Observational Cosmology

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Introduction

Schedule

Introduction

01	Monday	Introduction
02		History
03		Basic Facts

World Models

04		World Models

Classical Cosmology

05	Tuesday	Distance Determination, H_0

The Early Universe

06		Hot Big Bang Model Nucleosynthesis, Inflation, ...
07	Wednesday	Determination of Ω and Λ

Large Scale Structures

08	Thursday	Large Scale Structures, Structure Formation

Literature

1. Cosmology Textbooks

PEACOCK, J.A., 1999, *Cosmological Physics*, Cambridge: Cambridge Univ. Press, 49.50€

Very exhaustive, but difficult to read since the entropy per page is *very* high. . . still: a “must buy”.

LONGAIR, M.S., 1998, *Galaxy Formation*, Berlin: Springer, 53.45€

Clear and pedagogical treatment of structure formation, recommended.

BERGSTRÖM, L. & GOOBAR, A., 1999, *Cosmology and Particle Astrophysics*, New York: Wiley, 47.90€

Nice description of the physics relevant to cosmology and high energy astrophysics, focusing on concepts. Less detailed than Peacock, but easier to digest.

Literature

PADMANABHAN, T., 1996, *Cosmology and Astrophysics Through Problems*, Cambridge: Cambridge Univ. Press, $36.95

Large collection of standard astrophysical problems (with solutions) ranging from radiation processes and hydrodynamics to cosmology and general relativity

PADMANABHAN, T., 1993, *Structure Formation in the Universe*, Cambridge: Cambridge Univ. Press, 46.50€

Mathematical treatment of cosmology, focusing on the formation of structure . . . Less astrophysical than the book by Longair.

ISLAM, J.N., 2002, *An Introduction to Mathematical Cosmology*, Cambridge: Cambridge Univ. Press, 42.50€

Useful summary of the facts of classical theoretical cosmology, recently revised.

Literature

KOLB, E.W. & TURNER, M.S., 1990, *The Early Universe*, Reading: Addison-Wesley, 49.90€

Graduate-level text, the section on phase transitions and inflation in the early universe is especially recommended.

PEEBLES, P.J.E., 1993, *Principles of Physical Cosmology*, Princeton: Princeton Univ. Press (antiquarian only, do not pay more than $30!)

700p introduction to modern cosmology by one of its founders, in some parts quite readable, however, many forward references make the book very difficult to read for beginners.

Literature

2. Textbooks on General Relativity

WEINBERG, S., 1972, *Gravitation and Cosmology*, New York: Wiley, 129€

Classical textbook on GR, still one of the best introductions. Nice section on classical cosmology.

SCHUTZ, B.F., 1985, *A First Course in General Relativity*, Cambridge: Cambridge Univ. Press, 45.90€

Nice and modern introduction to GR. The cosmology section is very short, though.

MISNER, C.W., THORNE, K.S. & WHEELER, J.A., 1973, *Gravitation*, San Francisco: Freeman, 104.90€

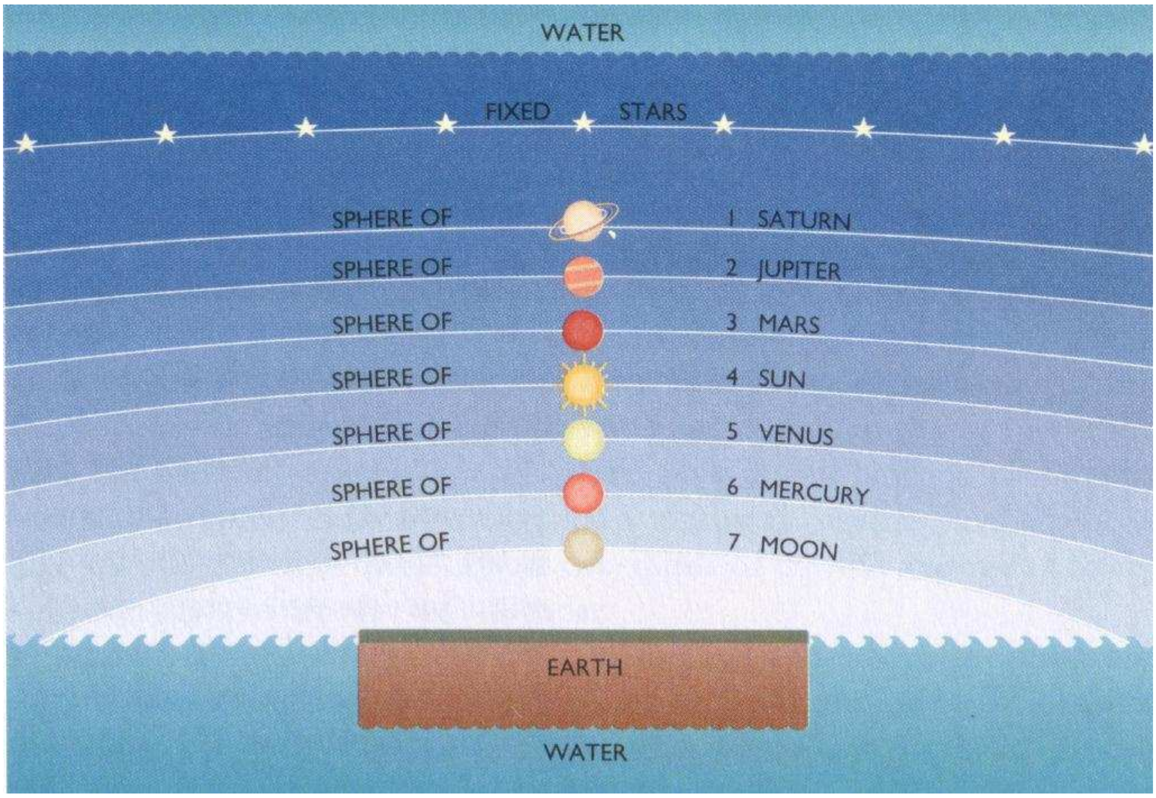
Commonly called MTW, this book is as heavy as the subject. . . Uses a weird notation. The cosmology section is outdated.

WALD, R.M., 1984, *General Relativity*, Chicago: Univ. Chicago Press (only antiquarian, ∼$40)

Modern introduction to GR for the mathematically inclined.

History

Babylon



(Aveni, 1993, p. 13)

Babylonian astronomy: Earliest astronomy with influence on us: $\sim$360 d year $\Longrightarrow$ sexagesimal system, 24h day, ...

Enuma Elish myth ($\sim$1100BC): Universe is place of battle between earth and sky, born from world parents.

Note similar myth in the Genesis...

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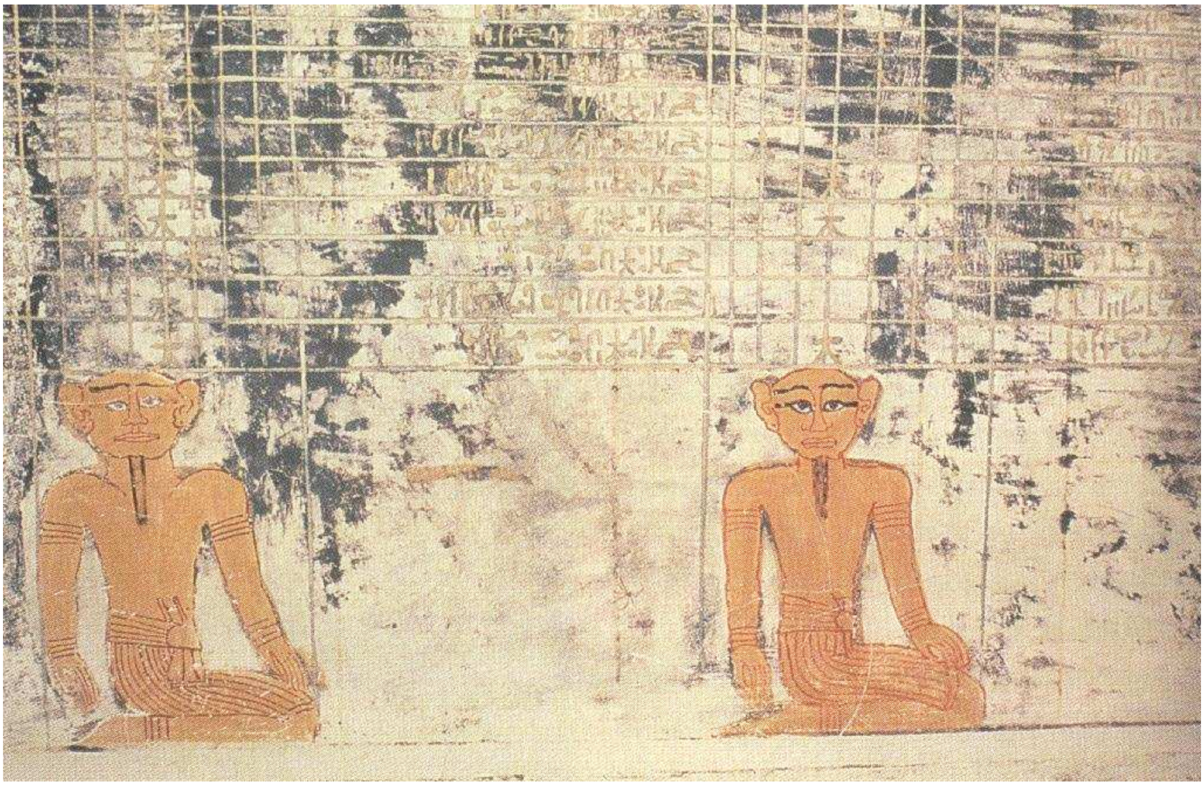
Egypt



(Aveni, 1993, p. 42)

Coffin lid showing two astronomers' assistants, 2000. . . 1500 BC; hieroglyphs list stars ("decans") whose rise defines the start of each hour of the night.

$\sim$2000 BC: 365 d calendar (12$\times$30 d plus 5 d extra), fixed to Nile flood (heliacal rising of Sirius), star clocks.

heliacal rising: first appearance of star in eastern sky at dawn, after it has been hidden by the sun.

Greek/Roman, I

Early Greek astronomy: folk tale astronomy (Hesiod, Weeks and Days), similar to Egypt.

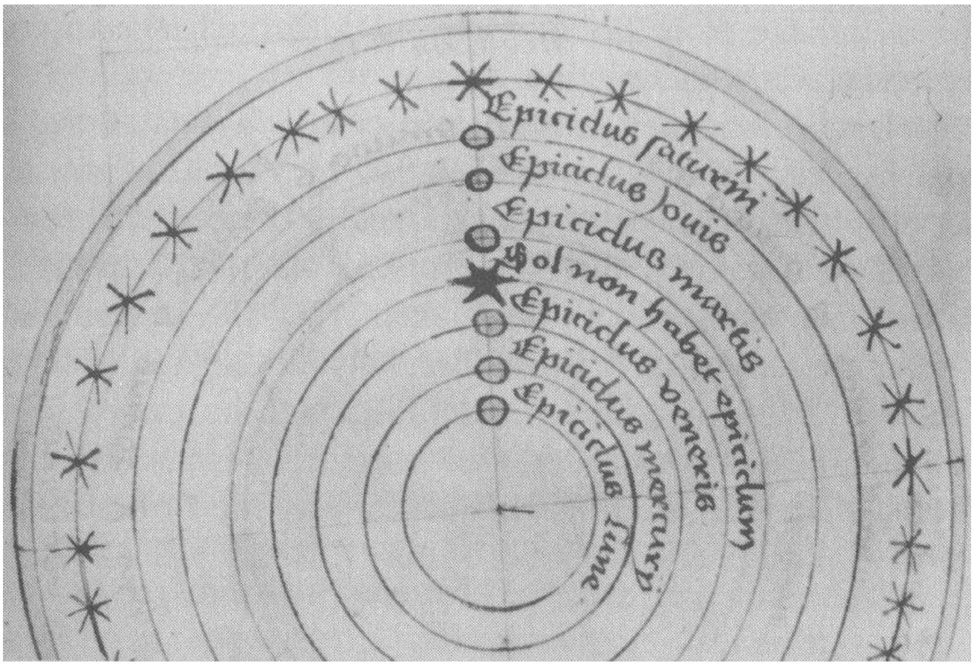
Thales (624–547 BC): Earth is flat, surrounded by water.

Anaxagoras (500–428 BC): Earth is flat, floats in nothingness, stars are far away, fixed on sphere rotating around us. Eclipses are due to shadow of Earth.

Eudoxus (408–355 BC): Geocentric model, planets affixed to concentric crystalline spheres, stars on outermost disk. First real model for planetary motions!

Aristarchus (310–230 BC): Determination of rel. distance to Moon and Sun: Moon is $\sim 20\times$ closer.

Greek/Roman, II



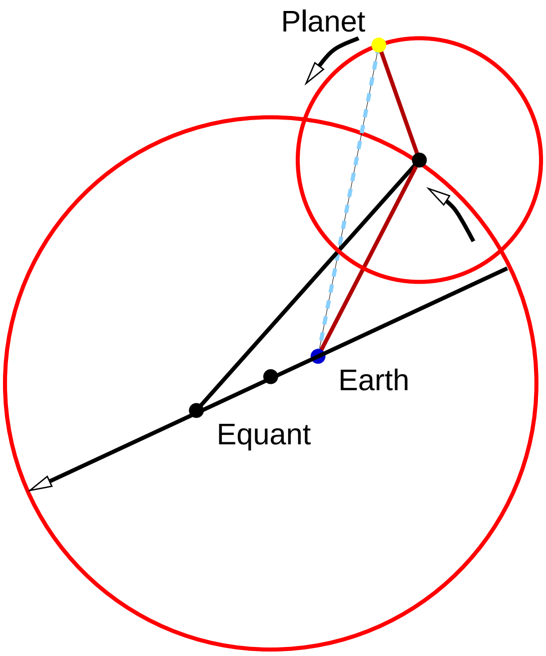
Aristotle (384–322 BC, *de caelo*): Refinement of Eudoxus model: add spheres to ensure smooth motion $\Longrightarrow$ Universe filled with crystalline spheres (nature abhors vacuum).

$\Longrightarrow$ Central philosophy until ∼1450AD!

Hipparchus (?? – ∼127 BC): Refinement of geocentric Aristotelian model into tool to make predictions.

Greek/Roman, III

(Aveni, 1993, p. 58)



Ptolemaeus (∼140AD): *Syntaxis*, aka Almagest: Refinement of Aristotelian theory into model useable for computations ⟹ Ptolemaic System.

Renaissance, I

Nicolaus Copernicus (1473–1543): Ptolemaic system is too complicated, a sun-centered system is more elegant: "In no other way do we perceive the clear harmonious linkage between the motions of the planets and the sizes of their orbs."

Copernicus principle: The Earth is not at the center of the universe.

Renaissance, II

(Gingerich, 1993, p. 165)

De revolutionibus orbium coelestium libri vi

Renaissance, III

Tycho Brahe (1546–1601): Visual planetary positions of highest precision reveal flaws in Ptolemaic positions $\Longrightarrow$ Refinement of ptolemaic system into a semi-Copernican form.

Renaissance, IV

Johannes Kepler (1571–1630): Planets orbit on ellipses around sun, not on circles, laws of motion.

Galileo Galilei (1564–1642): Moons of Jupiter (Kepler $\Longrightarrow$ similar to heliocentric model!)...


Dies diei eructat verbum
nocti indicat scientiam
Numerus
Mensura
Pondus
Videbo Cælos tuos, opera digitor tuor.
Non inclinabitur in seculum seculi
Ponderibus librata suis
Erigor dum corrigor
Cl. Ptolem.
F. Curtius Bon. Incid.

