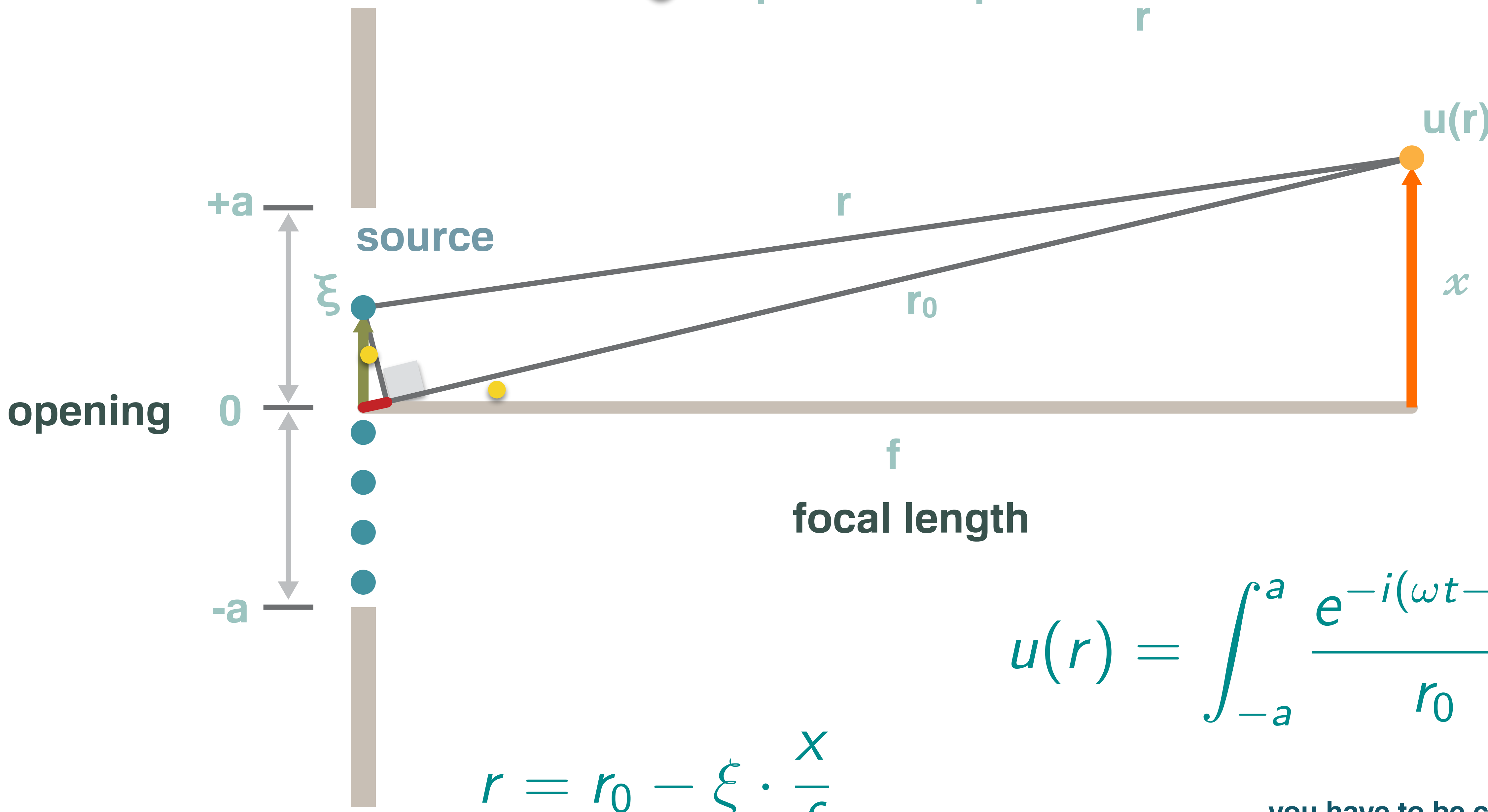
Fraunhofer diffraction

2 diffraction by a multiple slit

3 blazed grating

1 diffraction by a single slit

- $u_p(\xi) = e^{-i\omega t}$
- amplitude drops as $\frac{1}{r}$

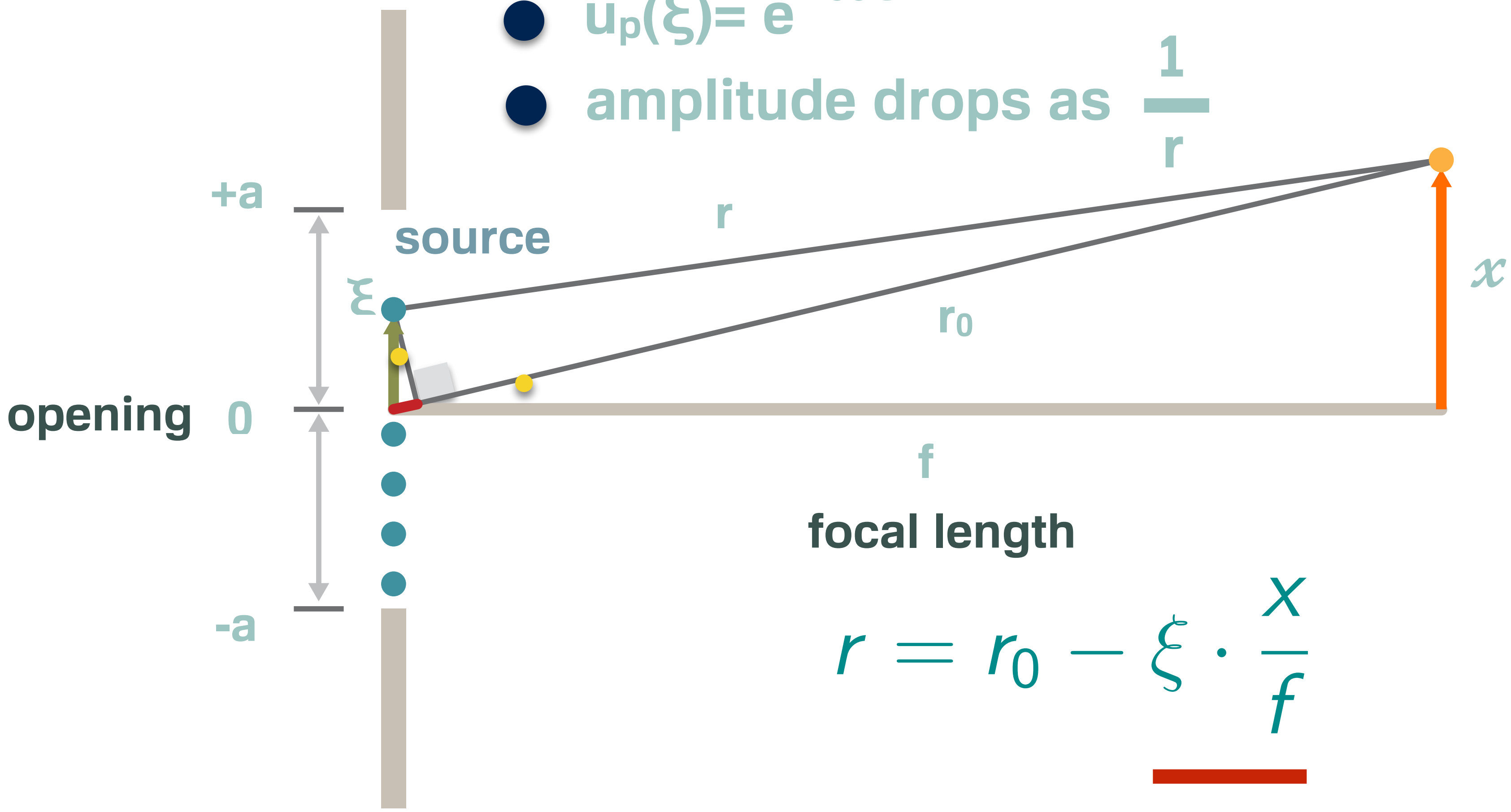


$$u(r) = \int_{-a}^{a} \frac{e^{-i(\omega t - kr)}}{r_0} d\xi$$

$$r = r_0 - \xi \cdot \frac{x}{f}$$

you have to be careful on phase part only

- $u_p(\xi) = e^{-i\omega t}$
- amplitude drops as $\frac{1}{r}$



$$r = r_0 - \xi \cdot \frac{x}{f}$$

$$u(r) = \int_{-a}^{+a} u_p(\xi) d\xi$$

$$= \frac{1}{r} \int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$

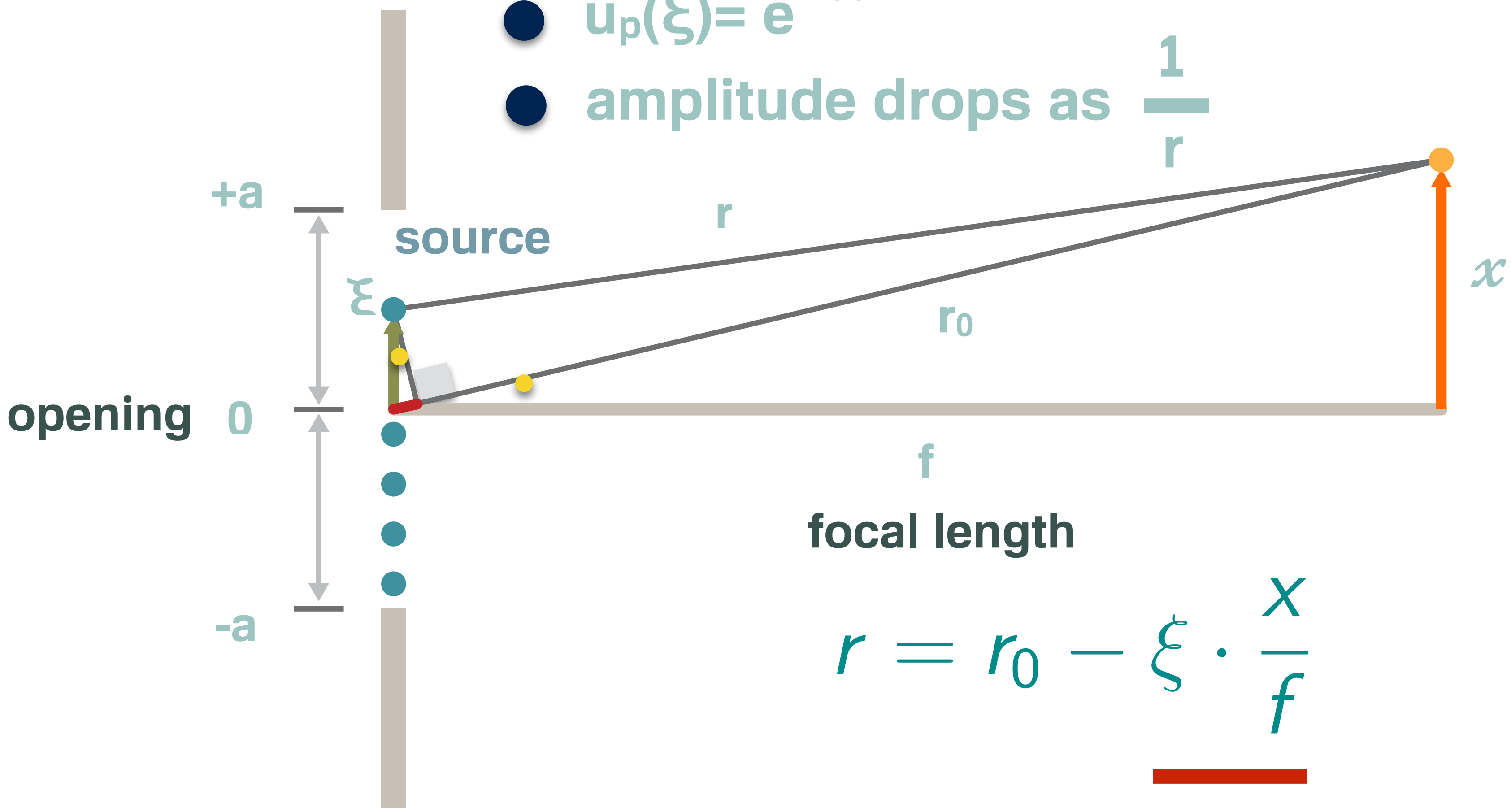
$$= \frac{e^{-i(\omega t - k r_0)}}{r} \int_{-a}^{+a} e^{-i\frac{k\xi x}{f}} d\xi$$

$$= \frac{e^{-i(\omega t - k r_0)}}{r} \frac{f}{-ikx} \left[e^{\frac{-ik\xi x}{f}}\right]_{-a}^{+a}$$

$$= \frac{e^{-i(\omega t - k r_0)}}{r} \frac{if}{kx} \left[e^{\frac{-ikax}{f}} - e^{\frac{ikax}{f}}\right]$$

$$\frac{e^{i\theta} - e^{-i\theta}}{2} = \sin\theta$$

- $u_p(\xi) = e^{-i\omega t}$
- amplitude drops as $\frac{1}{r}$



$$r = r_0 - \xi \cdot \frac{x}{f}$$

$$u(r) = \int_{-a}^{+a} u_p(\xi) d\xi$$

$$= \frac{1}{r} \int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$

$$= \frac{e^{-i(\omega t - k r_0)}}{r} \int_{-a}^{+a} e^{-i\frac{k\xi x}{f}} d\xi$$

