

# Astrophysik II: Galaxien und Kosmologie

WS17/18  
Übungsblatt 10

17.01.2018

## Aufgabe 1. Dynamical Friction

Under the assumption of an isotropic velocity distribution one can obtain Chandrasekhars dynamical friction formula:

$$\frac{d\mathbf{v}_M}{dt} = -16\pi^2 G^2 M m_a \ln \Lambda \left[ \int_0^{v_M} dv_a v_a^2 f(v_a) \right] \frac{\mathbf{v}_M}{v_M^3} \quad (1)$$

- (a) Describe the process of dynamical friction, using your own words (1 point).
- (b) Use equation (1) to derive a dynamical friction formula for small  $v_M$  (1 point).
- (c) Use equation (1) to derive a dynamical friction formula for large  $v_M$  (1 point). **Hint: For large  $v_M$  the integral from (1) gives the number density divided by a constant.**
- (d) Under the assumption of a maxwellian distribution for the velocity ( $f(v_a) = \frac{n}{(2\pi\sigma^2)^{3/2}} \exp(-v_a^2/2\sigma^2)$ ) derive a friction formula from equation 1 (1 point). Hint: The solution to this task is:  $\frac{d\mathbf{v}_M}{dt} = -\frac{4\pi M \rho \ln \Lambda}{v_M^3} \left[ \text{erf}(X) - \frac{2X}{\sqrt{\pi} \exp(-X^2)} \right]$  with  $X = v_M/(\sqrt{2}\sigma)$
- (e) Use the result of (d) to derive the effect of dynamical friction on a black hole in an isothermal sphere with the density profile  $\rho(r) = v_c^2/4\pi G r^2$ . Find the force on the black hole caused by dynamical friction for  $X=1$  (1 point).
- (f) Calculate the angular momentum loss of the black hole (1 point).
- (g) Use the time derivative of the angular momentum  $L = M r v_c$  to derive a differential equation of the form:

$$r \frac{dr}{dt} = -0.428 \ln \Lambda \frac{GM}{v_c} = -0.302 \ln \Lambda \frac{GM}{\sigma}. \quad (2)$$

Solve this equation under the assumption that  $\ln \Lambda$  is constant. This will lead to a formula for the friction time, which the black hole needs to fall to the center (1 point)

## Aufgabe 1. Quick Questions

- (a) Describe the process of star formation qualitatively (1 point)
- (b) Make a sketch of the Hertzsprung-Russel (HR) diagramm and explain the different types of stars using the HR-diagramm (1 point).
- (c) Use the sketch of (b) and overplot the Hayashi-line and describe its meaning (1 point)
- (d) What is the cosmic microwave background? (1 point)
- (e) Look up and describe the Sunyaev-Zeldovich effect (1 point)
- (f) Describe qualitatively how you can use the Sunyaev-Zeldovich effect to calculate the mass of a galaxy cluster (1 point)