Übersetzung der lateinischen Texte auf dem Stich von Ricolli (von oben nach unten):

Dies diei eructat uerbu. . .	Der Tag des Tags (=der jüngste Tag) wirft das Wort von sich *oder* Der Tag der Tage speit das Wort aus.
. . . et nox nocti indicat scientiam	Und eine Nacht teilt der anderen das Wissen mit *oder* Die Nacht der Nächte zeigt die Wissenschaft
Finger an Hand Gottes: Numerus, Mensura, Pondus	Weisheit Salomos (Apokryphe Schriften des Alten Testaments, Kapitel 11,21): "Aber du [Gott] hast alles nach *Maß*, *Zahl* und *Gewicht* geordnet" – Schöpfungstheologisch/kosmologische Kernstelle der Bibel
Videbo Caelos tuos, opera digitor tuor	Ich werde deine Himmel erkennen können, ich verteidige sehr würdig deine Werke *oder* ich werde deine Himmel sehen. . .
Non Inclinabitur in saeculum saeculi	Er wird mir in Ewigkeit keine andere (falsche) Richtung geben werden.
Erigo dum corrigar	Ich werde aufgerichtet/ermutigt werden indem ich verbessert werde
Ponderibus librata suis	Mit ihren Gewichten wird sie kräftig geschwungen *oder* Mit seinen Gewichten im Gleichgewicht gehalten.

Mit 10000fachem Dank an Papa Deetjen und Sohn, Marcus Kirsch, Eckart und Irene Goehler, und die Espressorunde!!!!

Weitere Übersetzungsvorschläge werden dankend entgegengenommen.

Newton

(Newton, 1730)

Isaac Newton (1642–1727): Newton's laws,
Physical cause for shape of orbits is gravitation
(*De Philosophiae Naturalis Principia Mathematica*, 1687).

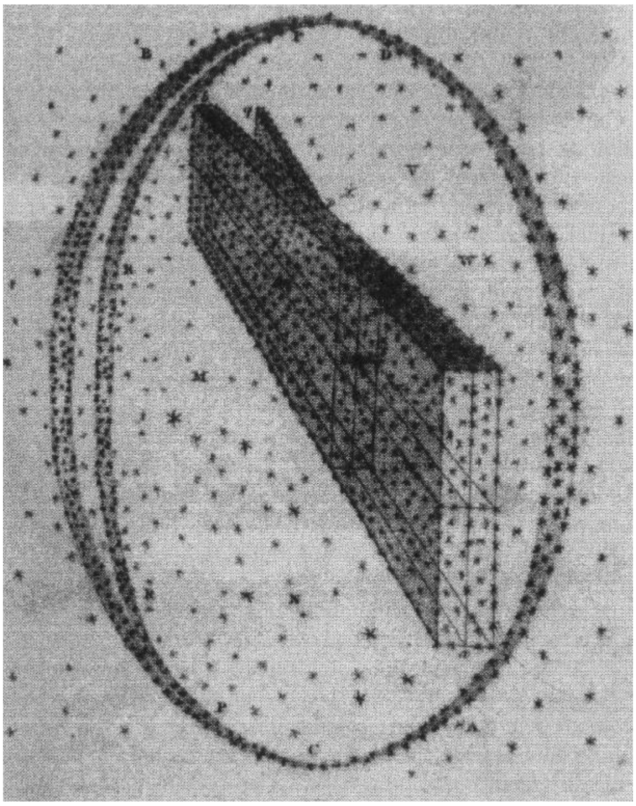
$\Longrightarrow$ Begin of modern cosmology.

Modern Cosmology

Galileo: Milky way consists of stars.

Newton: Stars are distant suns

William Herschel (1738–1822): Milky Way is a flattened disk of stars, sun is at center.



Friedrich Bessel (1784–1846): Distance to 61 Cyg (1838), positions of 50000 stars

Immanuel Kant: "Nebulae are galaxies" (disputed until the 1910s).

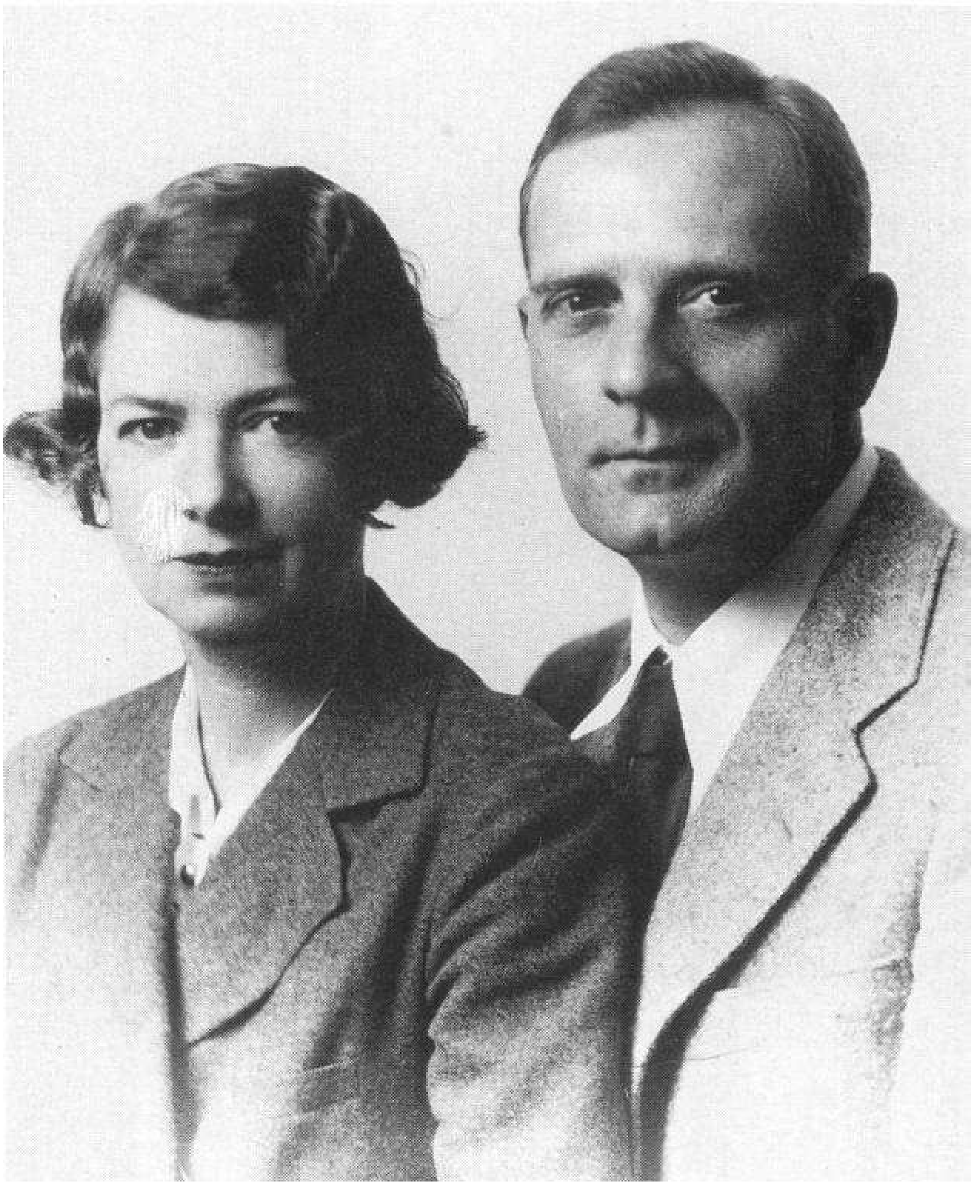
John Herschel (1792–1871): General Catalogue of Galaxies (1864, 5079 Objects)

Johan Dreyer (1852–1926): NGC+IC (15000 Objects)

Modern Cosmology

Albert Einstein (1879–1955): Theory of gravitation, applicability of theory to evolution of the universe as a whole.

Modern Cosmology



(Christianson, 1995, p. 165)

Edwin Hubble (1889–1953): Universe is expanding, realization of galaxies as being outside of the milky way: extragalactic astronomy

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Christianson, G. E., 1995, Edwin Hubble – Mariner of the Nebulae, Farrar, Straus and Giroux)

Gingerich, O., 1993, The Eye of Heaven – Ptolemy, Copernicus, Kepler, (New York: American Institute of Physics)

Newton, I., 1730, Opticks, Vol. 4th, (London: William Innys), reprint: Dover Publications, 1952

Basic Facts

Basic Facts

Cosmology deals with answering the questions about the universe as a whole.
The main question is:

How did the universe evolve into what it is now?

For this, *four major facts* need to be taken into account:

The universe is:
- expanding,
- isotropic,
- and homogeneous.

The isotropy and homogeneity of the universe is called the *cosmological principle*.
Perhaps (for us) the most important fact is:

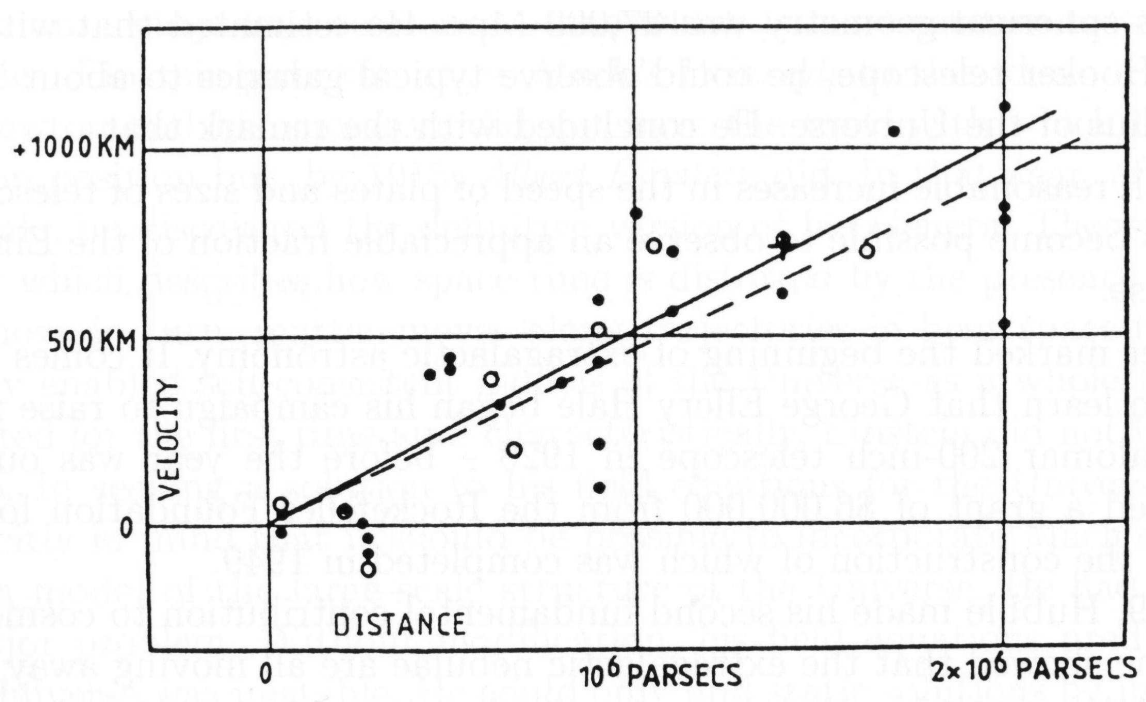
- The universe is habitable to humans.

i.e., the *anthropic principle*.

The one question cosmology does not attempt to answer is: How came the universe into being?

$\Longrightarrow$ Realm of theology!

Expansion, I



(Hubble, 1929, Fig. 1)

Hubble (1929): Velocity v (defined as $v/c := z = \Delta\lambda/\lambda$) for galaxy at distance r is

$$v(r) = H_0 r + X\cos\alpha\cos\delta + Y\sin\alpha\cos\delta + Z\sin\delta \quad (3.1)$$

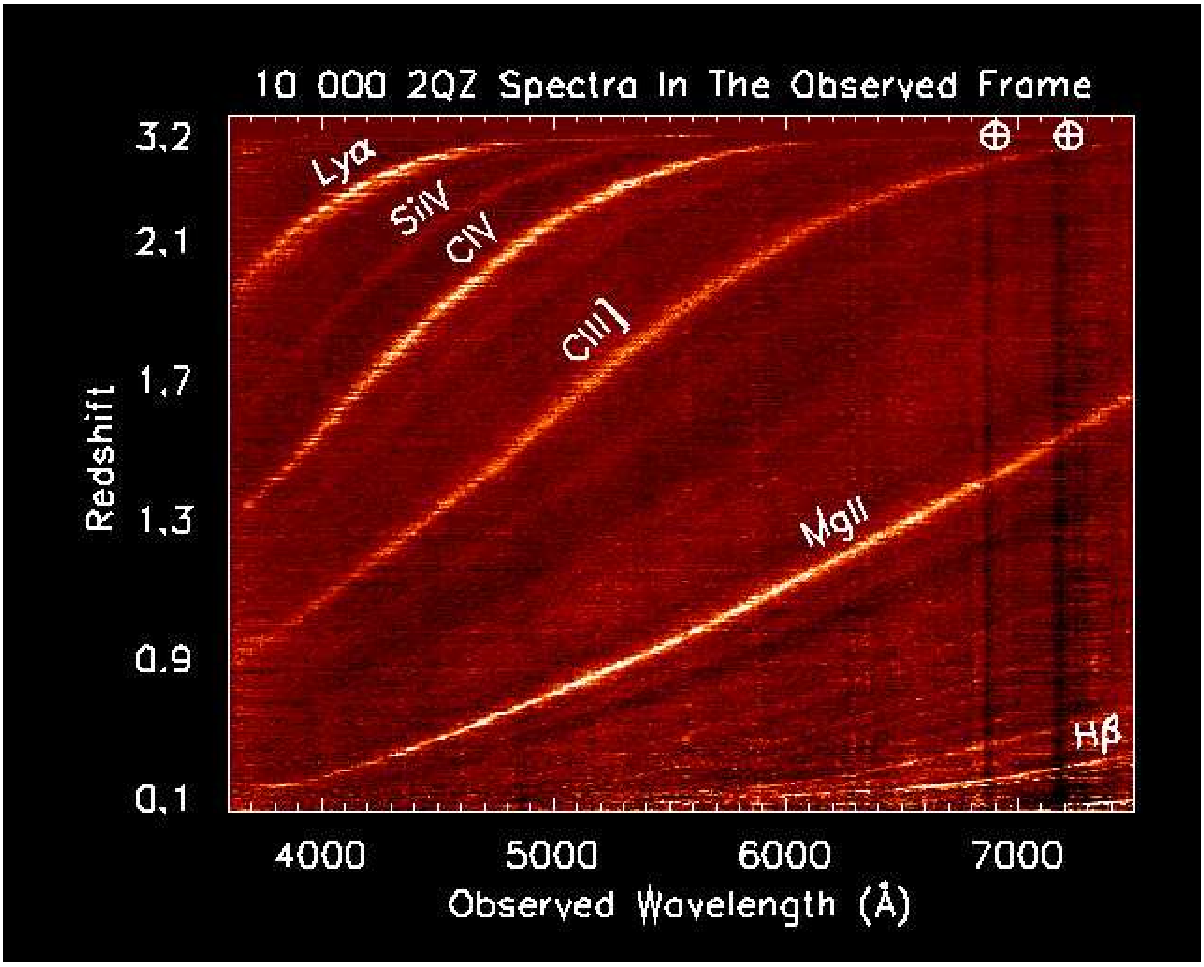
(X, Y, Z) velocity due to motion of solar system ($\sim$350 km/s towards $l = 264^\circ$, $b = 48^\circ$, Bennet et al., 1996)

H_0: *intrinsic* component of velocity due to *expansion* of the universe.