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$$= \frac{e^{-i(\omega t - k r_0)}}{r} \frac{if}{kx} \left[e^{\frac{-ikax}{f}} - e^{\frac{ikax}{f}} \right]$$

$$\frac{e^{i\theta} - e^{-i\theta}}{2} = \sin\theta$$

$$u(r) = \frac{e^{-i(\omega t - k r_0)}}{r} \frac{-i2f}{kx} \sin\left(\frac{kax}{f}\right)$$

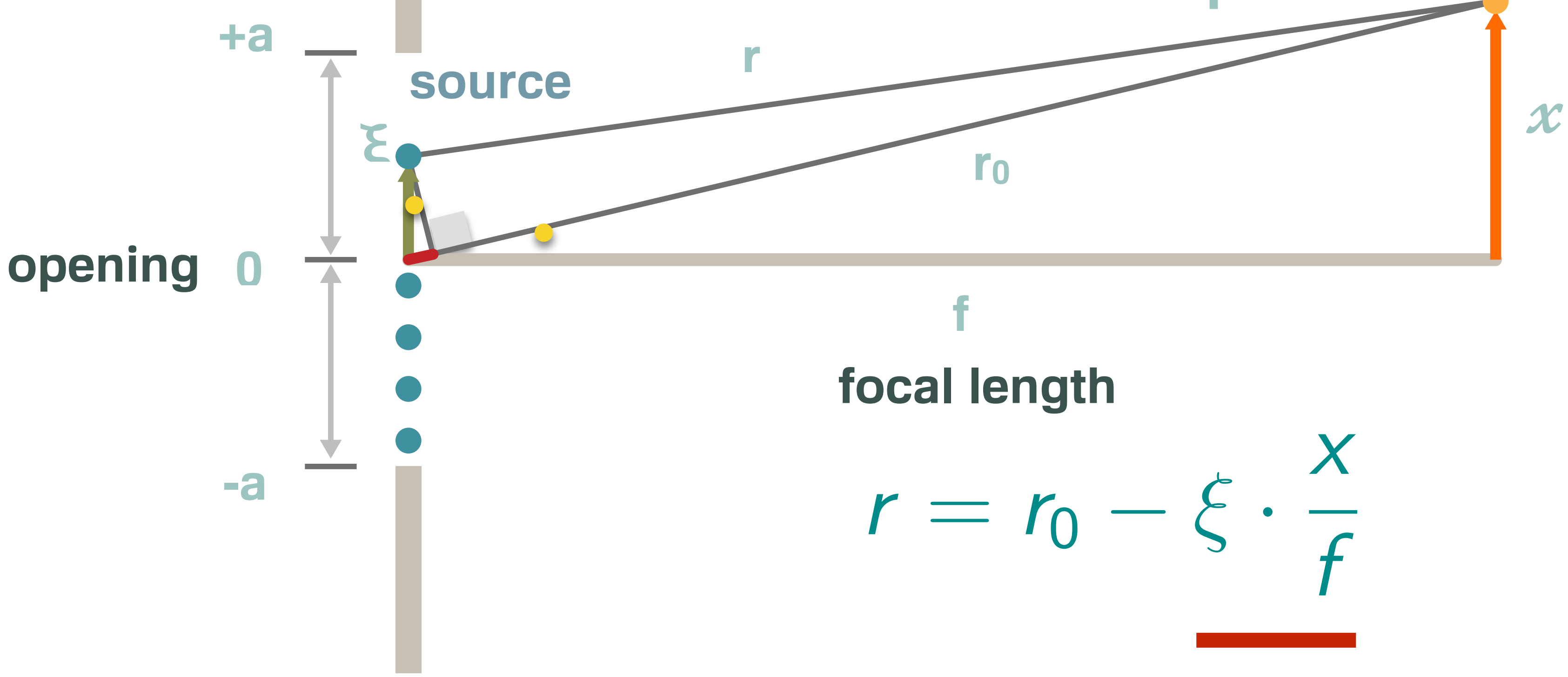
$$= \frac{e^{-i(\omega t - k r_0)}}{r} \frac{-i2f}{kx} \frac{kax}{f} \frac{\sin\left(\frac{kax}{f}\right)}{\frac{kax}{f}}$$

we want to make it

$$\frac{\sin x}{x}$$

$$= \frac{e^{-i(\omega t - k r_0)}}{r} (-i2a) \frac{\sin\left(\frac{kax}{f}\right)}{\frac{kax}{f}}$$

- $u_p(\xi) = e^{-i\omega t}$
- amplitude drops as $\frac{1}{r}$



$$r = r_0 - \xi \cdot \frac{x}{f}$$

$$u(r) = \int_{-a}^{+a} u_p(\xi)\, d\xi$$

$$= \frac{1}{r} \int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$

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$$= \frac{e^{-i(\omega t - k r_0)}}{r} \frac{if}{kx} \left[e^{\frac{-ikax}{f}} - e^{\frac{ikax}{f}} \right]$$

$$I = |u(r)|^2 = \frac{4a^2}{r^2} \left(\frac{\sin \frac{kax}{f}}{\frac{kax}{f}} \right)^2$$

we want to make it

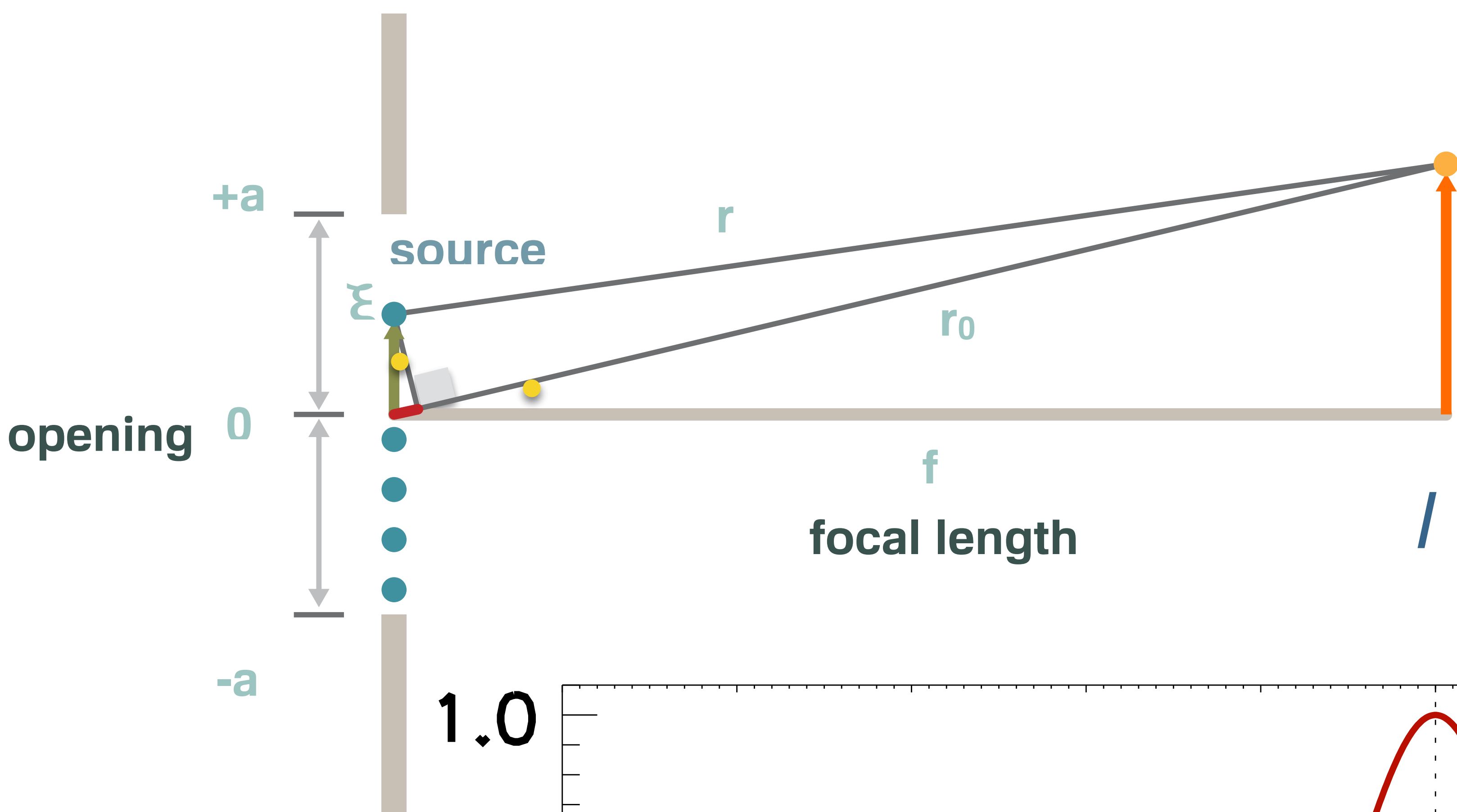
$$\frac{e^{i\theta} - e^{-i\theta}}{2} = \sin\theta$$

$$\frac{\sin x}{x}$$

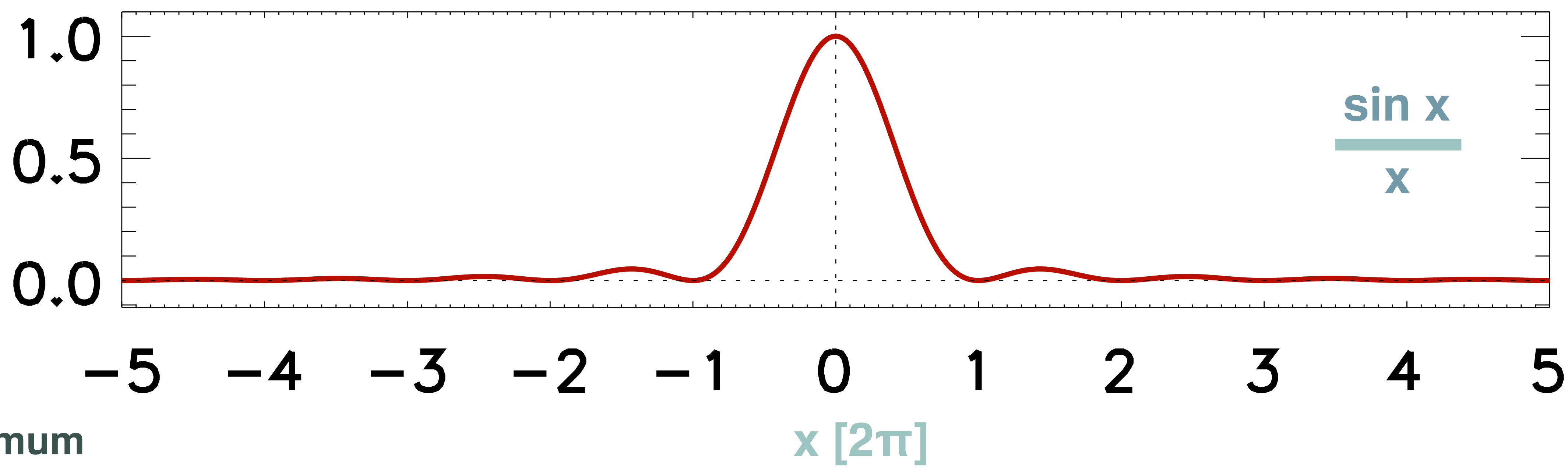
$$= \frac{4a^2}{r^2} \qquad (x = 0)$$

on axis intensity

$$u(r) = \frac{e^{-i(\omega t - k r_0)}}{r} (-i2a) \frac{\sin\left(\frac{kax}{f}\right)}{\frac{kax}{f}}$$



$$I = |u(r)|^2 = \frac{4a^2}{r^2}\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$



