Astrophysik II: Galaxien und Kosmologie

WS17/18

Übungsblatt 4

15.11.2017

Aufgabe 1. *Calculating the Jeans-Mass*

We consider the one dimensional collapse of an isothermal, homogenous, infinite, sphere which is slightly disturbed. Under this assumptions, the fluid follows the continuity equation, Eulers equation, with the density ρ, the velocity field u and the pressure P together with the Poisson equation for the potential Φ. This leads to the following equations, which describe the system completeley.

$$\frac{\partial \rho}{\partial t} = \frac{\partial(\rho u)}{\partial x}, \tag{1}$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right] = -\frac{\partial P}{\partial x} + \rho g, \tag{2}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = 4\pi G\rho \tag{3}$$

An equilibrium solution to this is given via, ρ_0, P_0, Φ_0, with $u_0 = 0$.

(a) In 1902, James Jeans considered $\Phi = const.$ to determine a condition for the collapse of the cloud. Make an easy physical argument, why this assumption can be a problem.

(b) Now defend the assumption Jeans made back in 1902. Hint: It is more less given in the description for the problem

(c) Now, that we solved this contradiction, use linear perturbation theory of the form:

$$A(x,t) = A_0(x,t) + A_1(x,t), \qquad \frac{A_1}{A_0} << 1. \tag{4}$$

Index zero indicates, the equilibrium solution and index one the small linear perturbation. Derive a system of eqautions for the linear perturabtions, with the form of equations (1) to (3).

(d) Additionally to this new system of equations lets consider the equation of state for the pressure fluctuation:

$$P_1 = \rho_1 \left(\frac{\partial P}{\partial \rho}\right)_{T=T_0} = \rho_1 c_s, \tag{5}$$

with the soundspeed $c_s = \sqrt{RT/\mu}$ and $T_0 = const.$ Now, calculate the dispersion relation of the gas cloud using this equation of state. The solution has the form:

$$-\frac{1}{c_s^2}\omega^2 + k^2 = \frac{4\pi G\rho_0}{c_s^2} \tag{6}$$

Hint: You need to find a second order partial differential equation for either the pressure or the density and then you need to solve it.

(e) Define the condition under which the fluctuations would rise and the collapse of the cloud sets in. Calculate the Jeans-Length from this condition.

(f) Finally, calculate the Jeans mass and discuss the result.

(g) Why does the result differ from the result you obtain with the virial theorem, which we discussed in the first tutorial.