H_0: Hubble parameter

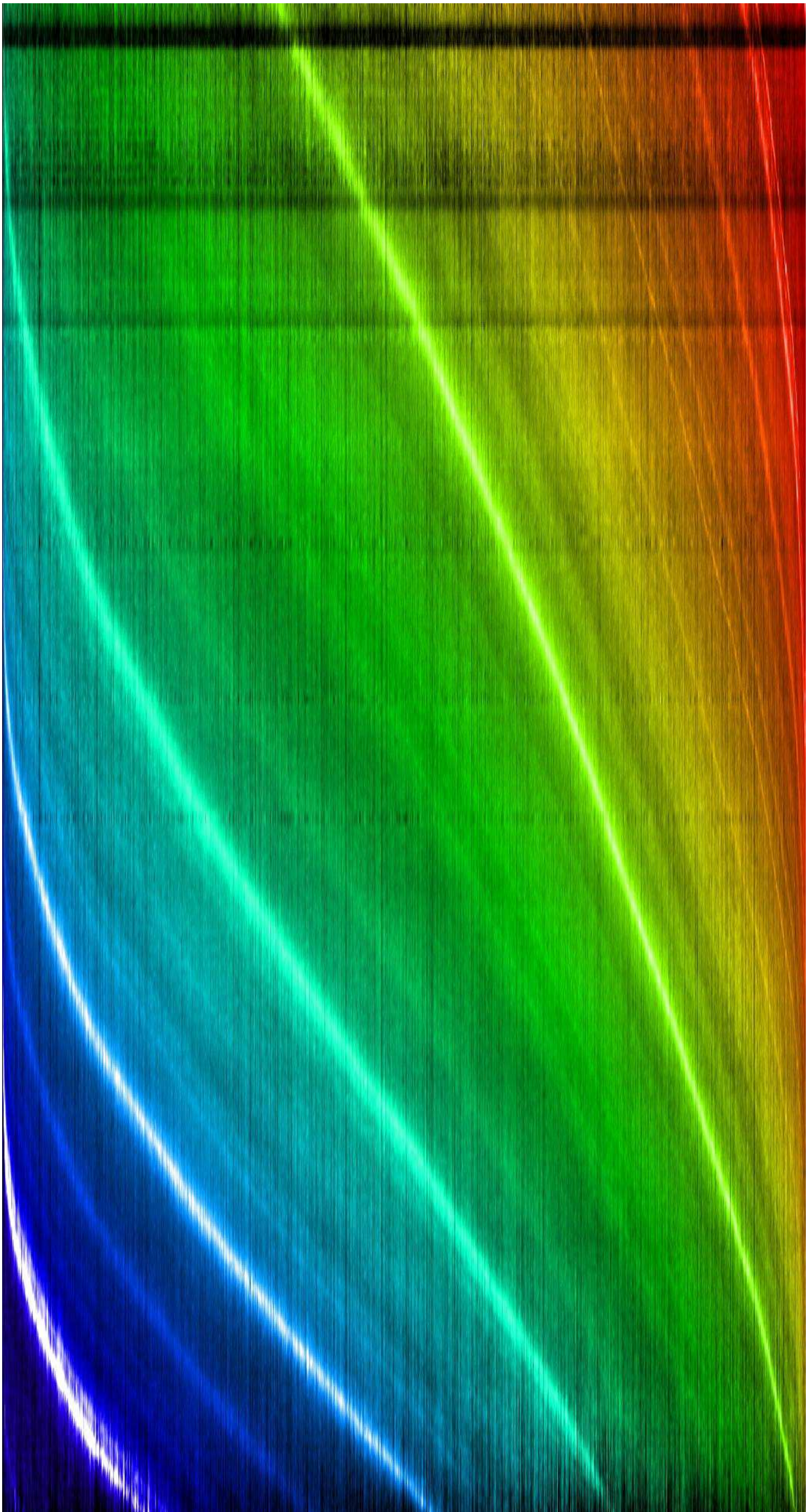
Old usage: "Hubble constant", but $H_0 \neq$ const. (cf. Eq. (4.38)).

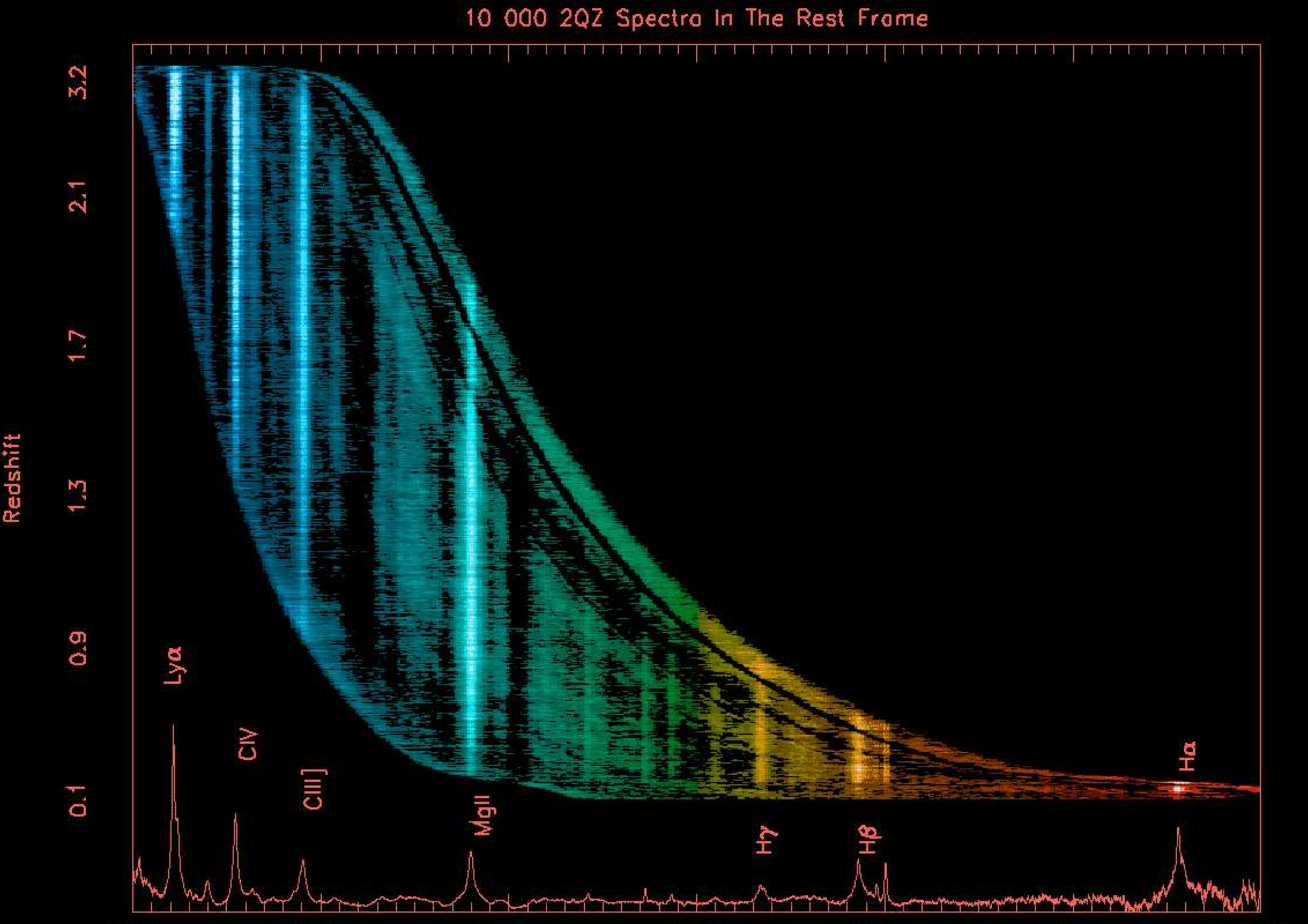
Expansion, II



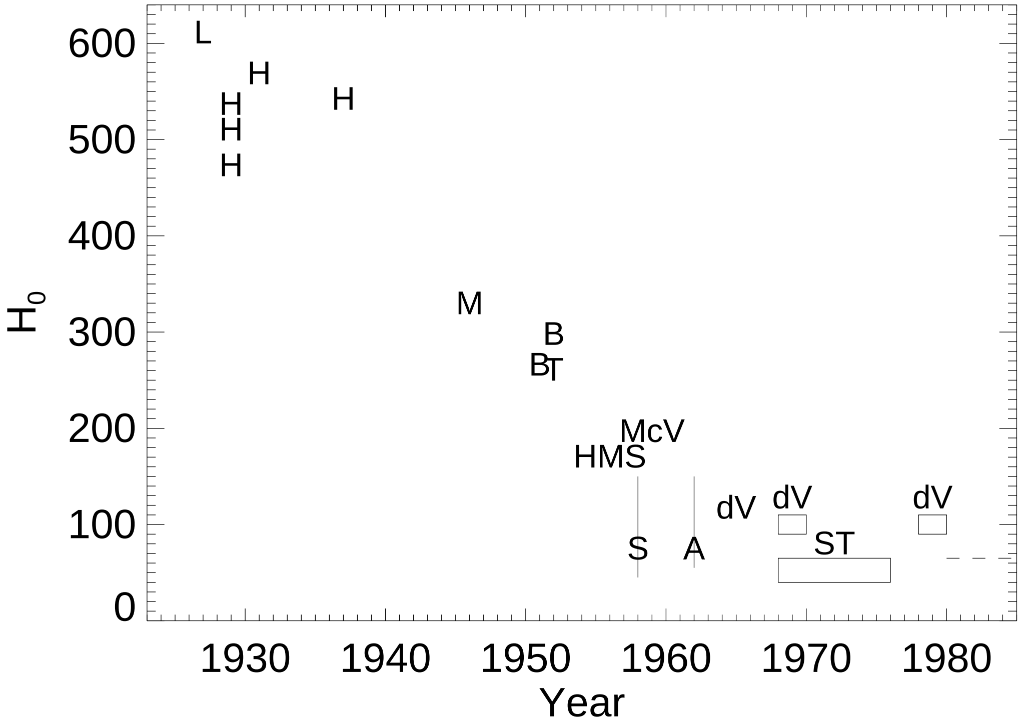
courtesy 2dF QSO Redshift survey

As a consequence of the cosmological redshift, for different z different parts of the spectrum of a distant source are visible.




10 000 2QZ Spectra In The Rest Frame
Redshift
3.2
2.1
1.7
1.3
0.9
0.1
Lyα
CIV
CIII]
MgII
Hγ
Hβ
Hα

Expansion, V



(after Trimble, 1997)

Currently accepted value: $H_0 \approx 75\,\mathrm{km/s/Mpc}$.

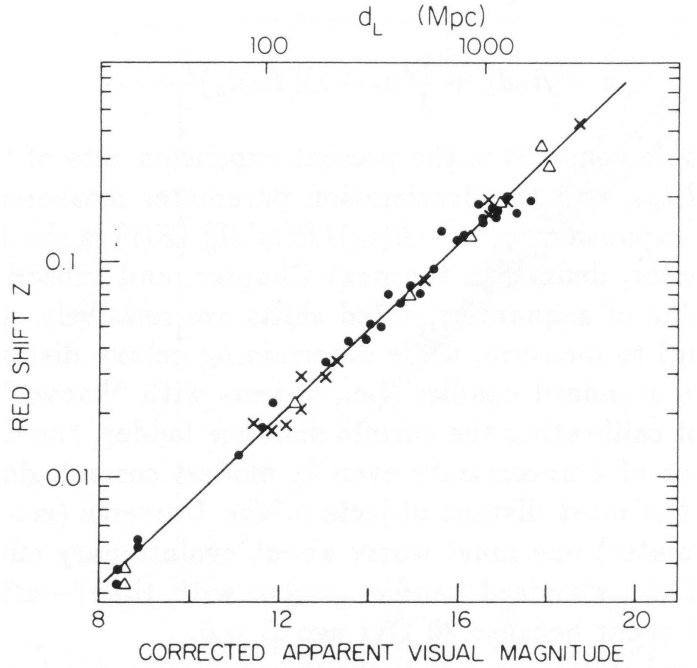
The systematic uncertainty of H_0 is $10\ldots20\,\mathrm{km/s/Mpc}$

$\Longrightarrow$ parameterize uncertainty in formulae by defining

$$\begin{aligned} H_0 &= 100\,\mathrm{km/s/Mpc} \cdot h \\ H_0 &= 75\,\mathrm{km/s/Mpc} \cdot h_{75} \end{aligned} \tag{3.2}$$

Note: H_0^{-1} has units of time: $H_0^{-1} = 9.78\,\mathrm{Gyr}/h$: Hubble-Time; for $h = 0.75$, the Hubble-Time is 13.2 Gyr

Expansion, VI



For standard candles, i.e., objects where the absolute luminosity L is known, the Hubble law can be written using observed quantities only:

Euclidean space $\Longrightarrow$ observed flux

$$f = \frac{L}{4\pi d_{\mathrm{L}}^2} \quad \Longleftrightarrow \quad d_{\mathrm{L}} = \left(\frac{L}{4\pi f}\right)^{1/2} \tag{3.3}$$

where d_{L} is the luminosity distance.
Using the Hubble law eq. (3.1)

$$H_0 d_{\mathrm{L}} = cz \quad \Longrightarrow \quad z \propto H_0 \left(\frac{L}{4\pi f}\right)^{1/2} \tag{3.4}$$

Since *magnitudes* are defined via $m \propto -2.5 \log f$:

$$\log z \propto \log H_0 + \frac{1}{2}(\log L - \log f) \tag{3.5}$$

$$\Longrightarrow \log z = a + b(m - M) \tag{3.6}$$

($m - M$: distance modulus)

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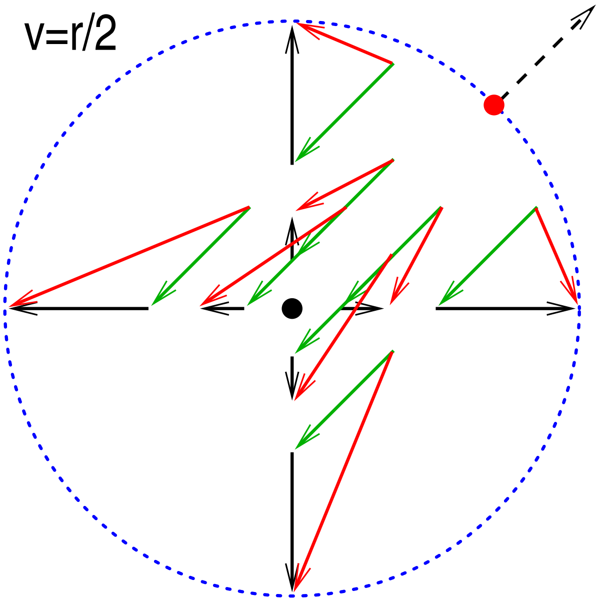
Expansion, VII



Expansion law
$\mathbf{v} = H_0\mathbf{r}$ is unchanged under rotation and translation: isomorphism.

Proof:

Rotation: Trivial.

Translation: Observations from place with position $\mathbf{r}'$ and velocity $\mathbf{v}'$: Observed distance is $\mathbf{r}_\mathrm{o} = \mathbf{r} - \mathbf{r}'$, observed velocity is $\mathbf{v}_\mathrm{o} = \mathbf{v} - \mathbf{v}'$. Because of the Hubble law,

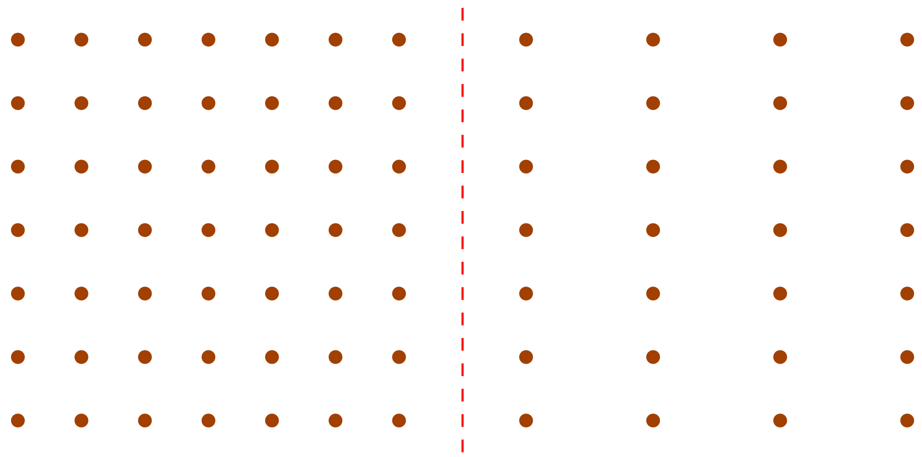
$$\mathbf{v}_\mathrm{o} = H_0\mathbf{r} - H_0\mathbf{r}' = H_0\left(\mathbf{r} - \mathbf{r}'\right) = H_0\mathbf{r}_\mathrm{o}$$

This isomorphism is a direct consequence of the homogeneity of the universe.

Despite everything receding from us, we are not at the center of the universe $\Longrightarrow$ Copernicus principle still holds.