I=0 when

$$\frac{kax}{f} = \pi$$

first minimum

$$\frac{2\pi ax}{f\lambda} = \pi \qquad k = \frac{2\pi}{\lambda}$$

$$\frac{x}{f} = \frac{\lambda}{2a}$$

$$\frac{x}{f} = \frac{\lambda}{R}$$

angular resolution | **wavelength / telescope aperture**

Fraunhofer diffraction

- far field
- opening ≫ λ

Fresnel diffraction

Sommerfeld solution

2 diffraction by a multiple slit

$$u(r) = \frac{1}{r}\int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$

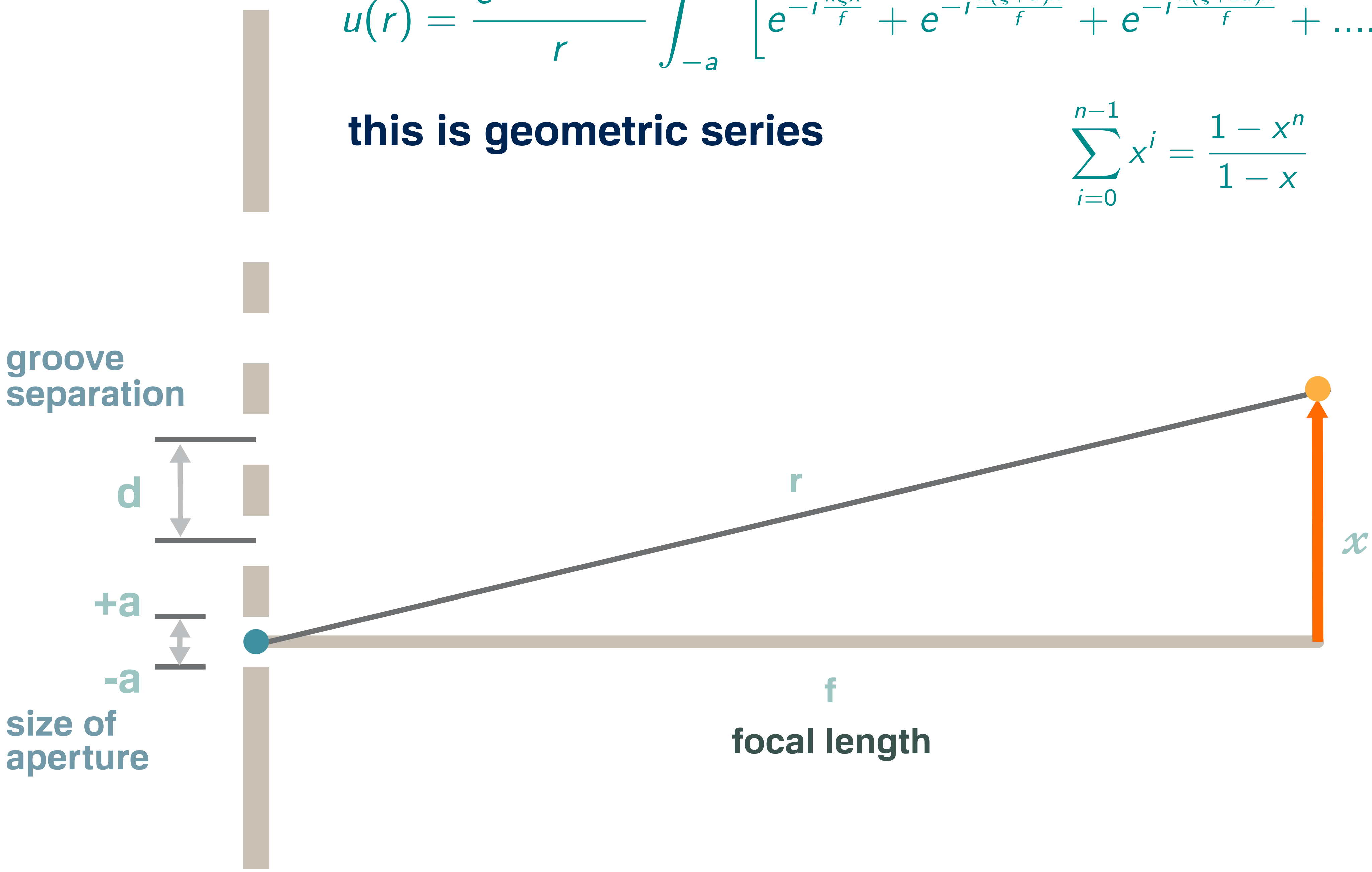
single slit

$$u(r) = \frac{e^{-i(\omega t - k r_0)}}{r}\int_{-a}^{+a}\left[e^{-i\frac{k\xi x}{f}} + e^{-i\frac{k(\xi+d)x}{f}} + e^{-i\frac{k(\xi+2d)x}{f}} + \dots. + e^{-i\frac{k(\xi+(N-1)d)x}{f}}\right] d\xi$$

N slits

this is geometric series

$$\sum_{i=0}^{n-1} x^i = \frac{1-x^n}{1-x} \qquad x = e^{-\frac{ikxd}{f}}$$



2 diffraction by a multiple slit

$$u(r) = \frac{1}{r}\int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$ single slit

$$u(r) = \frac{e^{-i(\omega t - k r_0)}}{r}\int_{-a}^{+a}\left[e^{-i\frac{k\xi x}{f}} + e^{-i\frac{k(\xi+d)x}{f}} + e^{-i\frac{k(\xi+2d)x}{f}} + \dots + e^{-i\frac{k(\xi+(N-1)d)x}{f}}\right] d\xi$$

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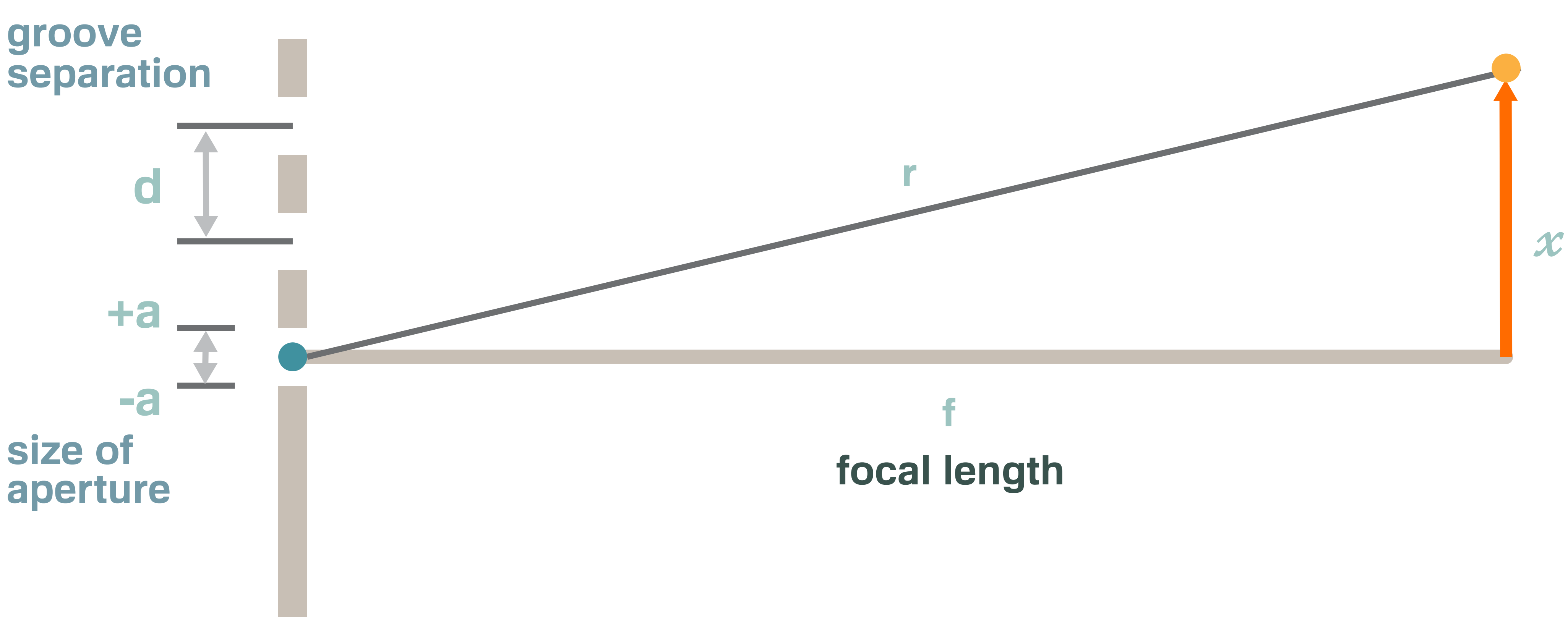
this part is same with single slit



2 diffraction by a multiple slit

$$u(r) = \frac{1}{r}\int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$ **single slit**

$$u(r) = \frac{e^{-i(\omega t - k r_0)}}{r}\int_{-a}^{+a}\left[e^{-i\frac{k\xi x}{f}} + e^{-i\frac{k(\xi+d)x}{f}} + e^{-i\frac{k(\xi+2d)x}{f}} + ... + e^{-i\frac{k(\xi+(N-1)d)x}{f}}\right] d\xi$$

N slits

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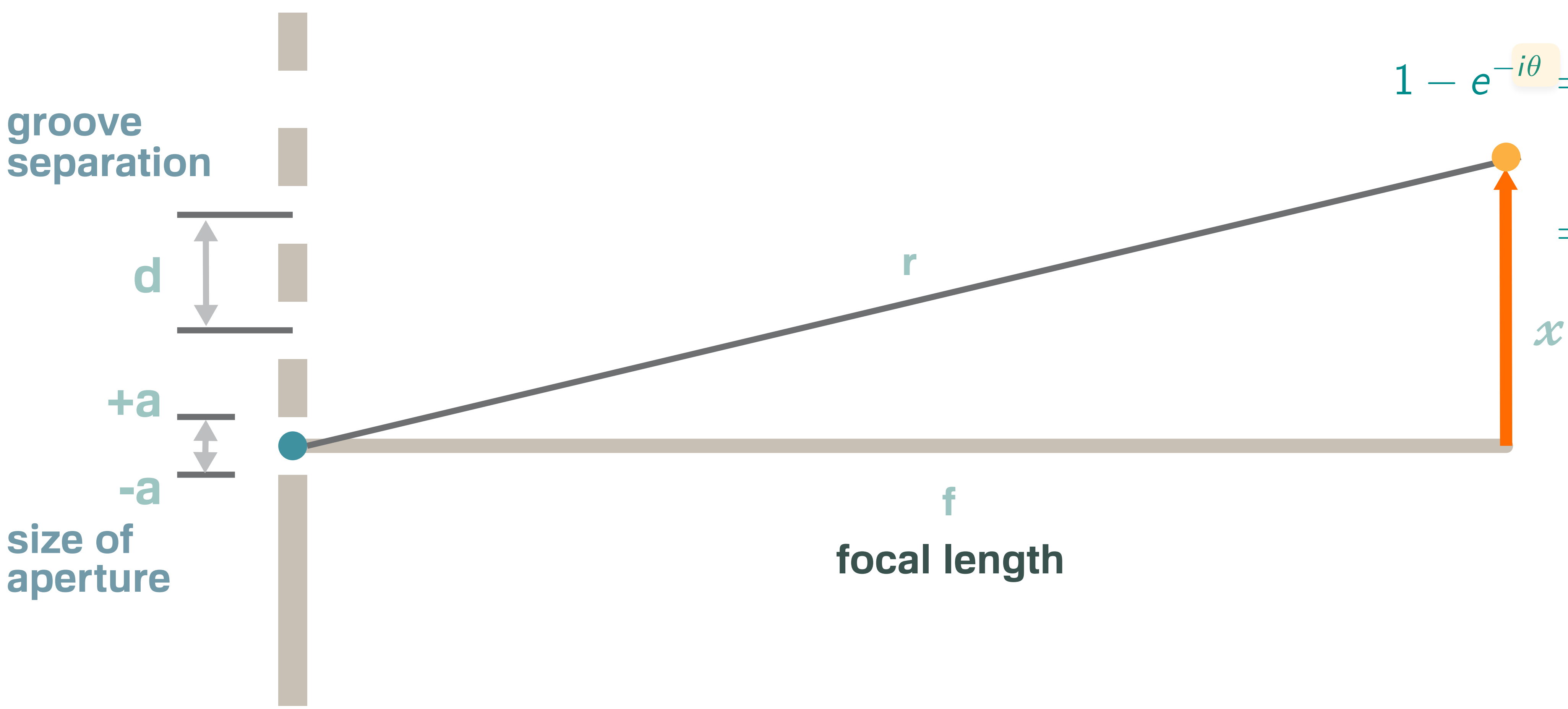
$$1 - e^{-i\theta} = e^{-\frac{i\theta}{2}}\left(e^{\frac{i\theta}{2}} - e^{-\frac{i\theta}{2}}\right)$$

$$= e^{-\frac{i\theta}{2}} \cdot 2\sin\left(\frac{\theta}{2}\right)$$

$$\theta = \frac{kxd}{f}$$



2 diffraction by a multiple slit

$$u(r) = \frac{1}{r}\int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$

one slit

$$u(r) = \frac{e^{-i(\omega t - kr_0)}}{r}\int_{-a}^{+a}\left[e^{-i\frac{k\xi x}{f}} + e^{-i\frac{k(\xi+d)x}{f}} + e^{-i\frac{k(\xi+2d)x}{f}} + \ldots + e^{-i\frac{k(\xi+(N-1)d)x}{f}}\right] d\xi$$

N slits

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$$\sum_{i=0}^{n-1} x^i = \frac{1-x^n}{1-x} \qquad x = e^{-\frac{ikxd}{f}}$$

