Astrophysik II: Galaxien und Kosmologie

WS17/18

Übungsblatt 9

10.01.2017

Aufgabe 1. *The density parameter*
Recall the first Friedmann equation in the form presented in the last assignment sheet.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3} \tag{1}$$

(a) Using the energy conservation equation,

$$\dot{\rho} = 3\rho c^2 \frac{\dot{a}}{a}(1+w) \tag{2}$$

where $P = \rho w c^2$ is the equation of state, and w is the state parameter. Determine how density evolves as a function of the scale parameter when $w = 0$ (matter domination), $w = 1/3$ (radiation domination), and $w = -1$ (cosmological constant domination). use these results to express the Friedmann equation in terms of present day quantities.

(b) What is the critical density? How might we use it to simplify the Friedmann equation further?

Aufgabe 2. *Evolution of the scale factor*

(a) Use the results of from 1.a and 1.b to show that the Friedmann equations take the form,

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = H_0^2\left(\frac{1-\Omega_0}{a^2} + \frac{\Omega_M}{a^3} + \frac{\Omega_R}{a^4} + \Omega_\Lambda\right)} \tag{3}$$

Where all densities are expressed in terms of their *present day values.*

(b) Consider a flat universe ($\Omega_0 = 1$), integrate the equation above to see how the scale factor evolves during the epoch where radiation dominates, matter dominates, and the cosmological constant dominates.

(c) Now, write a small program to numerically integrate a eq. (4). make a plot showing showing scale factor evolution with time under a few different cosmologies (Planck cosmology, empty universe, matter only, $\Omega_0 \neq 1$, etc.). Print these plots and bring them to the tutorial. Be creative! Use whichever programming language you prefer, but do *not* use any pre-built numerical integrators.

(d) Finally, think again about part (b), try to numerically integrate eq (4) to estimate the age of the Universe using *current* best measurements for the cosmological densities.