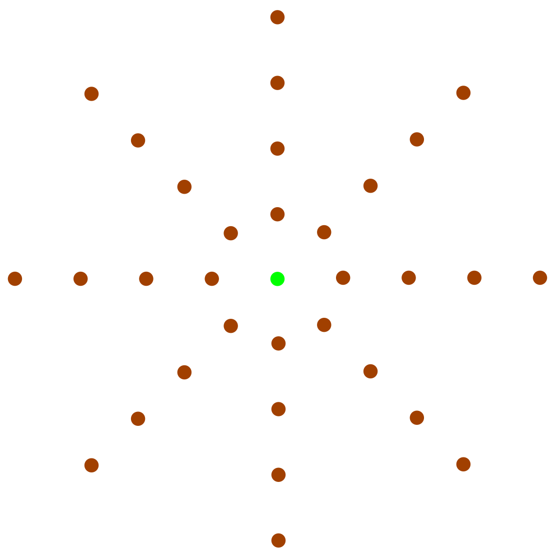
Copernicus principle: We are not at a special place in the universe in time or space.

Homogeneity and Isotropy



after Silk (1997, p. 8).

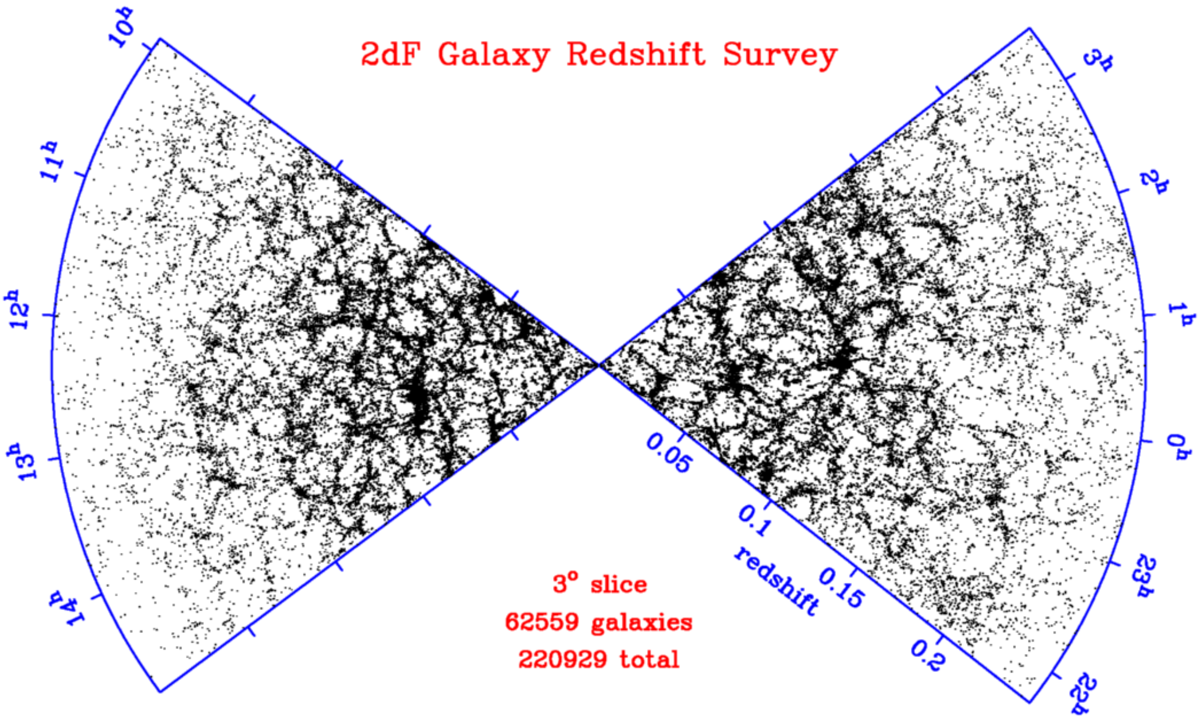
Note that homogeneity does not imply isotropy!



Neither does isotropy *around one point* imply homogeneity!

$\Longrightarrow$ *Both* assumptions need to be tested.

Homogeneity



2dF Survey, ∼220000 galaxies total

The universe is homogeneous $\Longleftrightarrow$ The universe looks the same everywhere in space

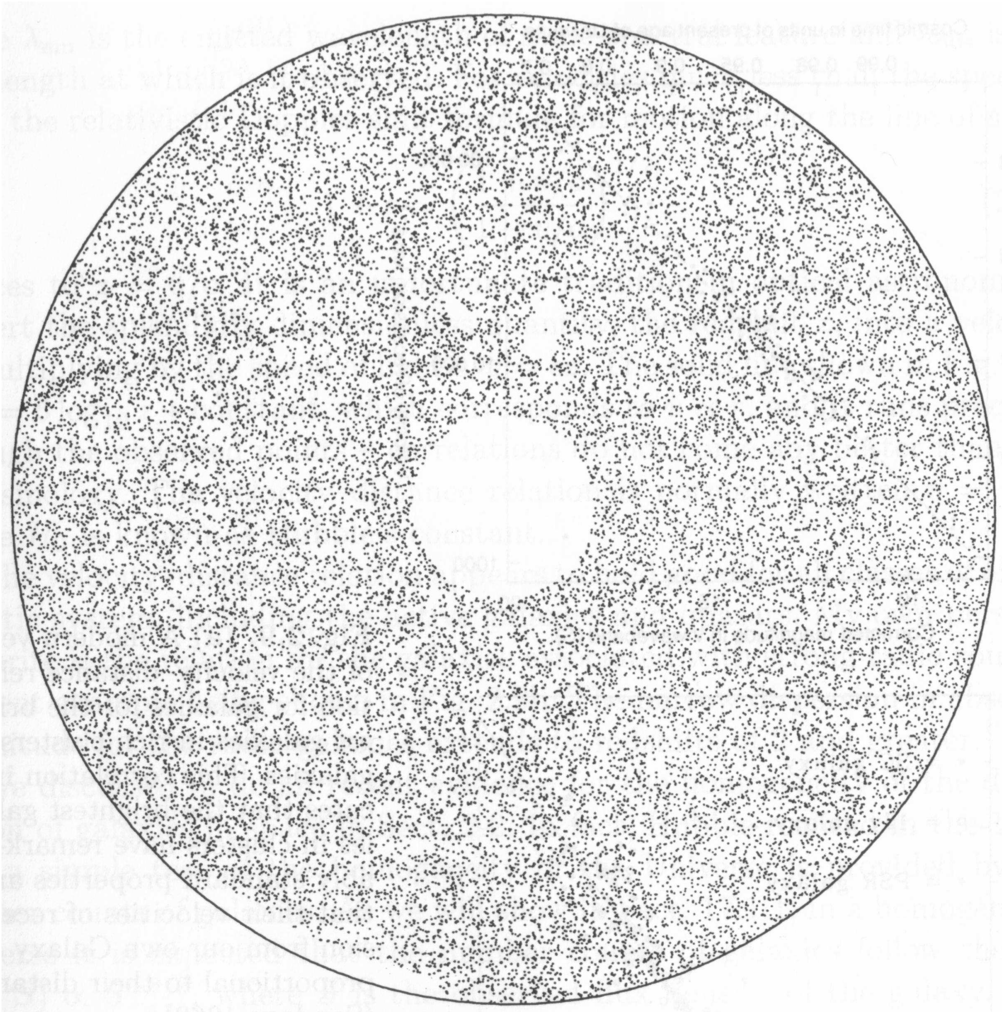
Testable by observing spatial distribution of galaxies.

On scales $\gg$ 100 Mpc the universe looks indeed the same.

Below that: structure.

Structures seen are galaxy clusters (gravitationally bound) and superclusters (larger structures, not [yet] gravitationally bound).

Isotropy



Peebles (1993): Distribution of 31000 objects at $\lambda = 6\,\mathrm{cm}$ from the Greenbank Catalogue.

The universe is isotropic $\Longleftrightarrow$ The universe looks the same in all directions

Radio galaxies are mainly quasars $\Longrightarrow$ Sample large space volume ($z \gtrsim 1$) $\Longrightarrow$ Clear isotropy.

Anisotropy in the image: galactic plane, exclusion region around Cyg A, Cas A, and the north celestial pole.

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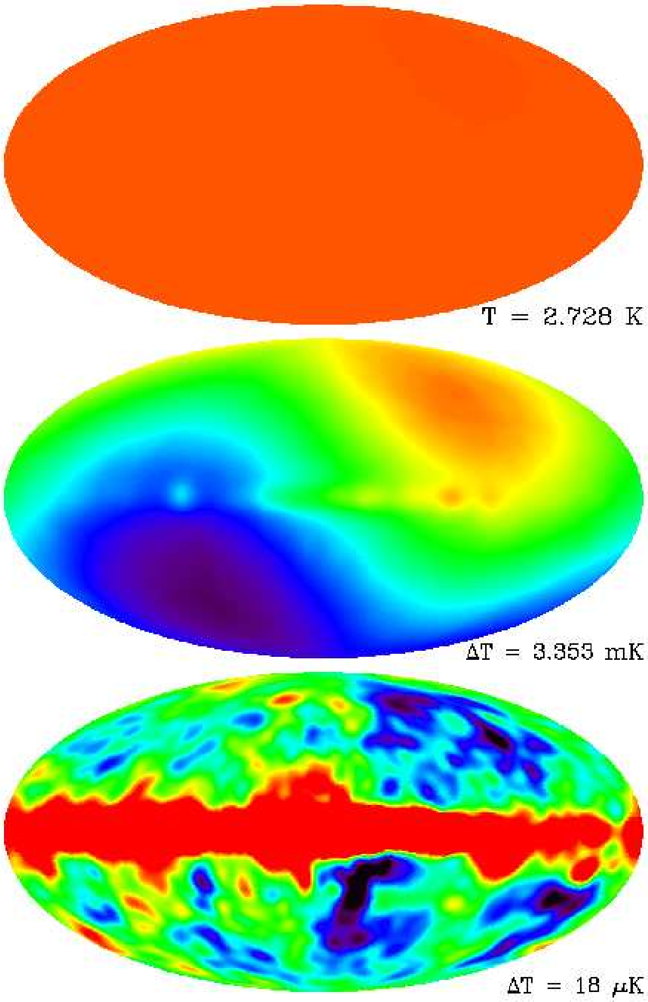
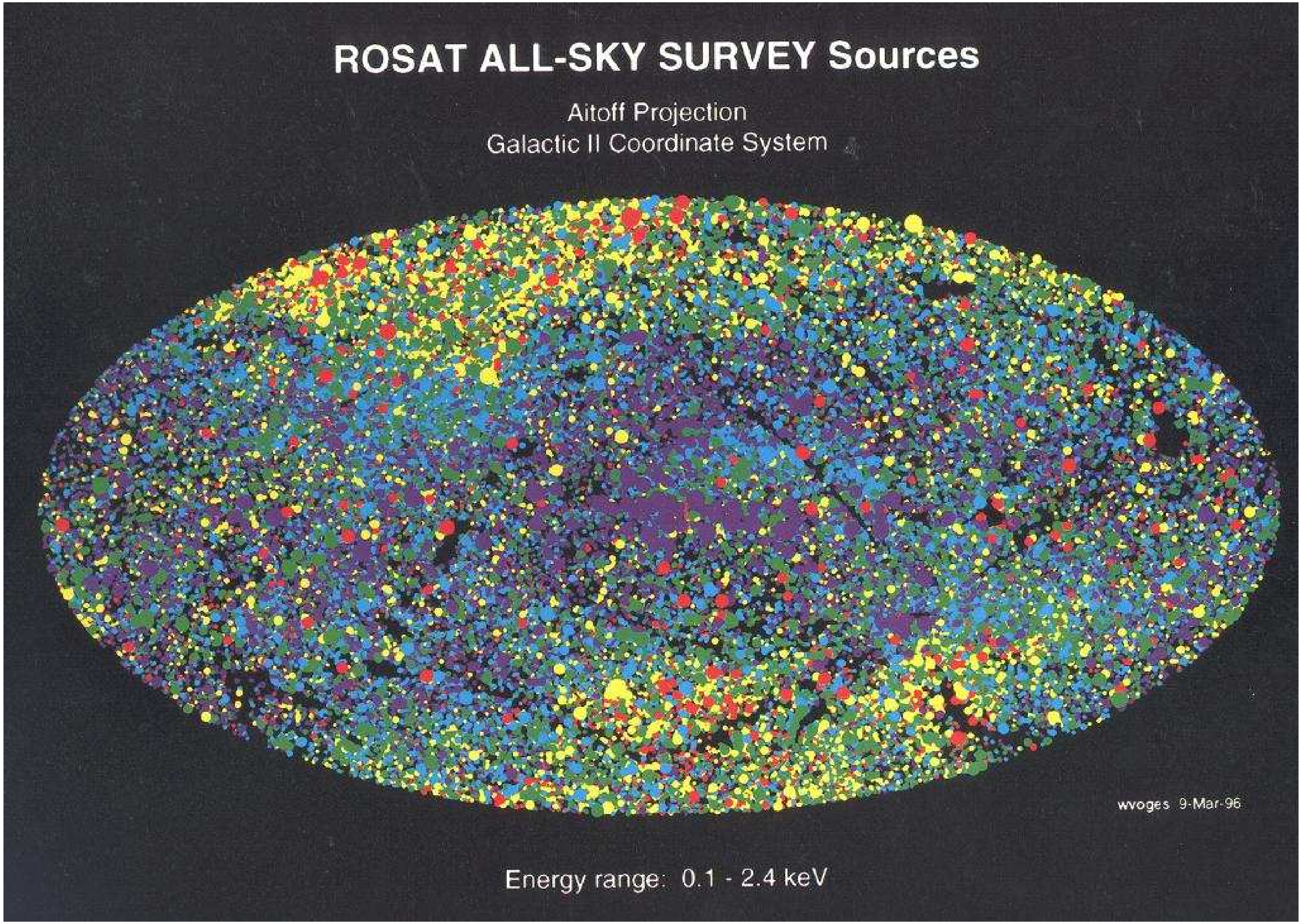
Isotropy



Best evidence for isotropy: Intensity of 3 K Cosmic Microwave Background (CMB) radiation.

First: dipole anisotropy due to motion of sun (see slide 3–3), after subtraction $\Longrightarrow$ $\Delta T/T \lesssim 10^{-4}$ on scales from $10''$ to 180°.

At level of 10^{-5}: structure in CMB due to structure of surface of last scattering of the CMB photons, i.e., structure at the time when Hydrogen recombined.



Also clear isotropy from X-ray source counts as seen in the ROSAT All Sky Survey (0.1. . . 2 keV), which mainly traces distribution of Active Galactic Nuclei (AGN) ("X-ray Background").

Bibliography

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Silk, J., 1997, A Short History of the Universe, Scientific American Library 53, (New York: W. H. Freeman)

Trimble, V., 1997, Space Sci. Rev., 79, 793

World Models

Structure

Observations: cosmological principle holds: The universe is homogeneous and isotropic.

$\Longrightarrow$ Need theoretical framework obeying the cosmological principle.

Use combination of

- General Relativity
- Thermodynamics
- Quantum Mechanics

$\Longrightarrow$ Complicated!

For 99% of the work, the above points can be dealt with separately:

1. Define metric obeying cosmological principle.
2. Obtain equation for evolution of universe using Einstein field equations.
3. Use thermo/QM to obtain equation of state.
4. Solve equations.

GRT vs. Newton

Before we can start to think about universe: Brief introduction to assumptions of general relativity.

$\Longrightarrow$ See theory lectures for the gory details, or check with the literature (Weinberg or MTW).

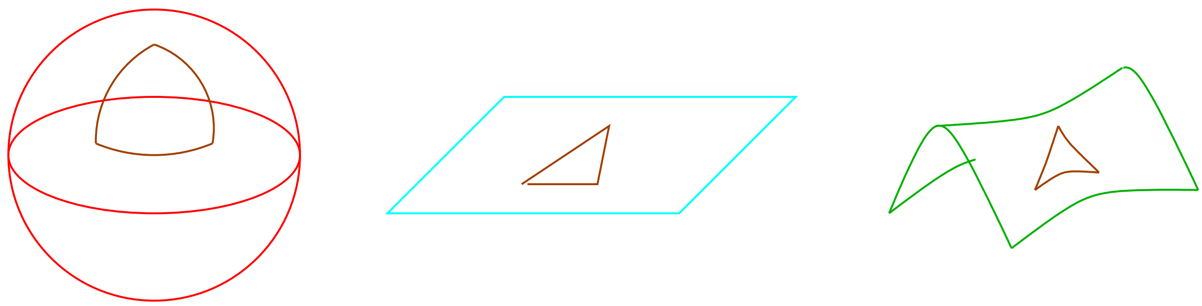
Assumptions of GRT:

- Space is 4-dimensional, might be curved
- Matter (=Energy) modifies space (Einstein field equation).
- Covariance: physical laws must be formulated in a coordinate-system independent way.
- Strong equivalence principle: There is no experiment by which one can distinguish between free falling coordinate systems and inertial systems.
- At each point, space is locally Minkowski (i.e., locally, SRT holds).