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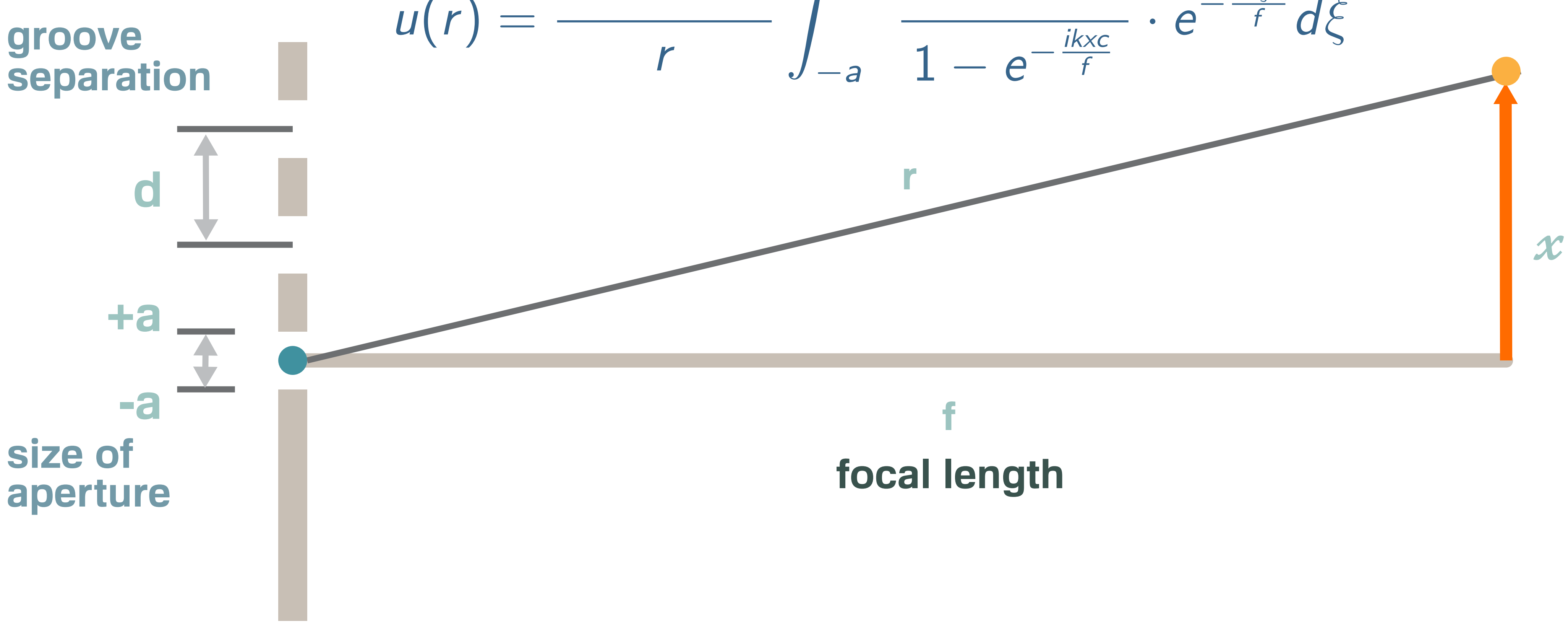
$$u(r) = \frac{e^{-i(\omega t - kr_0)}}{r}\int_{-a}^{+a}\frac{1 - e^{-\frac{ikNxd}{f}}}{1 - e^{-\frac{ikxc}{f}}} \cdot e^{-\frac{ik\xi x}{f}} d\xi$$



2 diffraction by a multiple slit

$$u(r) = \frac{1}{r}\int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$ **one slit**

$$u(r) = \frac{e^{-i(\omega t - kr_0)}}{r}\int_{-a}^{+a}\left[e^{-i\frac{k\xi x}{f}} + e^{-i\frac{k(\xi+d)x}{f}} + e^{-i\frac{k(\xi+2d)x}{f}} + ... + e^{-i\frac{k(\xi+(N-1)d)x}{f}}\right] d\xi$$

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$$1-e^{-i\theta} = e^{-\frac{i\theta}{2}}\left(e^{\frac{i\theta}{2}} - e^{-\frac{i\theta}{2}}\right)$$

$$= e^{-\frac{i\theta}{2}}\cdot 2\sin\left(\frac{\theta}{2}\right)$$

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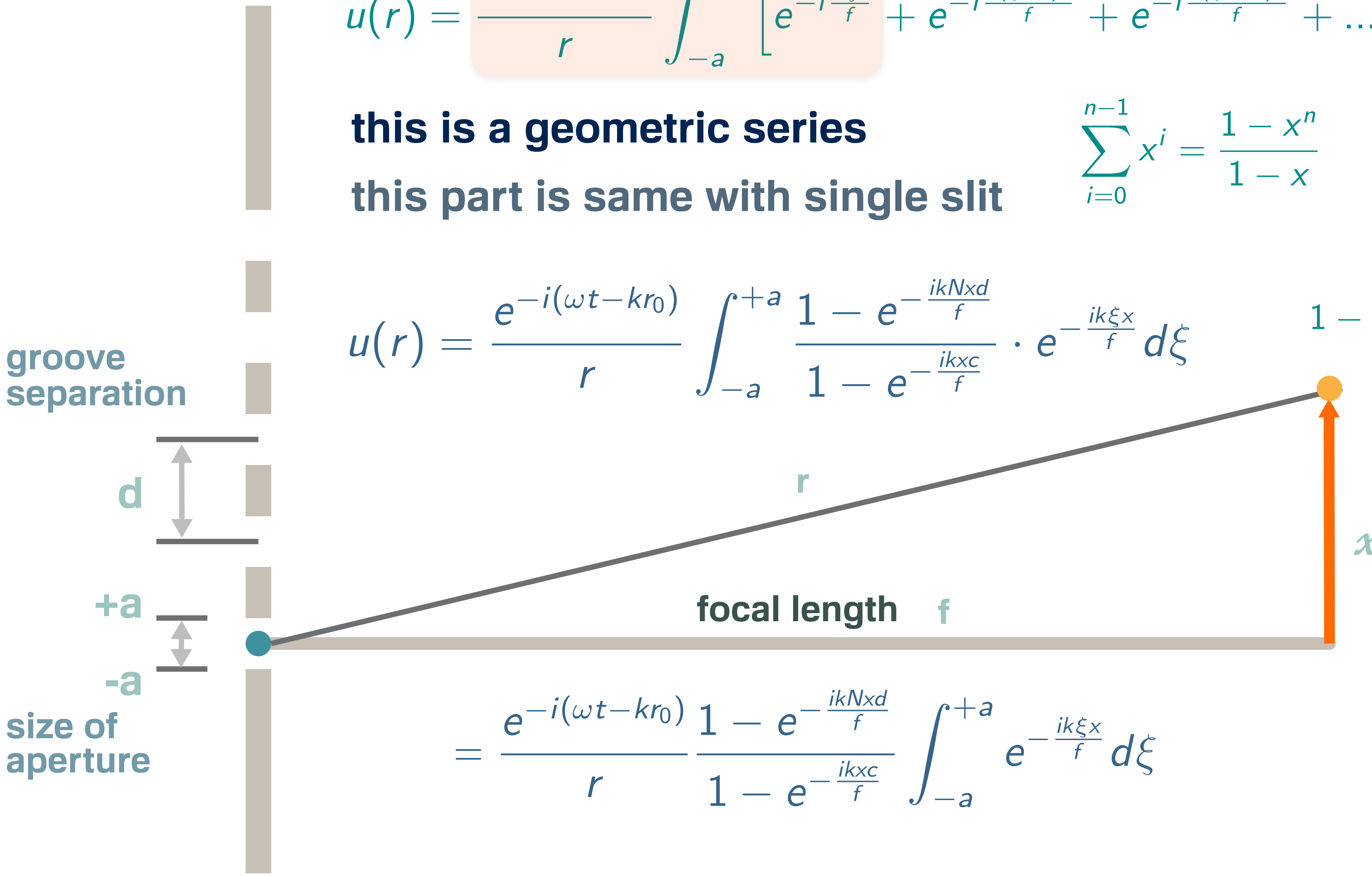

$$= \frac{e^{-i(\omega t - kr_0)}}{r}\,\frac{1-e^{-\frac{ikNxd}{f}}}{1-e^{-\frac{ikxc}{f}}}\int_{-a}^{+a} e^{-\frac{ik\xi x}{f}} d\xi$$

2 diffraction by a multiple slit

$$u(r) = \frac{1}{r}\int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$ **one slit**

$$u(r) = \frac{e^{-i(\omega t - kr_0)}}{r}\int_{-a}^{+a}\left[e^{-i\frac{k\xi x}{f}} + e^{-i\frac{k(\xi+d)x}{f}} + e^{-i\frac{k(\xi+2d)x}{f}} + ... + e^{-i\frac{k(\xi+(N-1)d)x}{f}}\right] d\xi$$

N slits

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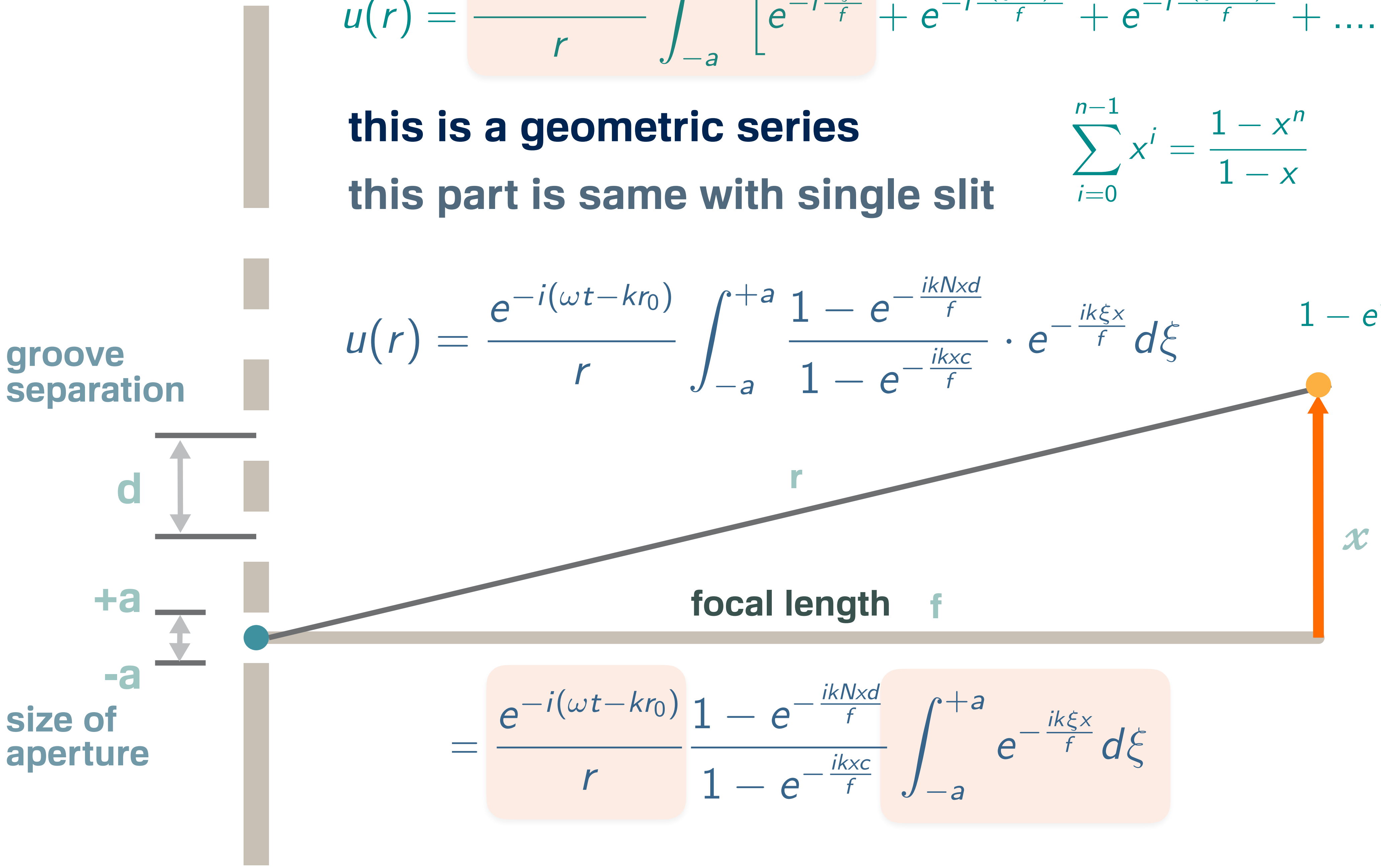

$$= \frac{e^{-i(\omega t - kr_0)}}{r}\frac{1-e^{-\frac{ikNxd}{f}}}{1-e^{-\frac{ikxc}{f}}}\int_{-a}^{+a} e^{-\frac{ik\xi x}{f}} d\xi$$

$$u(r) = \frac{e^{-i(\omega t - kr_0)}}{r}(-i2a)\frac{\sin\left(\frac{kax}{f}\right)}{\frac{kax}{f}}$$

2 diffraction by a multiple slit

$$u(r) = \frac{1}{r}\int_{-a}^{+a} e^{-i\left[\omega t - k\left(r_0 - \frac{\xi x}{f}\right)\right]} d\xi$$

one slit

$$u(r) = \frac{e^{-i(\omega t - kr_0)}}{r}\int_{-a}^{+a}\left[e^{-i\frac{k\xi x}{f}} + e^{-i\frac{k(\xi+d)x}{f}} + e^{-i\frac{k(\xi+2d)x}{f}} + \dots + e^{-i\frac{k(\xi+(N-1)d)x}{f}}\right] d\xi$$

N slits

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$$\sum_{i=0}^{n-1} x^i = \frac{1-x^n}{1-x} \qquad x = e^{-\frac{ikxd}{f}}$$

