Astrophysik II: Galaxien und Kosmologie

WS17/18

Übungsblatt 7

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Aufgabe 1. *Hydrostatic Equilibrium*
Use the equation

$$\frac{1}{r^2}\partial_r\left(\frac{r^2}{\rho_{gas}}\partial_r P\right) = -4\pi\rho_{DM}. \tag{1}$$

to derive the dark matter mass in terms of logarithmic gradients of density and temperature as presented in the lecture.

Aufgabe 2. *Bondi-accretion*
We want to derive an easy accretion model, for a stationary, spherically-symmetric flow on a star.

(a) Derive a form of the continuity equation under the assumptions given in the description for this easy accretion model. This means, you have to find a form of the continuity equation in spherical coordinates.

(b) Use the continuity equation to derive the mass per unit time that is flowing through the surface.

(c) Alongside the continuity equation, we need Bernoulli's law. The equation is given via:

$$\frac{v^2}{2} + \frac{c_s^2 - c_{s0}^2}{\gamma - 1} - \frac{GM}{r} = 0 \tag{2}$$

using a polytrope equation of state. A more general form of the equation holds:

$$\frac{v^2}{2} + \tilde{h} - \frac{GM}{r} = 0, \tag{3}$$

where $\tilde{h}$ is the free enthalpie per unit mass. For an isothermal gas you can calculate $\tilde{h}$ by:

$$\tilde{h} = \int \frac{dP}{\rho}. \tag{4}$$

Use the ideal equation of state to derive Bernoulli's law for isothermal gas.

(d) Use the substitutions $x = r/r_B$, $u = v/c_{s0}$ and $\alpha = \rho/\rho_0$, with the $r_B = GM/c_{s0}^2$ as the Bondi-radius, to derive a dimensionless equations for both forms of Bernoulli's equation, obtained in (c).

(e) Apply the substitutions from (d) on the result of (a) and derive a dimensionless form of the result of (a). Read the Bondi-accretion rate from it.

(f) Eliminate α for both types of Bernoulli's equation using the result of (e).

(g) Build the total differentials of the results of (e) and (f).

(h) Use the result of (g) to derive the following equation:

$$udu\left(1 - \frac{1}{Ma^2}\right) = \begin{cases} \frac{dx}{x}\left(2\alpha^{\gamma-1} - \frac{1}{x}\right) \\ \frac{dx}{x}\left(2 - \frac{1}{x}\right) \end{cases} \tag{5}$$

with the Machnumber $Ma = u^2/\alpha^{\gamma-1}$. Discuss the result.