$\Longrightarrow$ Understanding of geometry of space necessary to understand physics.

2D Metrics

Before describing the 4D geometry of the universe: first look at two-dimensional spaces (easier to visualize).



After Silk (1997, p. 107)

There are three classes of isotropic and homogeneous two-dimensional spaces:

- 2-sphere ($\mathscr{S}^2$) positively curved
- x-y-plane ($\mathbb{R}^2$) zero curvature
- hyperbolic plane ($\mathscr{H}^2$) negatively curved

(curvature $\approx \sum$angles in triangle $>$, $=$, or $<$ 180°)

We will now compute what the metric for these spaces looks like.

2D Metrics

The metric describes the local geometry of a space.

Differential distance, ds, in Euclidean space, $\mathbb{R}^2$:

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 \tag{4.1}$$

The metric tensor, $g_{\mu\nu}$, is defined via

$$\mathrm{d}s^2 = \sum_\mu \sum_\nu g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu =: g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \tag{4.2}$$

(Einstein's summation convention)

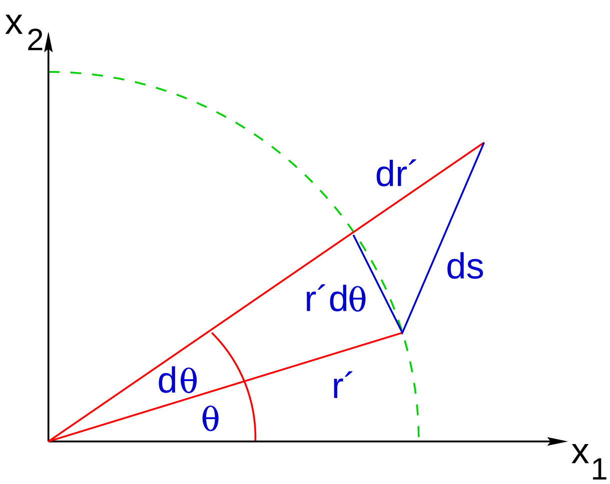
Thus, for the $\mathbb{R}^2$,

$$\begin{matrix} g_{11} = 1 & g_{12} = 0 \\ g_{21} = 0 & g_{22} = 1 \end{matrix} \tag{4.3}$$

But: Other coordinate-systems possible!

Changing to polar coordinates r', θ, defined by

$$x_1 =: r'\cos\theta \quad \text{and} \quad x_2 =: r'\sin\theta \tag{4.4}$$



it is easy to see that

$$\mathrm{d}s^2 = \mathrm{d}r'^2 + r'^2\,\mathrm{d}\theta^2 \tag{4.5}$$

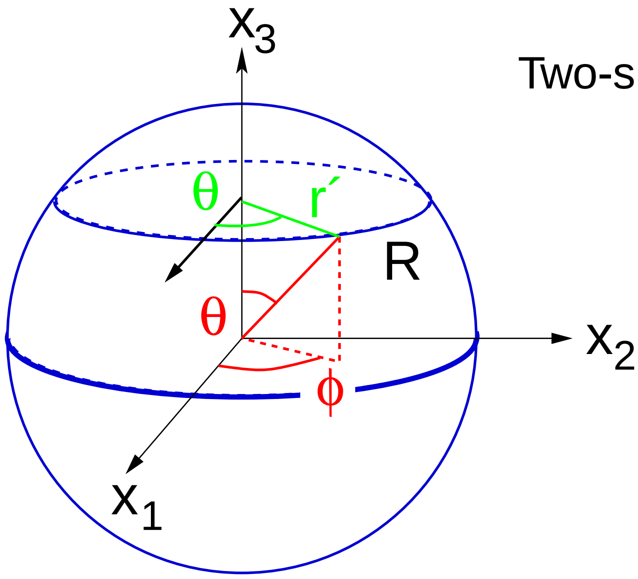
substituting $r' = Rr$,

(change of scale)

$$\mathrm{d}s^2 = R\{\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2\} \tag{4.6}$$

2D Metrics

A more complicated case occurs if space is curved. *Easiest case:* surface of three-dimensional sphere (a two-sphere).



After Kolb & Turner (1990, Fig. 2.1)

Two-sphere with radius R in $\mathbb{R}^3$:

$$x_1^2 + x_2^2 + x_3^2 = R^2 \quad (4.7)$$

Length element of $\mathbb{R}^3$:

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2$$

Eq. (4.7) gives

$$x_3 = \sqrt{R^2 - x_1^2 - x_2^2}$$

such that

$$dx_3 = \frac{\partial x_3}{\partial x_1}\,dx_1 + \frac{\partial x_3}{\partial x_2}\,dx_2 = -\frac{x_1\,dx_1 + x_2\,dx_2}{\sqrt{R^2 - x_1^2 - x_2^2}} \qquad (4.8)$$

Introduce again polar coordinates r', θ in x_3-plane:

$$x_1 =: r'\cos\theta \quad \text{and} \quad x_2 =: r'\sin\theta \qquad (4.4)$$

(note: r', θ only unique in upper or lower half-sphere)

The differentials are given by

$$\begin{aligned} dx_1 &= \cos\theta\,dr' - r'\sin\theta\,d\theta \\ dx_2 &= \sin\theta\,dr' + r'\cos\theta\,d\theta \end{aligned} \qquad (4.9)$$

2D Metrics

In cartesian coordinates, the length element on $\mathscr{S}^2$ is

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \frac{(x_1\,\mathrm{d}x_1 + x_2\,\mathrm{d}x_2)^2}{R^2 - x_1^2 - x_2^2} \tag{4.10}$$

inserting eq. (4.9) gives after some algebra

$$= r'^2\,\mathrm{d}\theta^2 + \frac{R^2}{R^2 - r'^2}\,\mathrm{d}r'^2 \tag{4.11}$$

finally, defining $r = r'/R$ (i.e., $0 \leq r \leq 1$) results in

$$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1 - r^2} + r^2\,\mathrm{d}\theta^2 \right\} \tag{4.12}$$

Alternatively, we can work in spherical coordinates on $\mathscr{S}^2$

$$\begin{aligned} x_1 &= R \sin\theta \cos\phi \\ x_2 &= R \sin\theta \sin\phi \\ x_3 &= R \cos\theta \end{aligned} \tag{4.13}$$

($\theta \in [0, \pi]$, $\phi \in [0, 2\pi]$).
Going through the same steps as before, we obtain after some tedious algebra

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 \right\} \tag{4.14}$$

2D Metrics

(Important) remarks:

1. The 2-sphere has no edges, has no boundaries, but has still a finite volume, $V = 4\pi R^2$.
2. Expansion or contraction of sphere caused by variation of $R \Longrightarrow R$ determines the *scale* of volumes and distances on $\mathscr{S}^2$.

   R is called the *scale factor*

3. Positions on $\mathscr{S}^2$ are defined, e.g., by r and θ, *independent* on the value of R

   r and θ are called *comoving coordinates*

4. Although the metrics Eq. (4.10), (4.12), and (4.14) look *very* different, they still describe the *same* space $\Longrightarrow$ that's why physics should be covariant.

2D Metrics

The hyperbolic plane, $\mathscr{H}^2$, is defined by

$$x_1^2 + x_2^2 - x_3^2 = -R^2 \tag{4.15}$$

If we work in Minkowski space, where

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 - \mathrm{d}x_3^2 \tag{4.16}$$

then

$$= \mathrm{d}x_1^2 + \mathrm{d}x_2^2 - \frac{(x_1\,\mathrm{d}x_1 + x_2\,\mathrm{d}x_2)^2}{R^2 + x_1^2 + x_2^2} \tag{4.17}$$

$\Longrightarrow$ substitute $R \to iR$ (where $i = \sqrt{-1}$) to obtain same form as for sphere (eq. 4.11)!

Therefore,

$$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1 + r^2} + r^2\,\mathrm{d}\theta^2 \right\} \tag{4.18}$$

2D Metrics

The analogy to spherical coordinates on the hyperbolic plane are given by

$$\begin{aligned} x_1 &= R \sinh\theta \cos\phi \\ x_2 &= R \sinh\theta \sin\phi \\ x_3 &= R \cosh\theta \end{aligned} \tag{4.19}$$

($\theta \in [-\infty, +\infty]$, $\phi \in [0, 2\pi]$).

A session with Maple (see handout) will convince you that these coordinates give

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \sinh^2\theta \, \mathrm{d}\phi^2 \right\} \tag{4.20}$$

Remark:
$\mathscr{H}^2$ is unbound and has an infinite volume.

Transcript of Maple session to obtain Eq. (4.20):

```
> x1:=r*sinh(theta)*cos(phi);
```

$$x1 := r \sinh(\theta) \cos(\phi)$$

```
> x2:=r*sinh(theta)*sin(phi);
```

$$x2 := r \sinh(\theta) \sin(\phi)$$

```
> x3:=r*cosh(theta);
```

$$x3 := r \cosh(\theta)$$

```
> dx1:=diff(x1,theta)*dtheta+diff(x1,phi)*dphi;
```

$$dx1 := r \cosh(\theta) \cos(\phi)\, dtheta - r \sinh(\theta) \sin(\phi)\, dphi$$

```
> dx2:=diff(x2,theta)*dtheta+diff(x2,phi)*dphi;
```

$$dx2 := r \cosh(\theta) \sin(\phi)\, dtheta + r \sinh(\theta) \cos(\phi)\, dphi$$

```
> ds2:=dx1*dx1+dx2*dx2-(x1*dx1+x2*dx2)^2/(r^2+x 1^2+x2^2);
```

$$ds2 := (r \cosh(\theta) \cos(\phi)\, dtheta - r \sinh(\theta) \sin(\phi)\, dphi)^2 + (r \cosh(\theta) \sin(\phi)\, dtheta + r \sinh(\theta) \cos(\phi)\, dphi)^2 - (r \sinh(\theta) \cos(\phi)\, (r \cosh(\theta) \cos(\phi)\, dtheta - r \sinh(\theta) \sin(\phi)\, dphi) + r \sinh(\theta) \sin(\phi)\, (r \cosh(\theta) \sin(\phi)\, dtheta + r \sinh(\theta) \cos(\phi)\, dphi))^2 \Big/ (r^2 + r^2 \sinh(\theta)^2 \cos(\phi)^2 + r^2 \sinh(\theta)^2 \sin(\phi)^2)$$

```
> expand(ds2);
```

$$r^2 \cosh(\theta)^2 \cos(\phi)^2\, dtheta^2 + r^2 \sinh(\theta)^2 \sin(\phi)^2\, dphi^2 + r^2 \cosh(\theta)^2 \sin(\phi)^2\, dtheta^2 + r^2 \sinh(\theta)^2 \cos(\phi)^2\, dphi^2 - \frac{r^4 \sinh(\theta)^2 \cos(\phi)^4 \cosh(\theta)^2\, dtheta^2}{\%1} - 2\, \frac{r^4 \sinh(\theta)^2 \cos(\phi)^2 \cosh(\theta)^2\, dtheta^2 \sin(\phi)^2}{\%1} - \frac{r^4 \sinh(\theta)^2 \sin(\phi)^4 \cosh(\theta)^2\, dtheta^2}{\%1}$$

$$\%1 := r^2 + r^2 \sinh(\theta)^2 \cos(\phi)^2 + r^2 \sinh(\theta)^2 \sin(\phi)^2$$

```
> simplify(",{cosh(theta)^2-sinh(theta)^2=1}, [cosh(theta)]);
```

$$r^2\, dtheta^2 + r^2 \sinh(\theta)^2\, dphi^2$$

2D Metrics

To summarize:

Sphere:

$$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1 - r^2} + r^2 \, \mathrm{d}\theta^2 \right\} \tag{4.12}$$

Plane:

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}r^2 + r^2 \, \mathrm{d}\theta^2 \right\} \tag{4.6}$$

Hyperbolic Plane:

$$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1 + r^2} + r^2 \, \mathrm{d}\theta^2 \right\} \tag{4.18}$$

$\Longrightarrow$ All three metrics can be written as

$$\mathrm{d}s^2 = R^2 \left\{ \frac{\mathrm{d}r^2}{1 - k\, r^2} + r^2 \, \mathrm{d}\theta^2 \right\} \tag{4.21}$$

where k defines the geometry:

$$k = \begin{cases} +1 & \text{spherical} \\ 0 & \text{planar} \\ -1 & \text{hyperbolic} \end{cases} \tag{4.22}$$

2D Metrics

For “spherical coordinates” we found:

Sphere:

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2 \right\} \tag{4.14}$$

Plane:

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \theta^2 \mathrm{d}\phi^2 \right\} \tag{4.6}$$

Hyperbolic plane:

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + \sinh^2\theta\, \mathrm{d}\phi^2 \right\} \tag{4.20}$$

$\Longrightarrow$ All three metrics can be written as

$$\mathrm{d}s^2 = R^2 \left\{ \mathrm{d}\theta^2 + S_k^2(\theta)\, \mathrm{d}\phi^2 \right\} \tag{4.23}$$

where

$$S_k(\theta) = \begin{cases} \sin\theta & \text{for } k = +1 \\ \theta & \text{for } k = 0 \\ \sinh\theta & \text{for } k = -1 \end{cases} \tag{4.24}$$

We will also need the $\cos$-like analogue

$$C_k(\theta) = \sqrt{1 - kS_k^2(\theta)} = \begin{cases} \cos\theta & \text{for } k = +1 \\ 1 & \text{for } k = 0 \\ \cosh\theta & \text{for } k = -1 \end{cases} \tag{4.25}$$

Note that, compared to the earlier formulae, some coordinates have been renamed. This is confusing, but legal. . .

RW Metric

- Cosmological principle + expansion $\Longrightarrow$ $\exists$ freely expanding cosmical coordinate system.
  - Observers $=:$ fundamental observers
  - Time $=:$ cosmic time

  This is the coordinate system in which the 3K radiation is isotropic, clocks can be synchronized, e.g., by adjusting time to the local density of the universe.

  $\Longrightarrow$ Metric has temporal and spatial part.

  This also follows directly from the equivalence principle.

- *Homogeneity and isotropy* $\Longrightarrow$ spatial part is spherically symmetric:

$$\mathrm{d}\psi^2 := \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 \tag{4.26}$$

- *Expansion:* $\exists$ scale factor, $R(t)$ $\Longrightarrow$ measure distances using comoving coordinates.

$\Longrightarrow$ metric looks like

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\left[f^2(r)\,\mathrm{d}r^2 + g^2(r)\,\mathrm{d}\psi^2\right] \tag{4.27}$$

where $f(r)$ and $g(r)$ are arbitrary.

RW Metric

Metrics of the form of eq. (4.27) are called Robertson-Walker (RW) metrics (1935).

Previously studied by Friedmann and Lemaître...