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$$u(r) = \frac{e^{-i(\omega t - kr_0)}}{r}\int_{-a}^{+a}\frac{1-e^{-\frac{ikNxd}{f}}}{1-e^{-\frac{ikxc}{f}}}\cdot e^{-\frac{ik\xi x}{f}} d\xi$$

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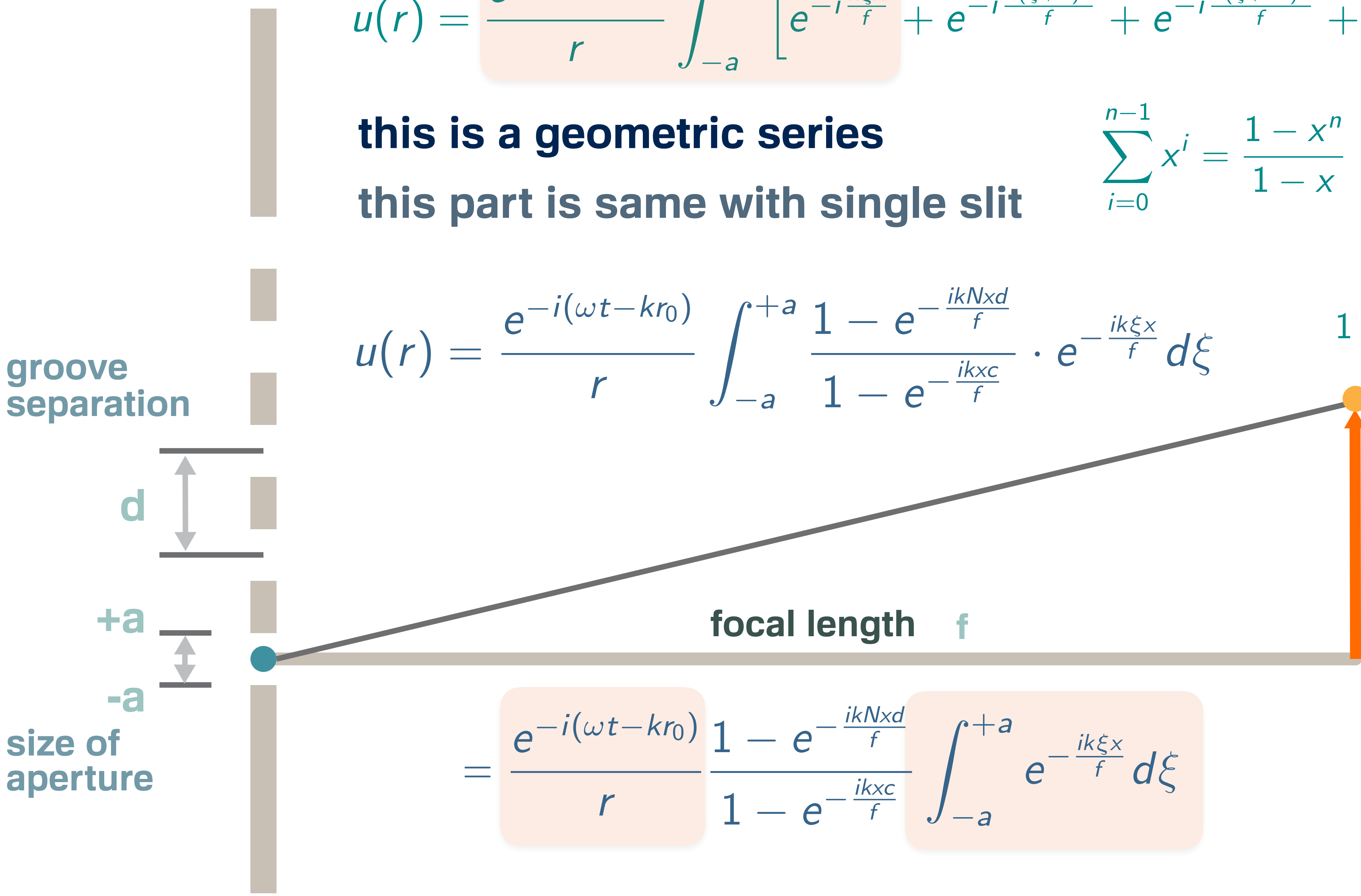


$$= \frac{e^{-i(\omega t - kr_0)}}{r}\frac{1-e^{-\frac{ikNxd}{f}}}{1-e^{-\frac{ikxc}{f}}}\int_{-a}^{+a} e^{-\frac{ik\xi x}{f}} d\xi$$

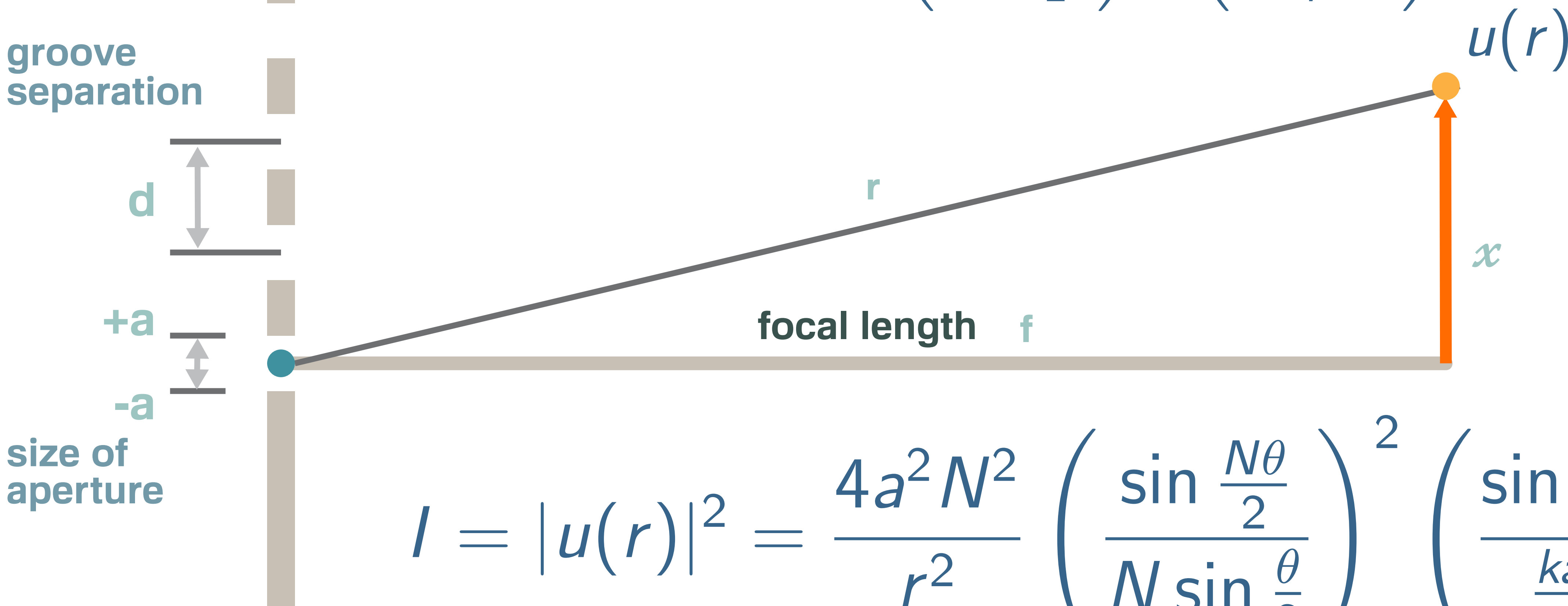
$$= \frac{e^{-i(\omega t - kr_0)}}{r}\frac{e^{-\frac{iN\theta}{2f}}\cdot 2\sin\frac{N\theta}{2}}{e^{-\frac{i\theta}{2f}}\cdot 2\sin\frac{\theta}{2}}\cdot(-i2a)\frac{\sin\left(\frac{kax}{f}\right)}{\frac{kax}{f}}$$

$$u(r) = \frac{e^{-i(\omega t - kr_0)}}{r}(-i2a)\frac{\sin\left(\frac{kax}{f}\right)}{\frac{kax}{f}}$$

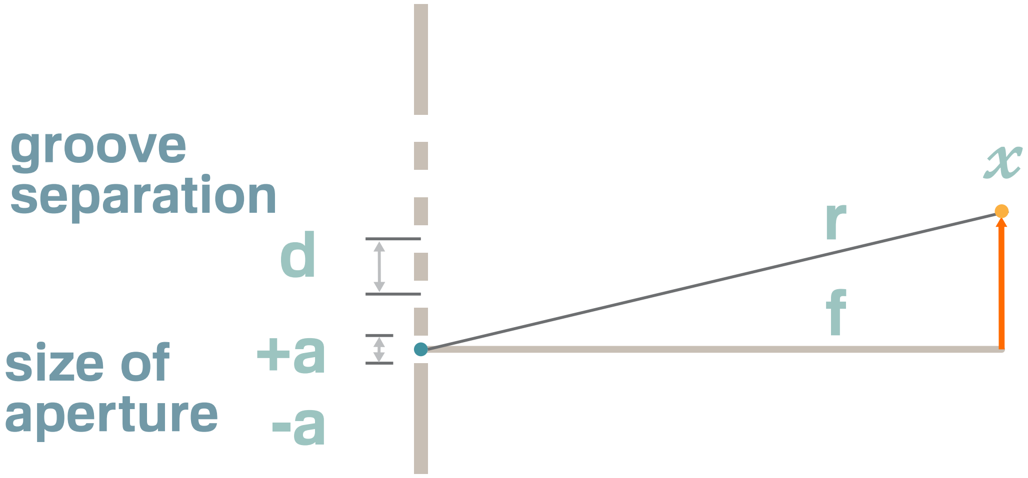
2 diffraction by a multiple slit

$$u(r) = \frac{e^{-i(\omega t - kr_0)}}{r} \frac{e^{-\frac{iN\theta}{2f}} \cdot 2\sin\frac{N\theta}{2}}{e^{-\frac{i\theta}{2f}} \cdot 2\sin\frac{\theta}{2}} \cdot (-i2a)\frac{\sin\left(\frac{kax}{f}\right)}{\frac{kax}{f}}$$

$$I = |u(r)|^2 = \frac{4a^2}{r^2}\left(\frac{\sin\frac{N\theta}{2}}{\sin\frac{\theta}{2}}\right)^2\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$



$$I = |u(r)|^2 = \frac{4a^2N^2}{r^2}\left(\frac{\sin\frac{N\theta}{2}}{N\sin\frac{\theta}{2}}\right)^2\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$

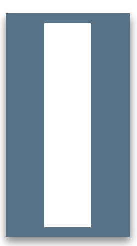


N slits

$$I = |u(r)|^2 = \frac{4a^2N^2}{r^2}\left(\frac{\sin\frac{N\theta}{2}}{N\sin\frac{\theta}{2}}\right)^2\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$

$$\frac{\sin Nx}{\sin x} \to N \quad (x \to 0)$$

single slit



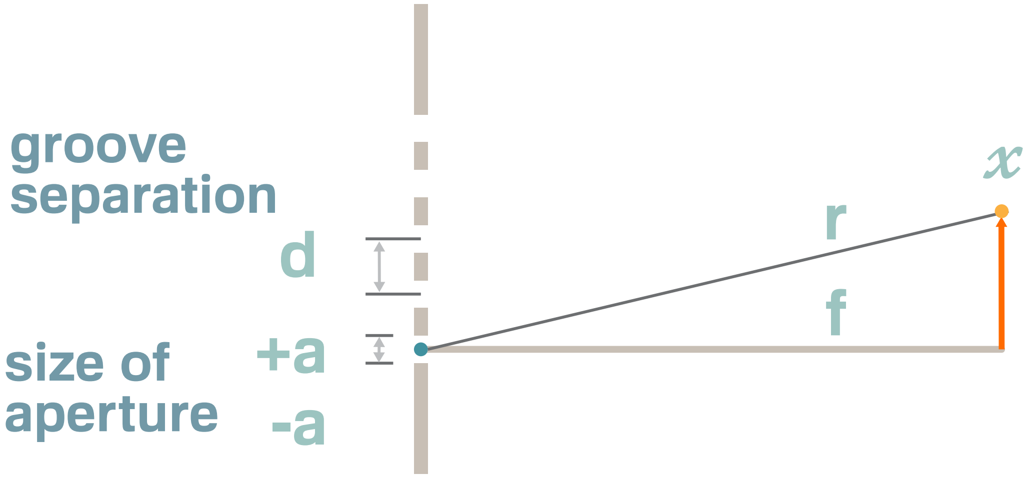
$$I = |u(r)|^2 = \frac{4a^2}{r^2}\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$