One common choice is

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\left[\mathrm{d}r^2 + S_k^2(r)\,\mathrm{d}\psi^2\right] \qquad (4.28)$$

where

$R(t)$**:** scale factor, containing the physics

t**:** cosmic time

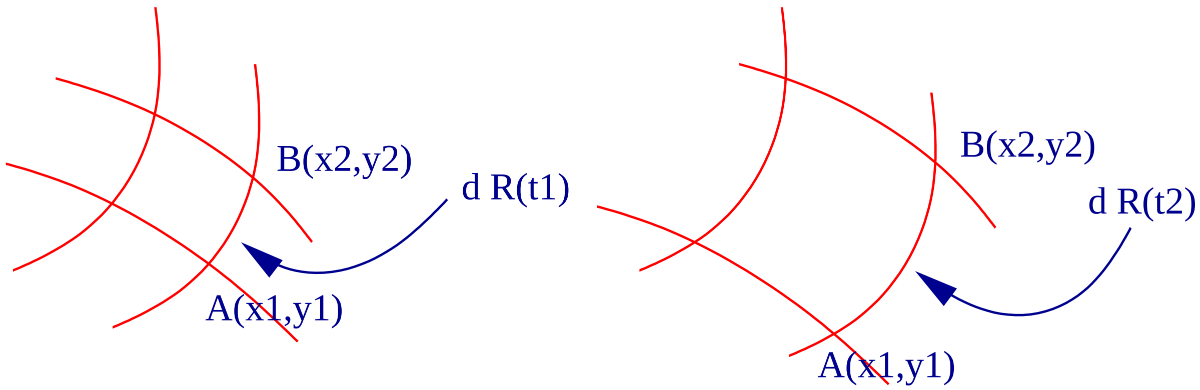
r**,** θ**,** ϕ**:** comoving coordinates

$S_k(r)$ was defined in Eq. (4.24).

Remark: θ and ϕ describe directions on sky, as seen from the arbitrary center of the coordinate system (=us), r can be interpreted as a radial coordinate.

RW Metric

The RW metric defines an universal coordinate system tied to expansion of space:



Scale factor $R(t)$ describes evolution of universe.

- d is called the comoving distance.
- $D(t) := d \cdot R(t)$ is called the proper distance,

(note that R is unitless, i.e., d and $dR(t)$ are measured in Mpc)

"World model": $R(t)$ from GRT *plus* assumptions about physics.

RW Metric

Other forms of the RW metric are also used:

1. Substitution $S_k(r) \longrightarrow r$ gives

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - R^2(t)\left\{\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\psi^2\right\} \tag{4.29}$$

   (i.e., other definition of comoving radius r).

2. A metric with a dimensionless scale factor,

$$a(t) := \frac{R(t)}{R(t_0)} = \frac{R(t)}{R_0} \tag{4.30}$$

   (where t_0=today, i.e., $a(t_0) = 1$), gives

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - a^2(t)\left\{\mathrm{d}r^2 + \frac{S_k^2(R_0 r)}{R_0^2}\,\mathrm{d}\psi^2\right\} \tag{4.31}$$

3. Using $a(t)$ *and* the substitution $S_k(r) \longrightarrow r$ is also possible:

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - a^2(t)\left\{\frac{\mathrm{d}r^2}{1-k\cdot(R_0 r)^2} + r^2\,\mathrm{d}\psi^2\right\} \tag{4.32}$$

   The units of $R_0 r$ are Mpc $\Longrightarrow$ *Used for observations!*

RW Metric

4. Replace cosmic time, t, by conformal time,
   $\mathrm{d}\eta = \mathrm{d}t/R(t) \Longrightarrow$ conformal metric,

$$\mathrm{d}s^2 = R^2(\eta)\left\{\mathrm{d}\eta^2 - \frac{\mathrm{d}r^2}{1-kr} - r^2\,\mathrm{d}\psi^2\right\} \tag{4.33}$$

   Theoretical importance of this metric: For $k=0$, i.e., a flat space, the RW metric $=$ Minkowski line element $\times$ $R^2(\eta) \Longrightarrow$ Equivalence principle!

5. Finally, the metric can also be written in the isotropic form,

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \frac{R(t)}{1+(k/4)r^2}\left\{\mathrm{d}r^2 + r^2\mathrm{d}\psi^2\right\} \tag{4.34}$$

   Here, the term in $\{\ldots\}$ is just the line element of a 3d-sphere $\Longrightarrow$ isotropy!

Note: There are as many notations as authors, e.g., some use $a(t)$ where we use $R(t)$, etc. $\Longrightarrow$ Be careful!

Note 2: Local homogeneity and isotropy (i.e., within a Hubble radius, $r = c/H_0$), do not imply *global* homogeneity and isotropy $\Longrightarrow$ Cosmologies with a non-trivial topology are possible (e.g., also with more dimensions...).

Hubble's Law

Hubble's Law follows from the variation of $R(t)$:

Small scales $\Longrightarrow$ Euclidean geometry

Proper distance between two observers:

$$D(t) = d \cdot R(t) \tag{4.35}$$

where d: comoving distance.

Expansion $\Longrightarrow$ proper separation changes:

$$\frac{\Delta D}{\Delta t} = \frac{R(t+\Delta t)d - R(t)d}{\Delta t} \tag{4.36}$$

Thus, for $\Delta t \to 0$,

$$v = \frac{\mathrm{d}D}{\mathrm{d}t} = \dot{R}\, d = \frac{\dot{R}}{R} D =: H\, D \tag{4.37}$$

$\Longrightarrow$ Identify local Hubble "constant" as

$$H = \frac{\dot{R}}{R} = \dot{a}(t) \tag{4.38}$$

($a(t)$ from Eq. 4.30, $a(\text{today}) = 1$)

Since $R = R(t) \Longrightarrow H$ is time-dependent!

For small v, interpreted classically the red-shift is

$$z = 1 + \frac{v}{c} \quad \Longrightarrow \quad z - 1 = \frac{Hd}{c} \tag{4.39}$$

Redshift, I

The cosmological redshift is a consequence of the expansion of the universe:

The comoving distance is constant, thus in terms of the proper distance:

$$d = \frac{D(t = \text{today})}{R(t = \text{today})} = \frac{D(t)}{R(t)} = \text{const.} \tag{4.40}$$

Set $a(t) = R(t)/R(t = \text{today})$, then eq. (4.40) implies

$$\lambda_{\text{obs}} = \frac{\lambda_{\text{emit}}}{a_{\text{emit}}} \tag{4.41}$$

(λ_{obs}: observed wavelength, λ_{emit}: emitted wavelength)

Thus the observed redshift is

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} - 1 \tag{4.42}$$

or

$$\boxed{1 + z = \frac{1}{a_{\text{emit}}} = \frac{R(t = \text{today})}{R(t)}} \tag{4.43}$$

Light emitted at $z = 1$ was emitted when the universe was half as big as today!

z: measure for *relative size* of universe at time the observed light was emitted.

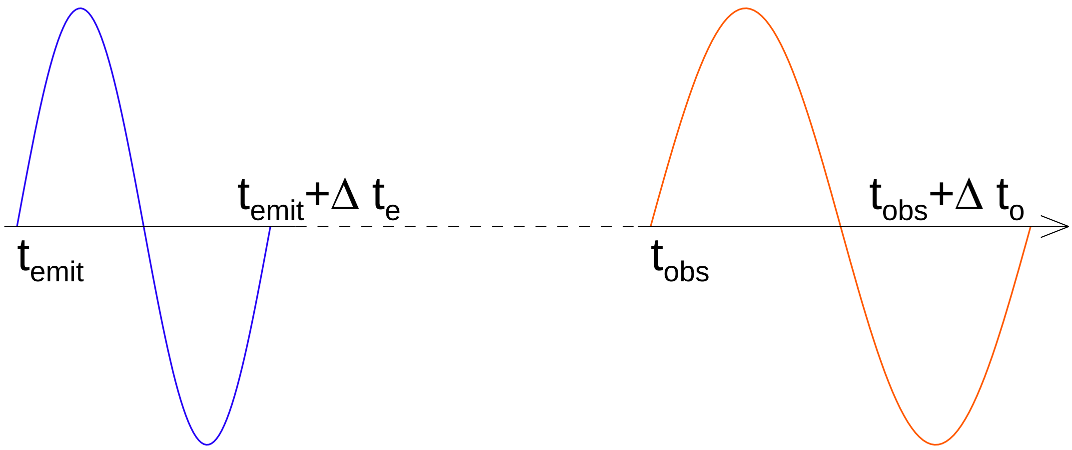
Because of $z = \nu_{\text{emit}}/\nu_{\text{obs}}$,

$$\frac{\nu_{\text{emit}}}{\nu_{\text{obs}}} = \frac{1}{a_{\text{emit}}} \tag{4.44}$$

An alternative derivation of the cosmological redshift follows directly from general relativity, using the basic GR fact that for photons $\mathrm{d}s^2 = 0$. Inserting this into the metric, and assuming without loss of generality that $\mathrm{d}\psi^2 = 0$, one finds

$$0 = c^2\,\mathrm{d}t^2 - R^2(t)\,\mathrm{d}r^2 \quad \Longrightarrow \quad \mathrm{d}r = \pm\frac{c\,\mathrm{d}t}{R(t)} \tag{4.45}$$

Since photons travel forward, we choose the $+$-sign.



The *comoving* distance traveled by photons emitted at cosmic times $t_{\rm emit}$ and $t_{\rm emit} + \Delta t_{\rm e}$ is

$$r_1 = \int_{t_{\rm emit}}^{t_{\rm obs}} \frac{c\,\mathrm{d}t}{R(t)} \quad \text{and} \quad r_2 = \int_{t_{\rm emit}+\Delta t_{\rm e}}^{t_{\rm obs}+\Delta t_{\rm o}} \frac{c\,\mathrm{d}t}{R(t)} \tag{4.46}$$

But the comoving distances are equal, $r_1 = r_2$! Therefore

$$0 = \int_{t_{\rm emit}}^{t_{\rm obs}} \frac{c\,\mathrm{d}t}{R(t)} - \int_{t_{\rm emit}+\Delta t_{\rm e}}^{t_{\rm obs}+\Delta t_{\rm o}} \frac{c\,\mathrm{d}t}{R(t)} \tag{4.47}$$

$$= \int_{t_{\rm emit}}^{t_{\rm emit}+\Delta t_{\rm e}} \frac{c\,\mathrm{d}t}{R(t)} - \int_{t_{\rm obs}}^{t_{\rm obs}+\Delta t_{\rm o}} \frac{c\,\mathrm{d}t}{R(t)} \tag{4.48}$$

If Δt small $\Longrightarrow R(t) \approx$ const.:

$$= \frac{c\,\Delta t_{\rm e}}{R(t_{\rm emit})} - \frac{c\,\Delta t_{\rm o}}{R(t_{\rm obs})} \tag{4.49}$$

For a wave: $c\Delta t = \lambda$, such that

$$\frac{\lambda_{\rm emit}}{R(t_{\rm emit})} = \frac{\lambda_{\rm obs}}{R(t_{\rm obs})} \quad \Longleftrightarrow \quad \frac{\lambda_{\rm emit}}{\lambda_{\rm obs}} = \frac{R(t_{\rm emit})}{R(t_{\rm obs})} \tag{4.50}$$

From this equation it is straightforward to derive Eq. (4.42).

Redshift, II

Outside of the local universe: Eq. (4.43) only valid interpretation of z.

$\Longrightarrow$ It is common to interpret z as in special relativity:

$$1 + z = \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{4.51}$$

This is WRONG

Redshift is due to expansion of space, not due to motion of galaxy.

What *is* true is that z is accumulation of many infinitesimal red-shifts à la Eq. (4.39), see, e.g., Peacock (1999).

Time Dilatation

Note the implication of Eq. (4.49) on the hand-out:

$$\frac{c\,\Delta t_{\mathrm{e}}}{R(t_{\mathrm{emit}})} = \frac{c\,\Delta t_{\mathrm{o}}}{R(t_{\mathrm{obs}})} \tag{4.49}$$

$\Longrightarrow$ dt/R is constant:

$$\frac{\mathrm{d}t}{R(t)} = \text{const.} \tag{4.52}$$

In other words:

$$\frac{\mathrm{d}t_{\mathrm{obs}}}{\mathrm{d}t_{\mathrm{emit}}} = \frac{R(t_{\mathrm{obs}})}{R(t_{\mathrm{emit}})} = 1 + z \tag{4.53}$$

$\Longrightarrow$ Time dilatation of events at large z.

This cosmological time dilatation has been observed in the light curves of supernova outbursts.

All other observables apart from z (e.g., number density $N(z)$, luminosity distance d_{L}, etc.) require explicit knowledge of $R(t)$ $\Longrightarrow$ Need to look at the dynamics of the universe.

Friedmann Equations, I

General relativistic approach: Insert metric into Einstein equation to obtain differential equation for $R(t)$:

Einstein equation:

$$\underbrace{R_{\mu\nu} - \frac{1}{2}\mathscr{R}g_{\mu\nu}}_{G_{\mu\nu}} = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{4.54}$$

where

$g_{\mu\nu}$: Metric tensor ($\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu$)

$R_{\mu\nu}$: Ricci tensor (function of $g_{\mu\nu}$)

$\mathscr{R}$: Ricci scalar (function of $g_{\mu\nu}$)

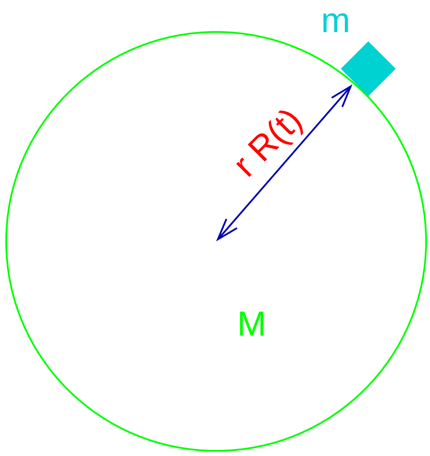
$G_{\mu\nu}$: Einstein tensor (function of $g_{\mu\nu}$)

$T_{\mu\nu}$: Stress-energy tensor, describing curvature of space due to fields present (matter, radiation,...)

Λ: Cosmological constant

$\Longrightarrow$ Messy, but doable

Friedmann Equations, II



Here, Newtonian derivation of Friedmann equations: Dynamics of a mass element on the surface of sphere of density $\rho(t)$ and comoving radius d, i.e., proper radius $d \cdot R(t)$ (after McCrea & Milne, 1934).

Mass of sphere:

$$M = \frac{4\pi}{3}(d\,R)^3\rho(t) = \frac{4\pi}{3}d^3\rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \tag{4.55}$$

Force on mass element:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}(d\,R(t)) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3}\frac{d\rho_0}{R^2(t)}m \tag{4.56}$$

Canceling $m \cdot d$ gives momentum equation:

$$\ddot{R} = -\frac{4\pi G}{3}\frac{\rho_0}{R^2} = -\frac{4\pi G}{3}\rho(t)R(t) \tag{4.57}$$

From energy conservation, or from multiplying Eq. (4.57) with $\dot{R}$ and integrating, we obtain the energy equation,

$$\begin{aligned} \frac{1}{2}\dot{R}^2 &= +\frac{4\pi G}{3}\frac{\rho_0}{R(t)} + \text{const.} \\ &= +\frac{4\pi G}{3}\rho(t)R^2(t) + \text{const.} \end{aligned} \tag{4.58}$$

where the constant can only be obtained from GR.

Friedmann Equations, III

Problems with the Newtonian derivation:

1. Cloud is implicitly assumed to have $r_{\text{cloud}} < \infty$ (for $r_{\text{cloud}} \rightarrow \infty$ the force is undefined)
   $\Longrightarrow$ violates cosmological principle.
2. Particles move *through* space
   $\Longrightarrow v > c$ possible
   $\Longrightarrow$ violates SRT.

Why do we get correct result?

GRT $\longrightarrow$ Newton for small scales and mass densities; since universe is isotropic $\Longrightarrow$ scale invariance on Mpc scales $\Longrightarrow$ Newton sufficient (classical limit of GR).

(In fact, point 1 above *does* hold in GR: Birkhoff's theorem).

UWarwick

Friedmann Equations, IV

The exact GR derivation of Friedmanns equation gives:

$$\ddot{R} = -\frac{4\pi G}{3}R\left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right]$$
$$\dot{R}^2 = +\frac{8\pi G\rho}{3}R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right] \tag{4.59}$$

Notes:

1. For $k = 0$: Eq. (4.59) $\longrightarrow$ Eq. (4.58).
2. $k \in \{-1, 0, +1\}$ determines the curvature of space.
3. The density, ρ, includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. There is energy associated with the vacuum, parameterized by the parameter Λ.