$$= \frac{4a^2}{r^2} \quad (x = 0)$$

2 **show this.**

$$= \frac{4a^2N^2}{r^2} \quad (x = 0)$$

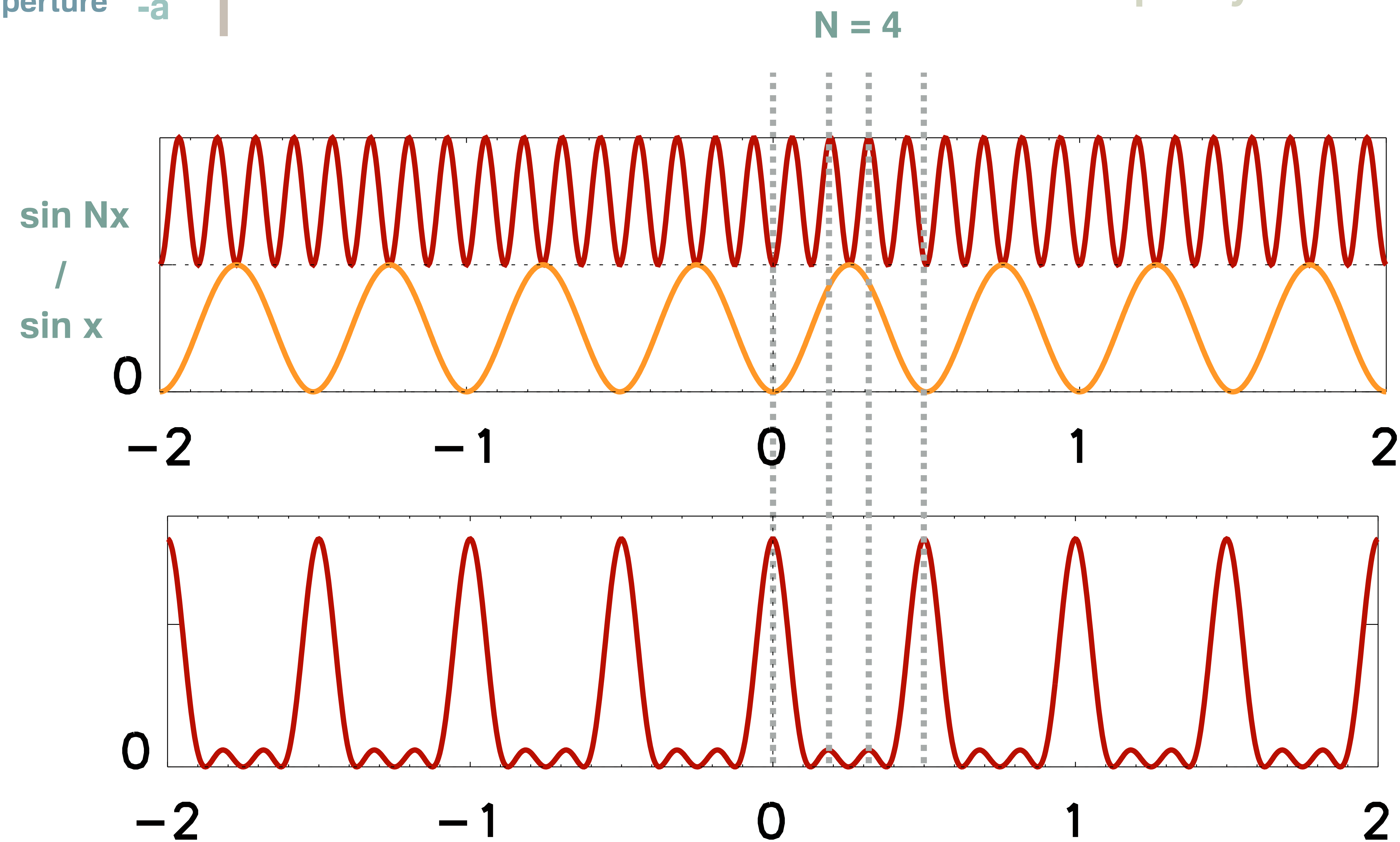
Intensity goes N^2 instead of N

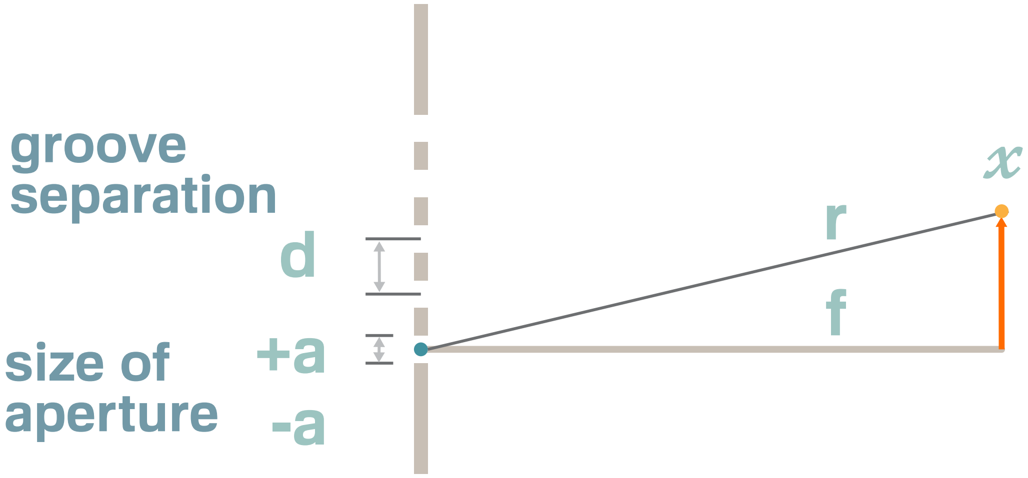


fast frequency

$$I = |u(r)|^2 = \frac{4a^2}{r^2}\left(\frac{\sin\frac{N\theta}{2}}{\sin\frac{\theta}{2}}\right)^2\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$

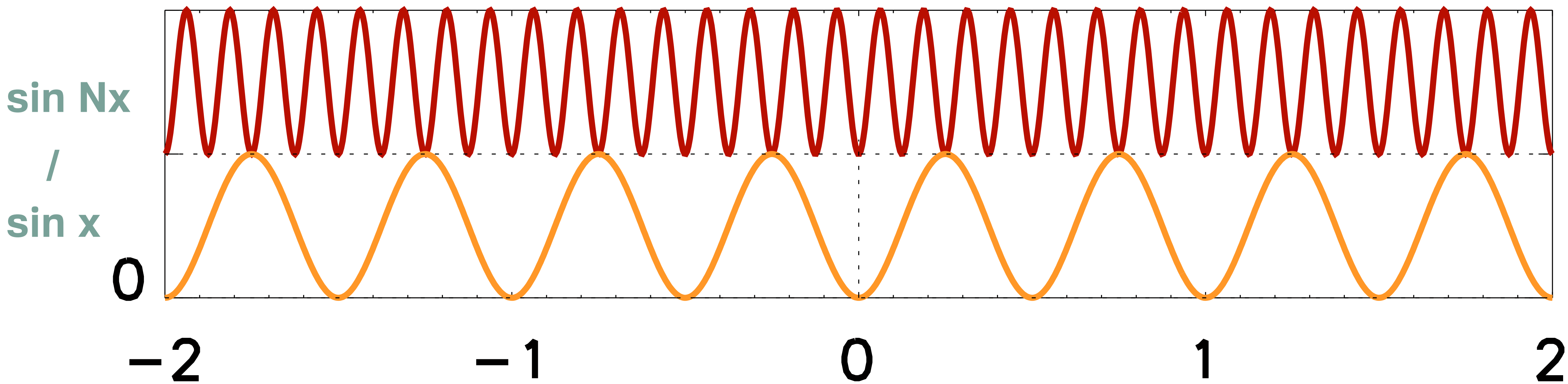
slow frequency

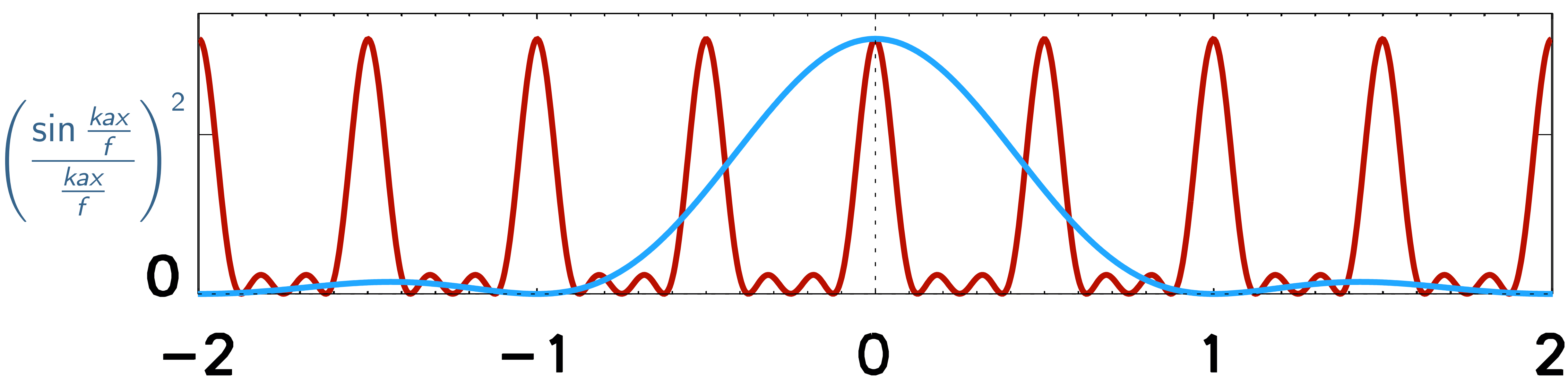


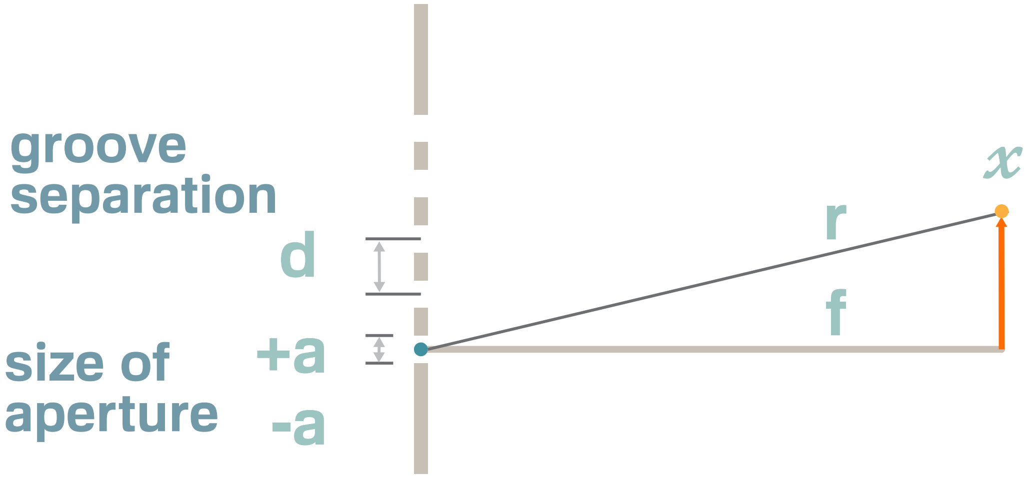


$$I = |u(r)|^2 = \frac{4a^2}{r^2}\left(\frac{\sin\frac{N\theta}{2}}{\sin\frac{\theta}{2}}\right)^2\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$