The evolution of the Hubble parameter is ($\Lambda = 0$):

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2(t) = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} \tag{4.60}$$

The Critical Density, I

Solving Eq. (4.60) for k:

$$\frac{R^2}{c}\left(\frac{8\pi G}{3}\rho - H^2\right) = k \qquad (4.61)$$

$\Longrightarrow$ Sign of curvature parameter k only depends on density, ρ:

Defining

$$\rho_c = \frac{3H^2}{8\pi G} \quad \text{and} \quad \Omega = \frac{\rho}{\rho_c} \qquad (4.62)$$

it is easy to see that:

$$\begin{aligned} \Omega > 1 &\Longrightarrow k > 0 \quad \text{closed} \\ \Omega = 1 &\Longrightarrow k = 0 \quad \text{flat} \\ \Omega < 1 &\Longrightarrow k < 0 \quad \text{open} \end{aligned}$$

thus ρ_c is called the critical density.

For $\Omega \leq 1$ the universe will expand until ∞,
for $\Omega > 1$ we will see the “big crunch”.

Current value of ρ_c: $\sim 1.67 \times 10^{-24}\,\mathrm{g/cm^3}$,
(3...10 H-atoms/m^3).
Measured: $\Omega = 0.1 \ldots 0.3$.

(but note that Λ can influence things ($\Omega_\Lambda = 0.7$)!).

The Critical Density, II

Ω has a second order effect on the expansion:

Taylor series of $R(t)$ around $t = t_0$:

$$\frac{R(t)}{R(t_0)} = \frac{R(t_0)}{R(t_0)} + \frac{\dot{R}(t_0)}{R(t_0)}(t - t_0) + \frac{1}{2}\frac{\ddot{R}(t_0)}{R(t_0)}(t - t_0)^2 \tag{4.63}$$

The Friedmann equation Eq. (4.57) can be written

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\rho = -\frac{4\pi G}{3}\Omega\frac{3H^2}{8\pi G} = -\frac{\Omega H^2}{2} \tag{4.64}$$

Since $H(t) = \dot{R}/R$ (Eq. 4.38), Eq. (4.63) is

$$\frac{R(t)}{R(t_0)} = 1 + H_0\,(t - t_0) - \frac{1}{2}\frac{\Omega_0}{2}H_0^2\,(t - t_0)^2 \tag{4.65}$$

where $H_0 = H(t_0)$ and $\Omega_0 = \Omega(t_0)$.

The subscript 0 is often omitted in the case of Ω.

Often, Eq. (4.65) is written using the deceleration parameter:

$$q := \frac{\Omega}{2} = -\frac{\ddot{R}(t_0)R(t_0)}{\dot{R}^2(t_0)} \tag{4.66}$$

Equation of state, I

For the evolution of the universe, need to look at three different kinds of equation of state:

Matter: Normal particles get diluted by expansion of the universe:

$$\rho_\mathrm{m} \propto R^{-3} \tag{4.67}$$

Matter is also often called dust by cosmologists.

Radiation: The energy density of radiation decreases because of volume expansion and because of the cosmological redshift (Eq. 4.50: $\lambda_\mathrm{o}/\lambda_\mathrm{e} = \nu_\mathrm{e}/\nu_\mathrm{o} = R(t_\mathrm{o})/R(t_\mathrm{e})$) $\Longrightarrow$

$$\rho_\mathrm{r} \propto R^{-4} \tag{4.68}$$

Vacuum: The vacuum energy density (=Λ) is independent of R:

$$\rho_\mathrm{v} = \text{const.} \tag{4.69}$$

Inserting these equations of state into the Friedmann equation and solving with the boundary condition $R(t=0)=0$ then gives a specific world model.

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Equation of state, II

Current scale factor is determined by H_0 and Ω_0: Friedmann for $t = t_0$:

$$\dot{R}_0^2 - \frac{8\pi G}{3}\rho R_0^2 = -kc^2 \quad (4.70)$$

Insert Ω and note $H_0 = \dot{R}_0/R_0$

$$\Longleftrightarrow H_0^2 R_0^2 - H_0^2 \Omega_0 R_0^2 = -kc^2 \quad (4.71)$$

And therefore

$$R_0 = \frac{c}{H_0}\sqrt{\frac{k}{\Omega - 1}} \quad (4.72)$$

For $\Omega \longrightarrow 0$, $R_0 \longrightarrow c/H_0$, the Hubble length.

For $\Omega = 1$, R_0 is arbitrary.

We now have everything we need to solve the Friedmann equation and determine the evolution of the universe. Three cases: $k = 0, +1, -1$.

$k = 0$, Matter dominated

For the matter dominated, flat case (the Einstein-de Sitter case), the Friedmann equation is

$$\dot{R}^2 - \frac{8\pi G}{3}\frac{\rho_0 R_0^3}{R^3} R^2 = 0 \tag{4.73}$$

For $k = 0$: $\Omega = 1$ and

$$\frac{8\pi G\rho_0}{3} = \Omega_0 H_0^2 R_0^3 = H_0^2 R_0^3 \tag{4.74}$$

Therefore, the Friedmann eq. is

$$\dot{R}^2 - \frac{H_0^2 R_0^3}{R} = 0 \quad \Longrightarrow \quad \frac{\mathrm{d}R}{\mathrm{d}t} = H_0 R_0^{3/2} R^{-1/2} \tag{4.75}$$

Separation of variables and setting $R(0) = 0$,

$$\int_0^{R(t)} R^{1/2}\,\mathrm{d}R = H_0 R_0^{3/2} t \quad \Longleftrightarrow \quad \frac{2}{3}R^{3/2}(t) = H_0 R_0^{3/2} t \tag{4.76}$$

Such that

$$R(t) = R_0 \left(\frac{3H_0}{2}\,t\right)^{2/3} \tag{4.77}$$

For $k = 0$, the universe expands until ∞, its current age ($R(t_0) = R_0$) is given by

$$t_0 = \frac{2}{3H_0} \tag{4.78}$$

Reminder: The Hubble-Time is $H_0^{-1} = 9.78\,\mathrm{Gyr}/h$.

$k = +1$, Matter dominated, I

For the matter dominated, closed case, Friedmanns equation is

$$\dot{R}^2 - \frac{8\pi G}{3}\frac{\rho_0 R_0^3}{R} = -c^2 \quad \Longleftrightarrow \quad \dot{R}^2 - \frac{H_0^2 R_0^3 \Omega_0}{R} = -c^2 \tag{4.79}$$

Inserting R_0 from Eq. (4.72) gives

$$\dot{R}^2 - \frac{H_0^2 c^3 \Omega_0}{H_0^3 (\Omega - 1)^{3/2}} \frac{1}{R} = -c^2 \tag{4.80}$$

which is equivalent to

$$\frac{\mathrm{d}R}{\mathrm{d}t} = c\left(\frac{\xi}{R} - 1\right)^{1/2} \quad \text{with} \quad \xi = \frac{c}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \tag{4.81}$$

With the boundary condition $R(0) = 0$, separation of variables gives

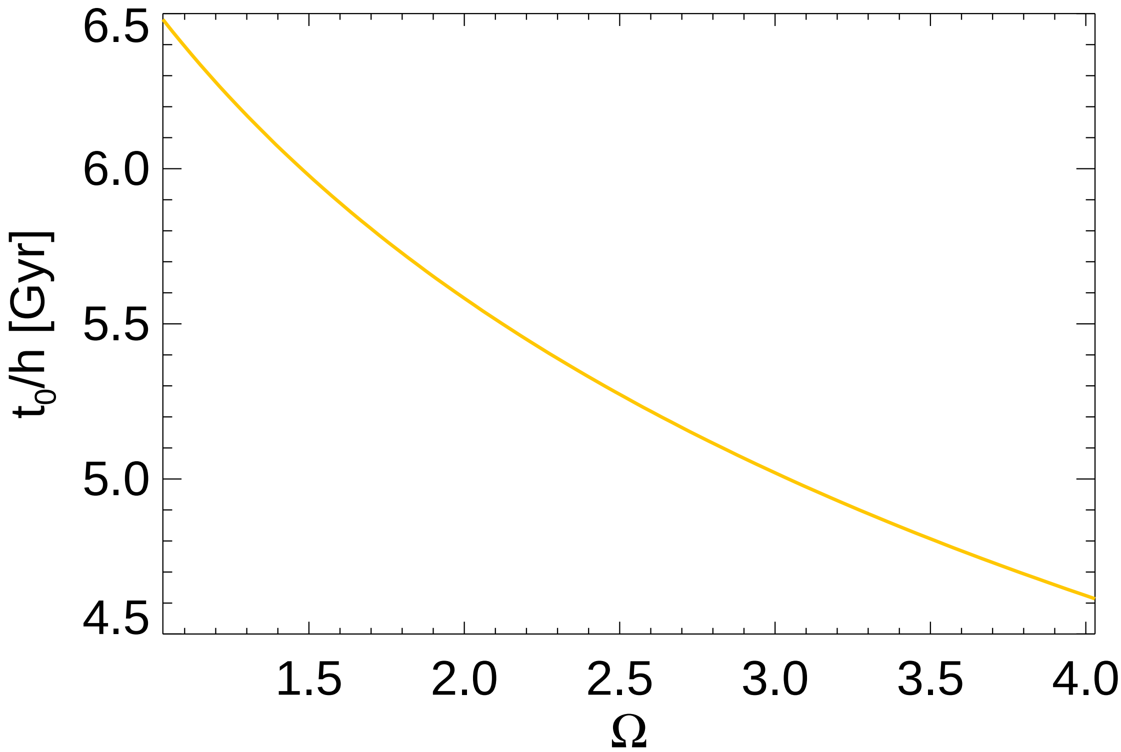
$$ct = \int_0^{R(t)} \frac{\mathrm{d}R}{(\xi/R - 1)^{1/2}} = \int_0^{R(t)} \frac{\sqrt{R}\,\mathrm{d}R}{(\xi - R)^{1/2}} \tag{4.82}$$

Integration by substitution gives

$$R = \xi \sin^2\frac{\theta}{2} = \frac{\xi}{2}(1 - \cos\theta)$$

$$\Longrightarrow \quad ct = \frac{\xi}{2}(\theta - \sin\theta) \tag{4.83}$$

$k = +1$, Matter dominated, II



The age of the universe, t_0, is obtained by solving

$$R_0 = \frac{c}{H_0(\Omega_0 - 1)^{1/2}}$$
$$= \frac{\xi}{2}(1 - \cos\theta_0) = \frac{1}{2}\frac{c}{H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}(1 - \cos\theta_0) \quad (4.84)$$

(remember Eq. 4.72!). Therefore

$$\cos\theta_0 = \frac{2 - \Omega_0}{\Omega_0} \iff \sin\theta_0 = \frac{2}{\Omega_0}\sqrt{\Omega_0 - 1} \quad (4.85)$$

Inserting this into Eq. (4.83) gives

$$t_0 = \frac{1}{2H_0}\frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}\left[\arccos\left(\frac{2 - \Omega_0}{\Omega_0}\right) - \frac{2}{\Omega_0}\sqrt{\Omega_0 - 1}\right] \quad (4.86)$$

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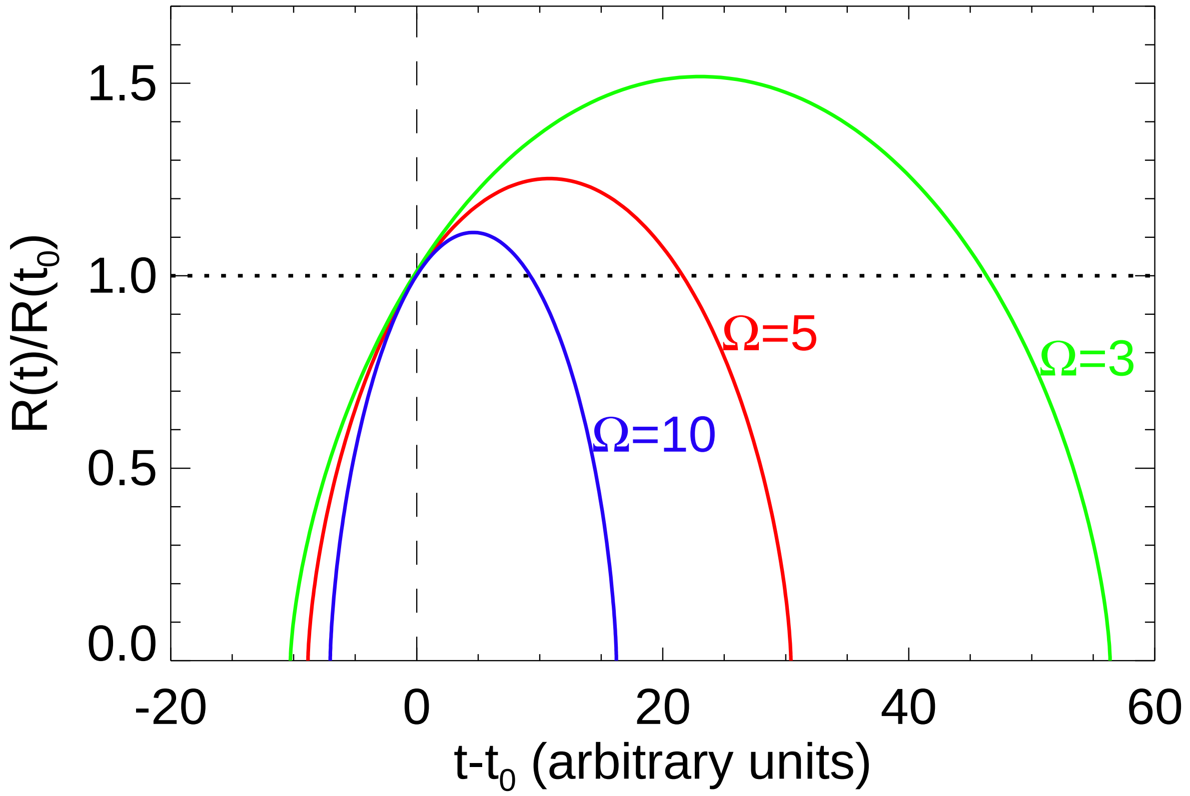
$k = +1$, Matter dominated, III



Since R is a cyclic function $\Longrightarrow$ The closed universe has a finite lifetime.

Max. expansion at $\theta = \pi$, with a maximum scale factor of

$$R_{\max} = \xi = \frac{c}{H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \tag{4.87}$$

After that: contraction to the big crunch at $\theta = 2\pi$.

$\Longrightarrow$ The lifetime of the closed universe is

$$t = \frac{\pi}{H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}} \tag{4.88}$$

$k = -1$, Matter dominated, I

Finally, the matter dominated, open case. This case is very similar to the case of $k = +1$:

For $k = -1$, the Friedmann equation becomes

$$\frac{\mathrm{d}R}{\mathrm{d}t} = c\left(\frac{\zeta}{R} + 1\right)^{1/2} \tag{4.89}$$

where

$$\zeta = \frac{c}{H_0}\frac{\Omega_0}{(1-\Omega_0)^{3/2}} \tag{4.90}$$

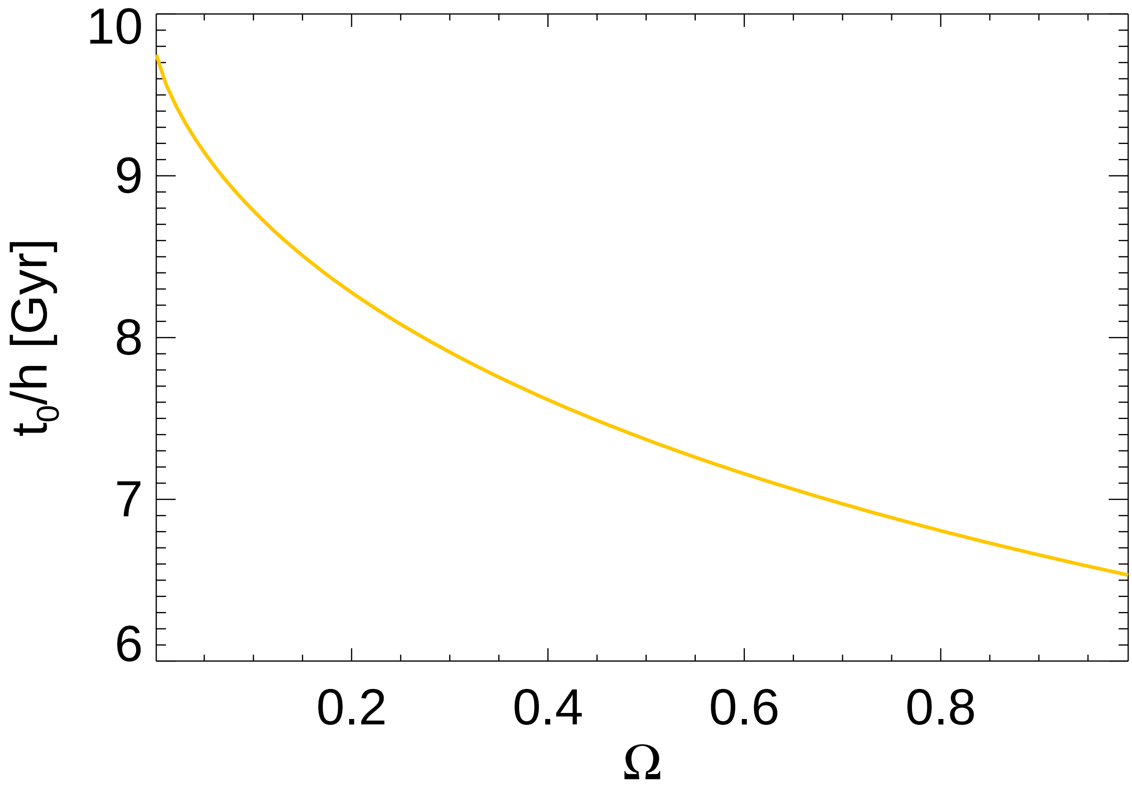
Separation of variables gives after a little bit of algebra

$$\begin{aligned} R &= \frac{\zeta}{2}(\cosh\theta - 1) \\ ct &= \frac{\zeta}{2}(\sinh\theta - 1) \end{aligned} \tag{4.91}$$

where the integration was again performed by substitution.