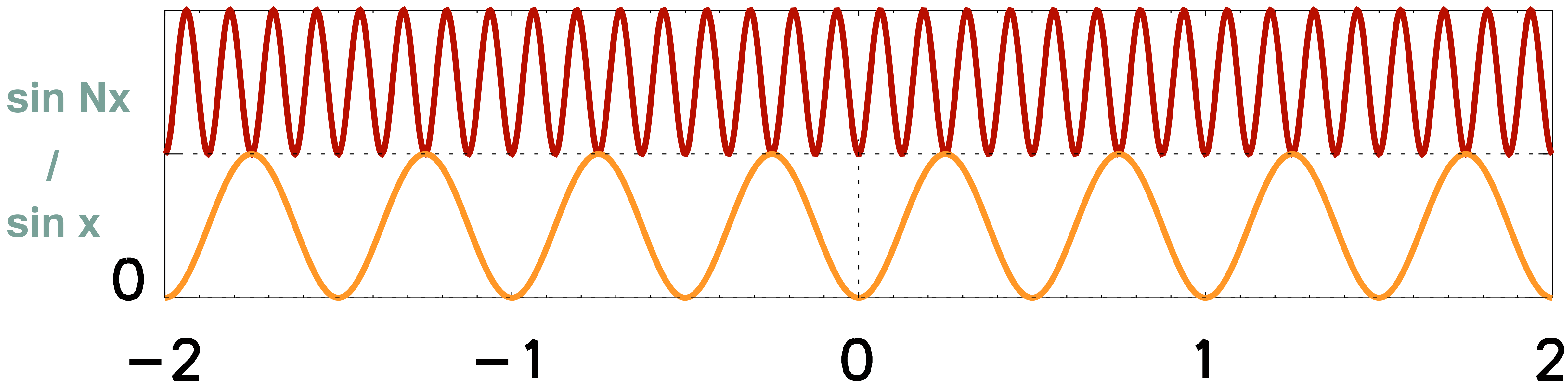
convolution

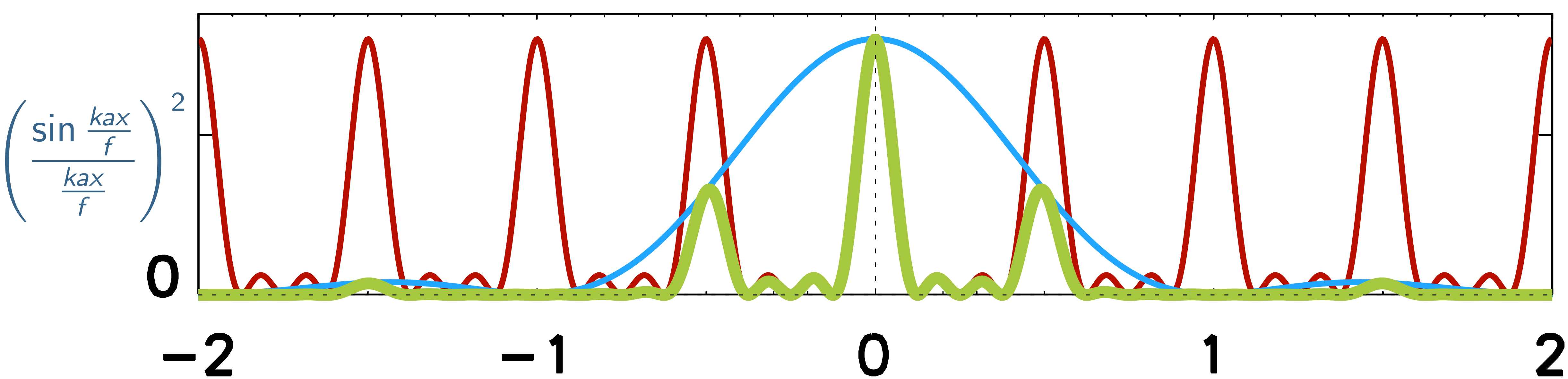


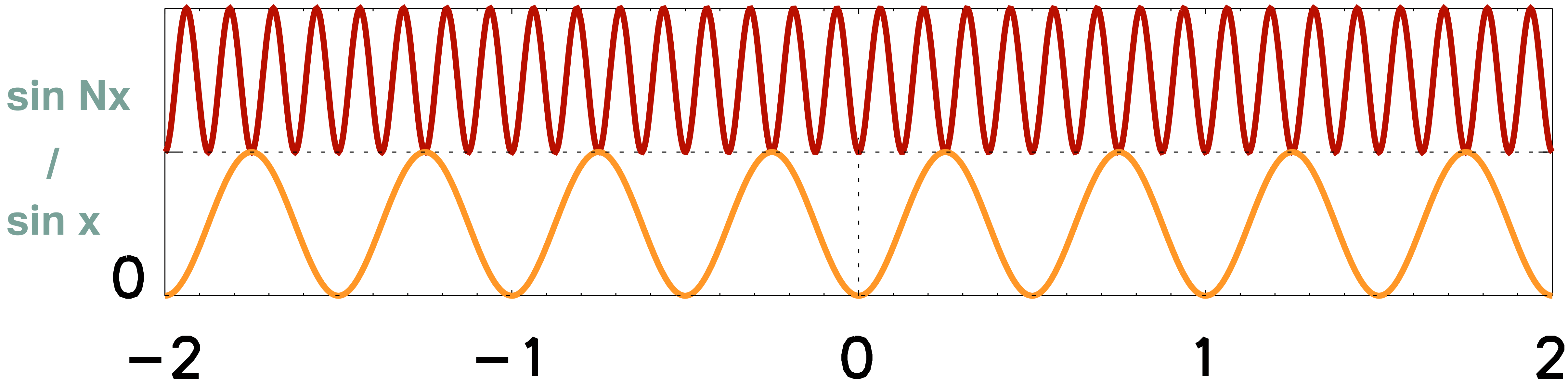


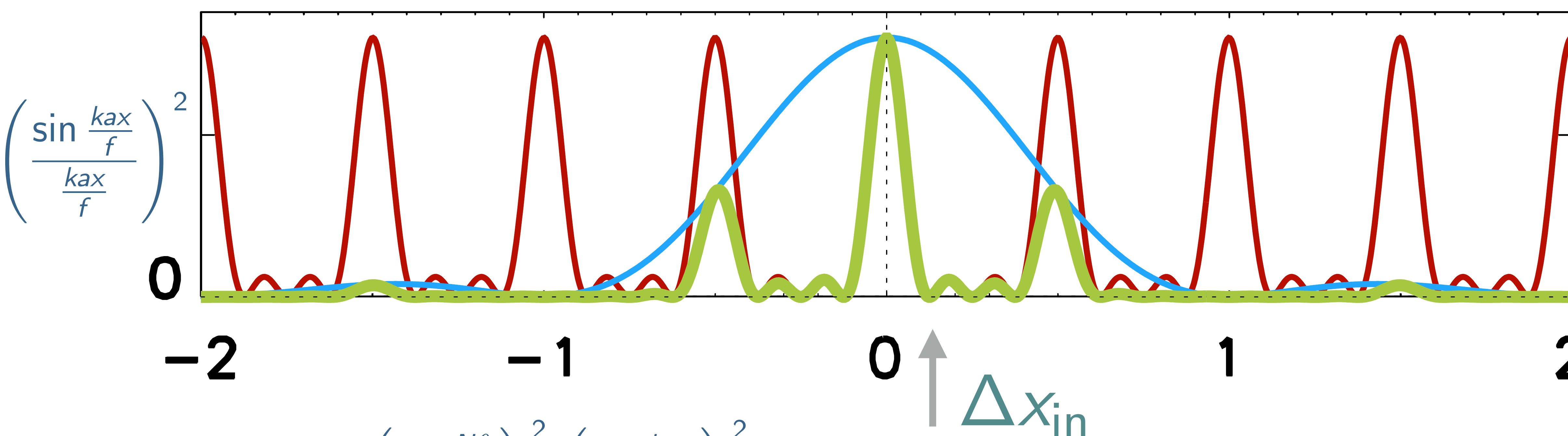


$$I = |u(r)|^2 = \frac{4a^2}{r^2}\left(\frac{\sin\frac{N\theta}{2}}{\sin\frac{\theta}{2}}\right)^2\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$









first null

$$I = |u(r)|^2 = \frac{4a^2}{r^2}\left(\frac{\sin \frac{N\theta}{2}}{\sin \frac{\theta}{2}}\right)^2 \left(\frac{\sin \frac{kax}{f}}{\frac{kax}{f}}\right)^2$$

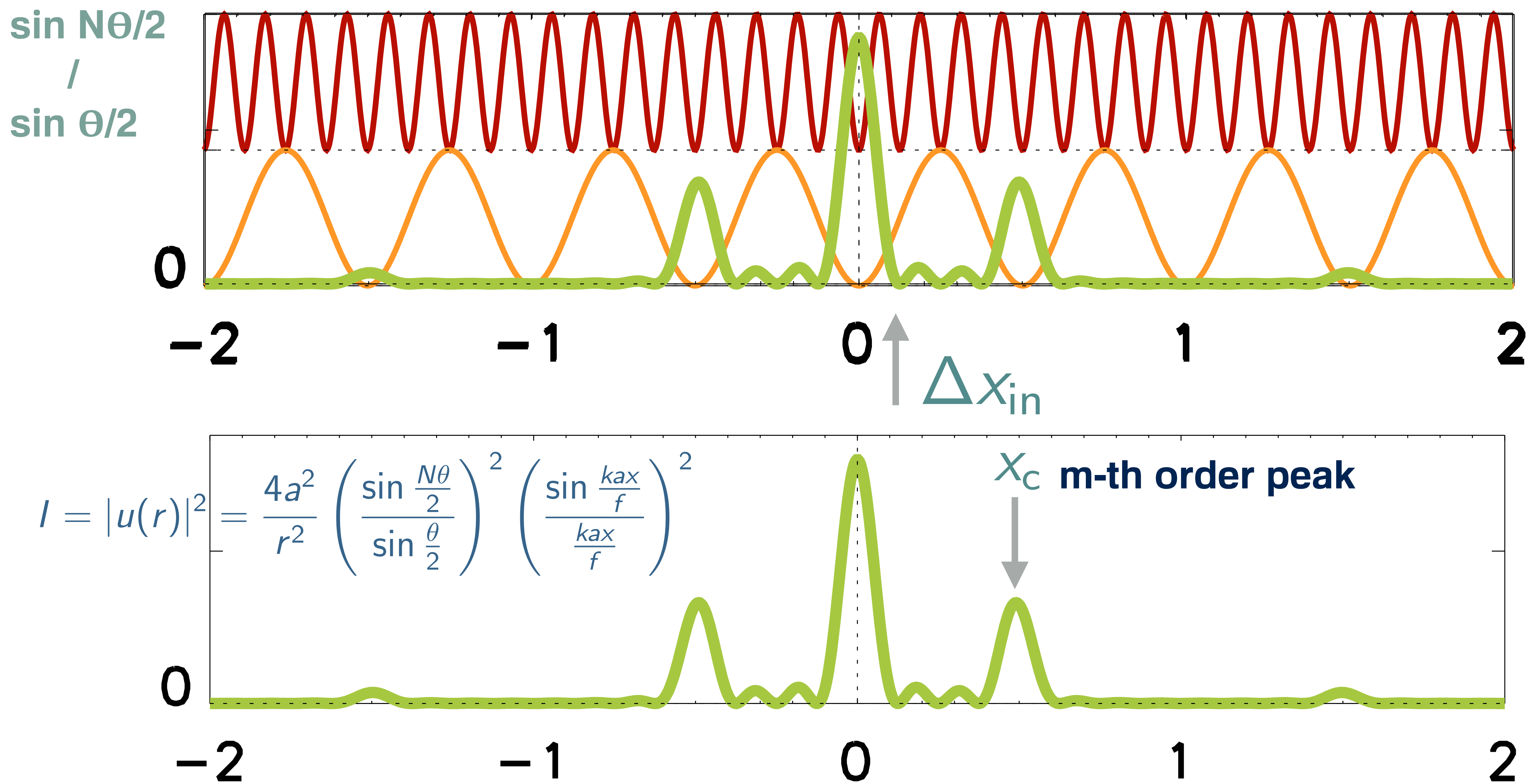
$$\Delta x_{in} = \frac{2f}{N}\frac{\pi}{kd} = \frac{2f}{N}\frac{\lambda\pi}{2\pi d} = \frac{f}{N}\frac{\lambda}{d}$$

$$\frac{N\theta}{2} = \pi$$

line becomes sharper with N

$$\theta = \frac{kxd}{f} \qquad \frac{N}{2}\frac{kxd}{f} = \pi$$

I ~ N²



$$I = |u(r)|^2 = \frac{4a^2}{r^2} \left(\frac{\sin \frac{N\theta}{2}}{\sin \frac{\theta}{2}} \right)^2 \left(\frac{\sin \frac{kax}{f}}{\frac{kax}{f}} \right)^2$$

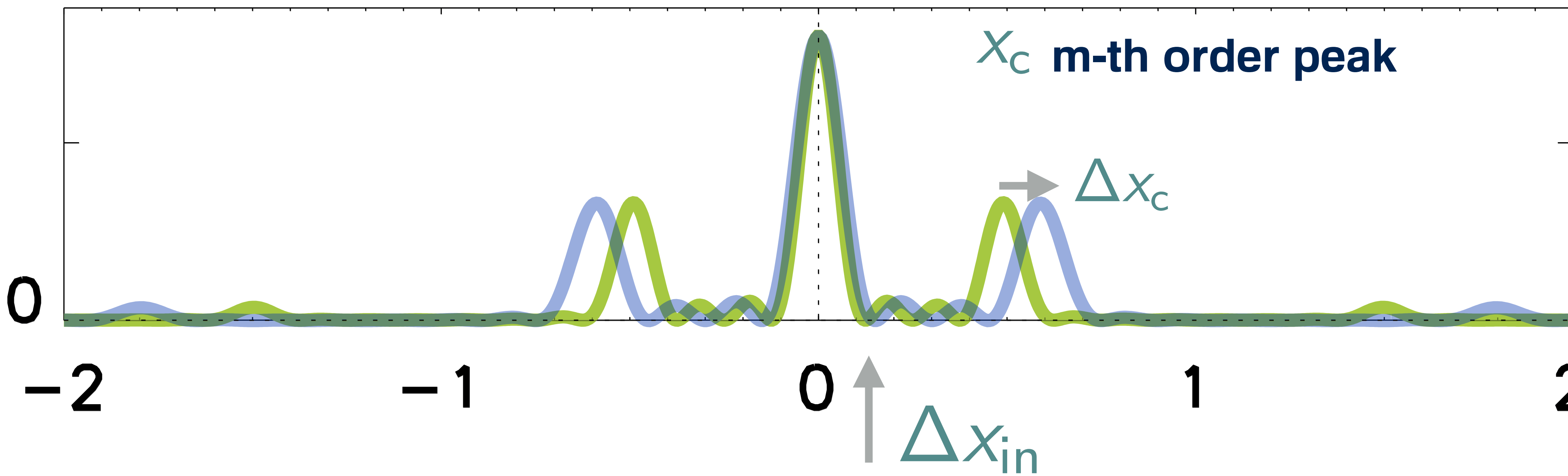
$$\frac{\theta}{2} = m \cdot \pi$$

$$\theta = \frac{kxd}{f}$$

peak position is color dependent

$$\frac{kx_c d}{2f} = m \cdot \pi$$

$$x_c = m \cdot \frac{2\pi f}{kd} = m \cdot \frac{2\pi \lambda f}{2\pi d} = m \cdot \frac{\lambda f}{d}$$



peak position is color dependent

$$x_c = m \cdot \frac{\lambda f}{d}$$

at different wavelength

peak is slightly off

$$\Delta x_c = m \cdot \frac{\Delta \lambda f}{d}$$

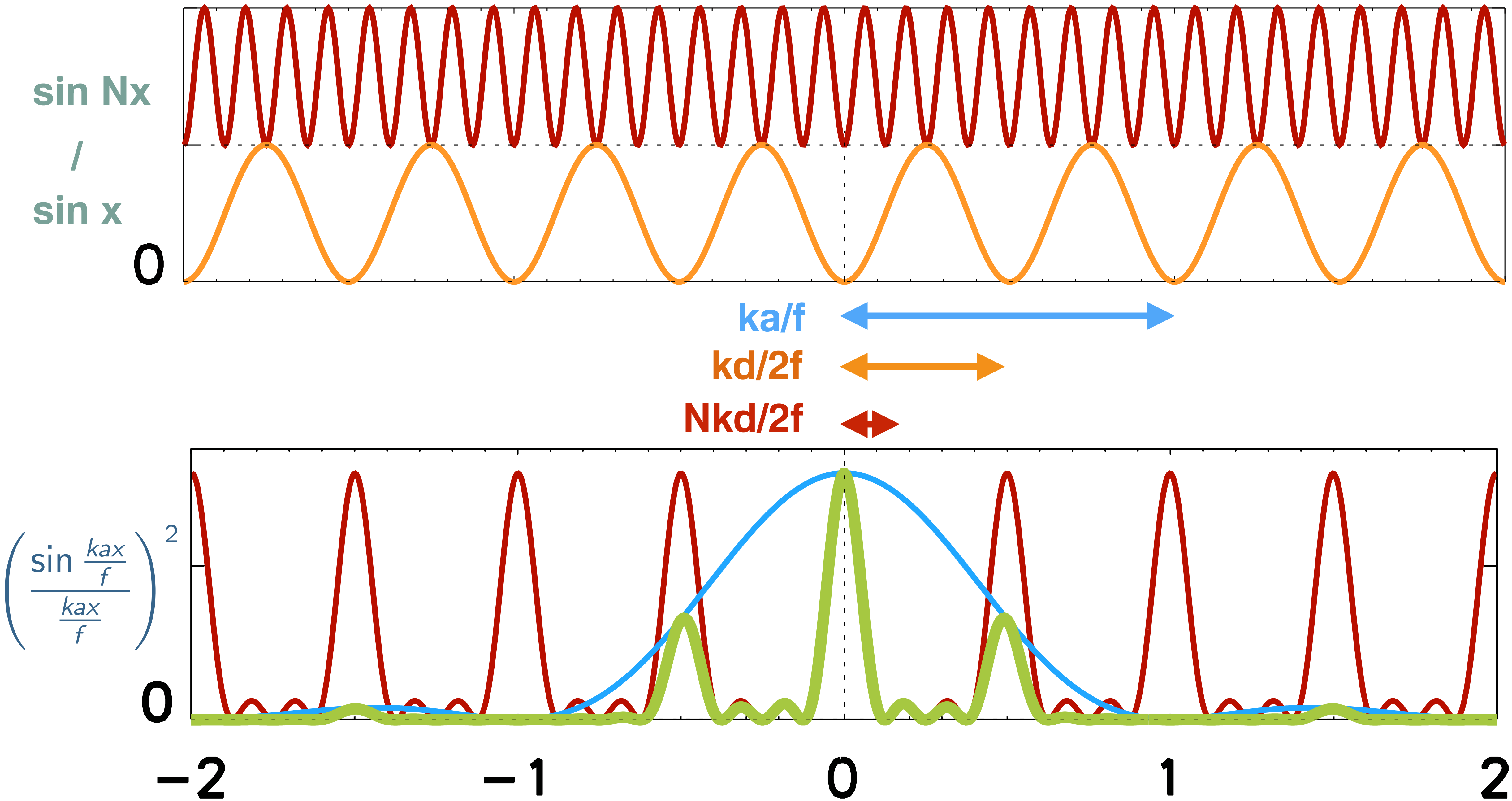
in order to have a line resolved

$$\Delta x_{in} < \Delta x_c$$

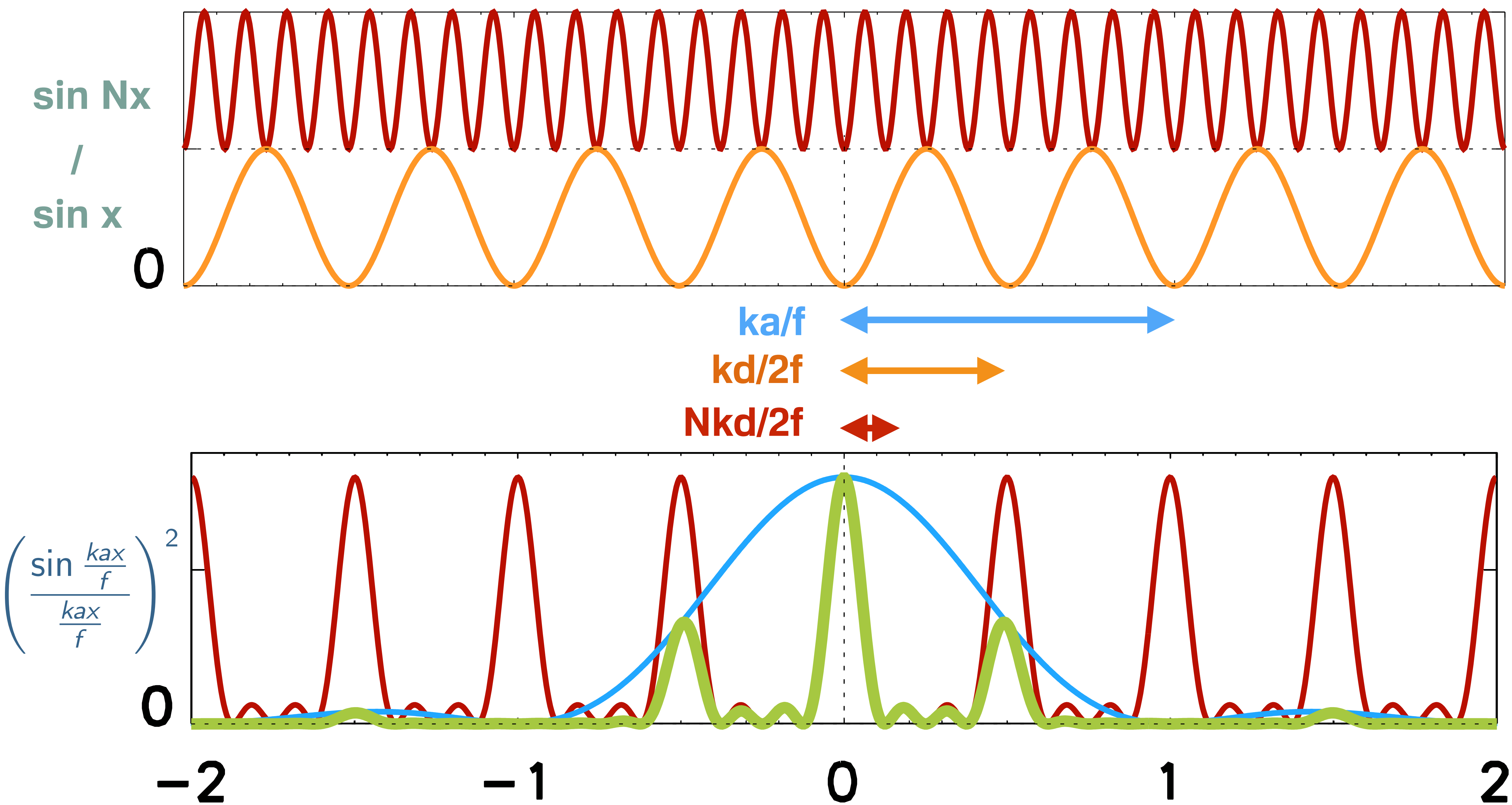
intrinsic | dispersion

$$\frac{f}{N}\frac{\lambda}{d} < m \cdot \frac{\Delta \lambda f}{d}$$

$$\frac{\lambda}{\Delta \lambda} < mN$$



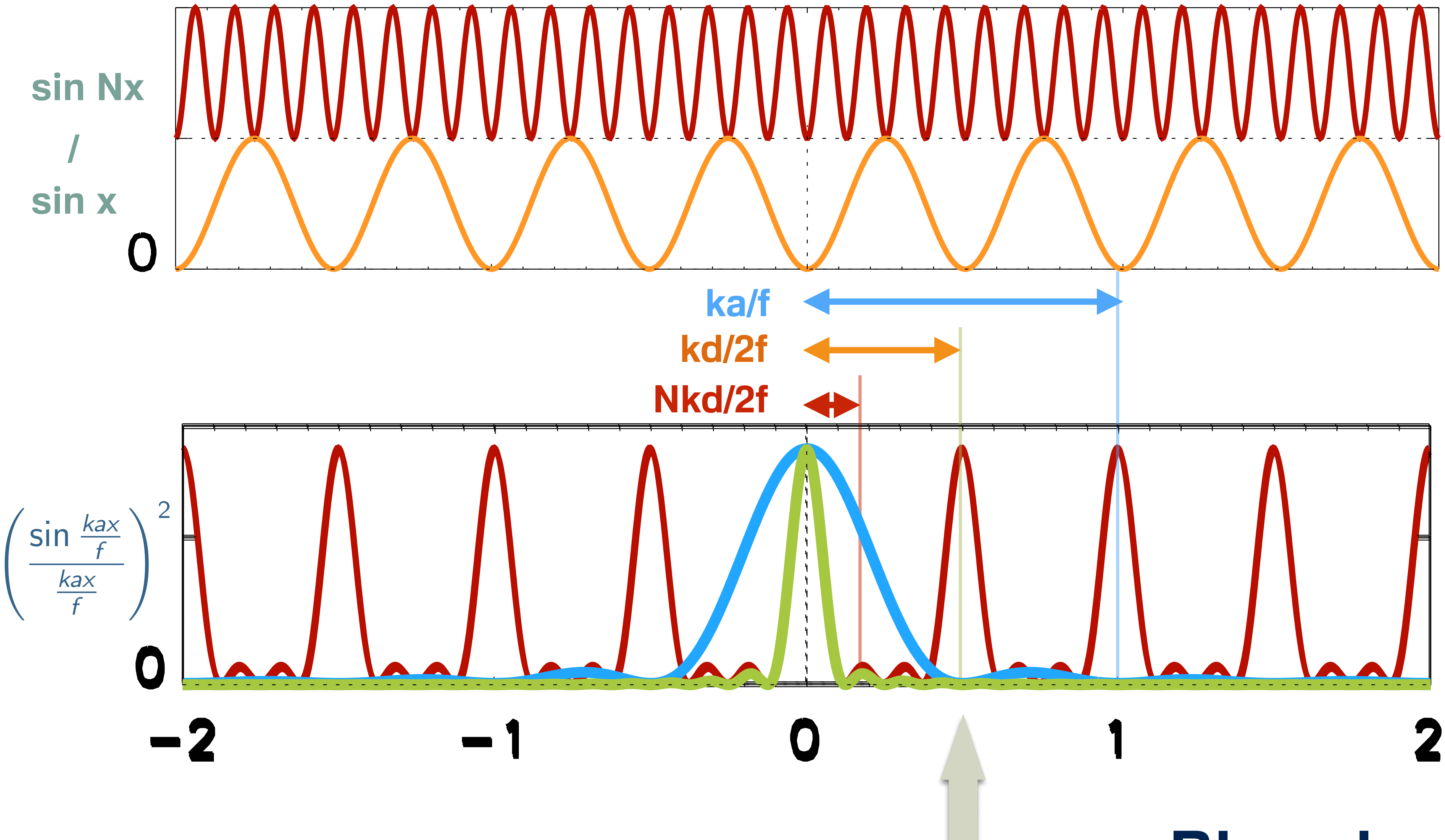
$$I = \frac{4a^2N^2}{r^2}\left(\frac{\sin\frac{Nkxd}{2f}}{N\sin\frac{kxd}{2f}}\right)^2\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$



$$\frac{kdx}{2f} = \ell\pi \qquad \boxed{} = 1$$

$$\frac{kax}{f} = m\pi \qquad \boxed{} = 0$$

$$I = \frac{4a^2N^2}{r^2}\left(\frac{\sin\frac{Nkxd}{2f}}{N\sin\frac{kxd}{2f}}\right)^2\left(\frac{\sin\frac{kax}{f}}{\frac{kax}{f}}\right)^2$$



$\frac{\mathbf{kdx}}{\mathbf{2f}} = \ell\pi$ □ = 1

$\frac{\mathbf{kax}}{\mathbf{f}} = m\pi$ □ = 0

d = 2a no side peak any more

all missing order

except x = 0 straight reflection

what kind of grating?

Blazed grating