Note: θ here has *nothing* to do with the coordinate angle θ!

$k = -1$, Matter dominated, II



To obtain the age of the universe, note that at the present time,

$$\cosh\theta_0 = \frac{2-\Omega_0}{\Omega_0}$$
$$\sinh\theta_0 = \frac{2}{\Omega_0}\sqrt{1-\Omega_0} \tag{4.92}$$

(identical derivation as that leading to Eq. 4.84) such that

$$t_0 = \frac{1}{2H_0}\frac{\Omega_0}{(1-\Omega_0)^{3/2}} \cdot \left\{ \frac{2}{\Omega_0}\sqrt{1-\Omega_0} - \ln\left(\frac{2-\Omega_0+2\sqrt{1-\Omega_0}}{\Omega_0}\right)\right\} \tag{4.93}$$

Summary

For the matter dominated case, our results from Eqs. (4.83), and (4.91) can be written with the functions S_k and C_k (Eqs. 4.24 and 4.25):

$$\begin{aligned} R &= k\mathscr{R}\left(1 - C_k(\theta)\right) \\ ct &= k\mathscr{R}\left(\theta - S_k(\theta)\right) \end{aligned} \tag{4.94}$$

where

$$S_k(\theta) = \begin{cases} \sin\theta \\ \theta \\ \sinh\theta \end{cases} \quad \text{and} \quad C_k(\theta) = \begin{cases} \cos\theta & \text{for } k = +1 \\ 1 & \text{for } k = 0 \\ \cosh\theta & \text{for } k = -1 \end{cases} \tag{4.24, 4.25}$$

Eq. (4.94) is called the cycloid solution.
The characteristic radius, $\mathscr{R}$, is given by

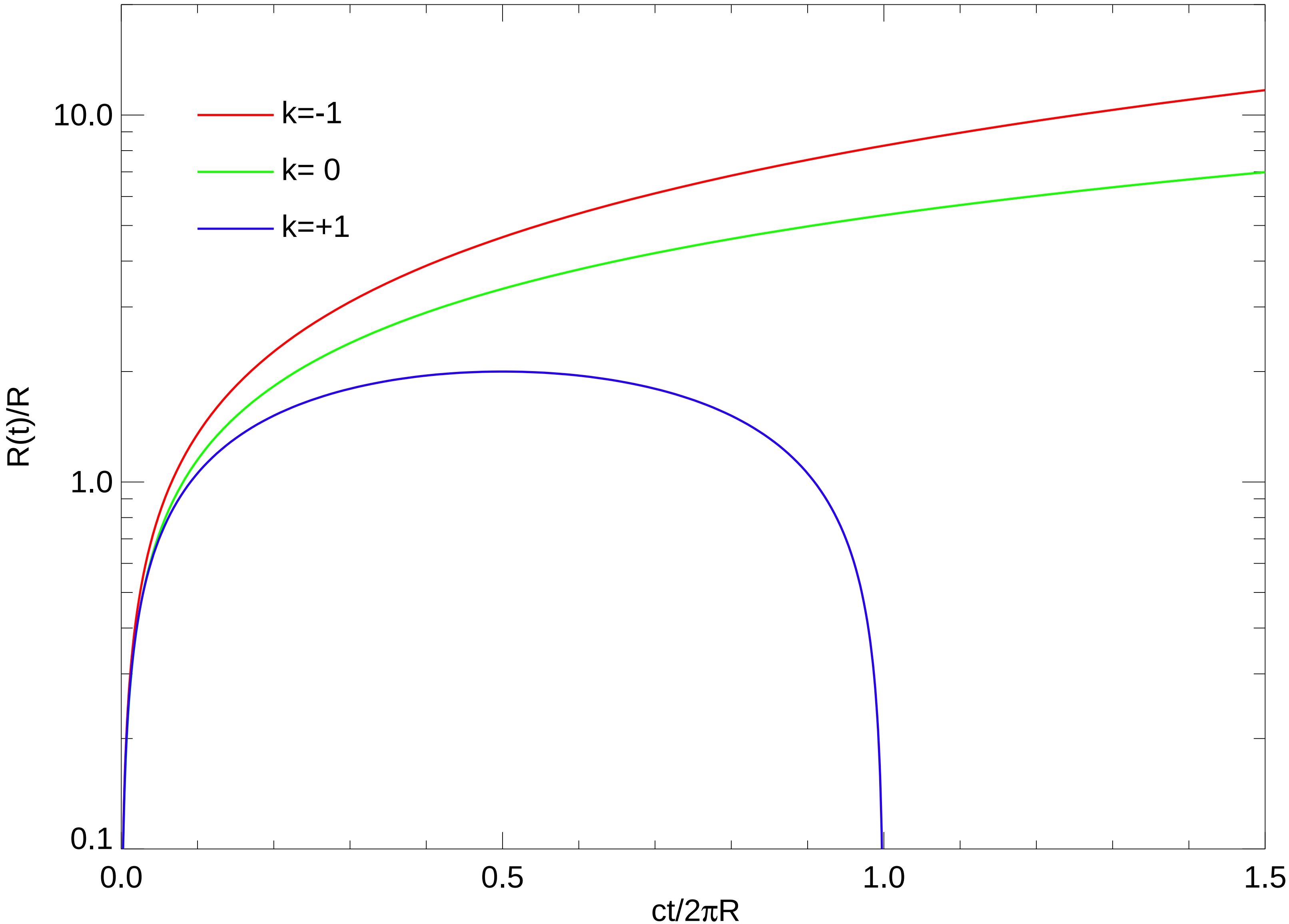
$$\mathscr{R} = \frac{c}{H_0} \frac{\Omega_0/2}{\left(k\left(\Omega_0 - 1\right)\right)^{3/2}} \tag{4.95}$$

(note typo in Eq. 3.42 of Peacock, 1999).

Notes:

1. Eq. (4.94) can also be derived as the result of the Newtonian collapse/expansion of a spherical mass distribution.
2. θ is called the development angle, it can be shown to be equal to the *conformal time* of Eq. (4.33).

Summary



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Classical Cosmology

Classical Cosmology

To understand what universe we live in, we need to determine observationally the following numbers:

1. The Hubble constant, H_0
   $\Longrightarrow$ Requires distance measurements.
2. The current density parameter, Ω_0
   $\Longrightarrow$ Requires measurement of the mass density.
3. The cosmological constant, Λ
   $\Longrightarrow$ Requires acceleration measurements.
4. The age of the universe, t_0, for consistency checks
   $\Longrightarrow$ Requires age measurements.

The determination of these numbers is the realm of classical cosmology.

First part: Distance determination and H_0!

Introduction, I

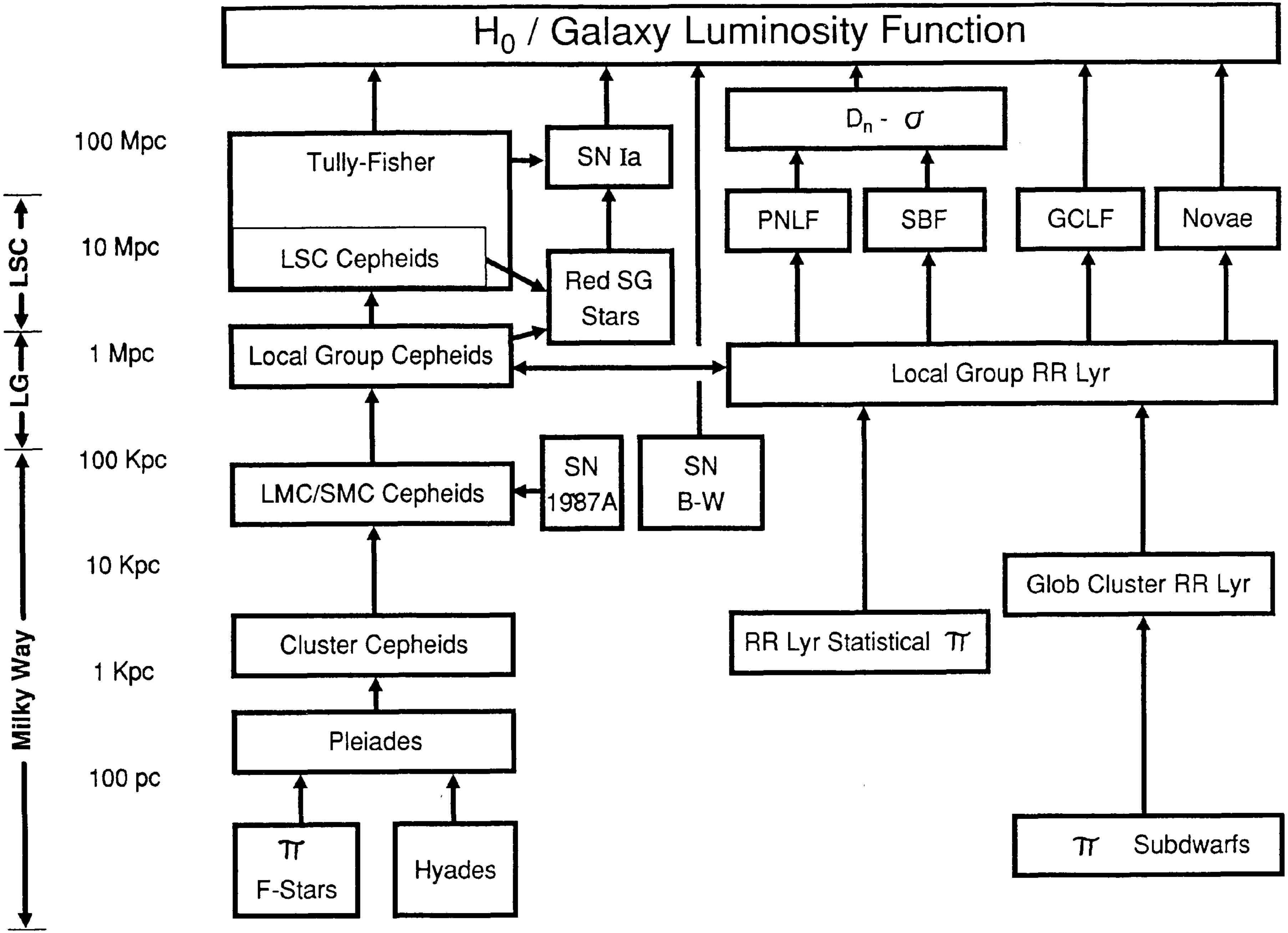
Distances are required for determination of H_0.
$\Longrightarrow$ Need to measure distances out to $\sim$200 Mpc to obtain reliable values.

To get this far: cosmological distance ladder.

1. Trigonometric Parallax
2. Moving Cluster
3. Main Sequence Fitting
4. RR Lyr
5. Baade-Wesselink
6. Cepheids
7. Light echos
8. Luminosity function of planetary nebulae
9. Brightest Stars
10. Type Ia Supernovae
11. Tully-Fisher
12. D_n-σ for ellipticals
13. Brightest Cluster Galaxies
14. Gravitational Lenses

The best reference is

Rowan-Robinson, M., 1985, The Cosmological Distance Ladder, New York: Freeman



Pathways to Extragalactic Distances

(Jacoby et al., 1992, Fig. 1)

Units

Basic unit of length in astronomy: Astronomical Unit (AU).

Colloquial Definition: 1 AU=mean distance Earth–Sun.

Measurement: (Venus) radar ranging, interplanetary satellite positions, χ^2 minimization of N-body simulations of solar system

$$\boxed{1\,\text{AU} \sim 149.6 \times 10^6\,\text{km}}$$

In the astronomical system of units (IAU 1976), the AU is defined via Gaussian gravitational constant (k).

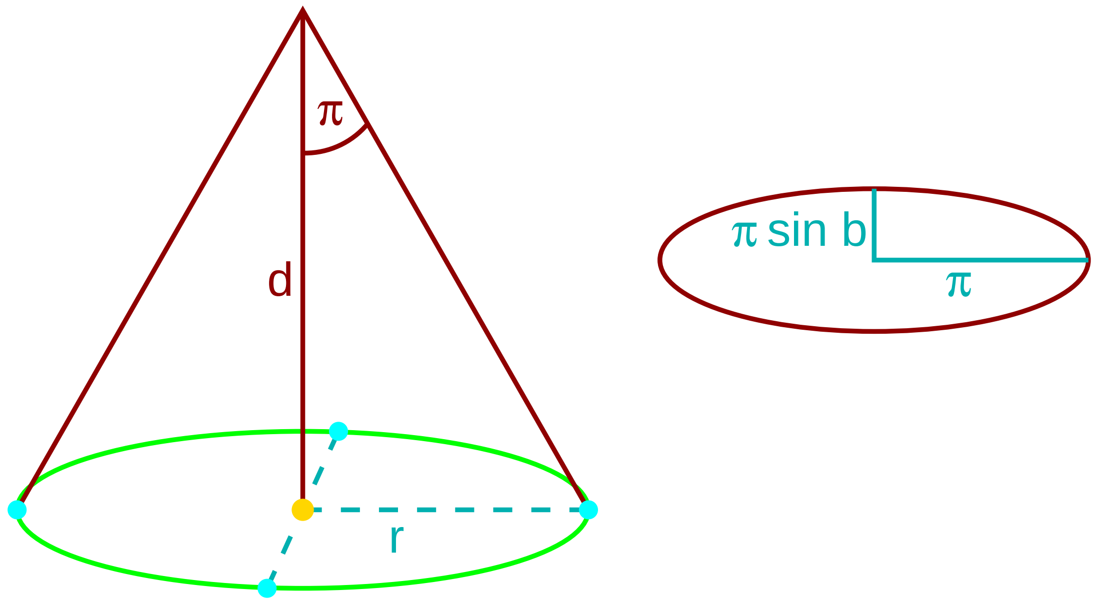
Acceleration:

$$\ddot{\mathbf{r}} = -\frac{k^2(\mathbf{1}+m)\mathbf{r}}{r^3}$$

where $k = 0.01720209895$, leading to
$a_{♁} = 1.00000105726665$, and
1 AU=$1.4959787066 \times 10^{11}$ m (Seidelmann, 1992).

Reason for this definition: k much better known than G.

Trigonometric Parallax, I



after Rowan-Robinson (1985, Fig. 2.1)

Motion of Earth around Sun $\Longrightarrow$ Parallax produces apparent motion by amount

$$\tan\pi \sim \pi = \frac{r_{\oplus}}{d} \tag{5.1}$$

π is called the trigonometric parallax, and *not* 3.141!

If star is at ecliptic latitude b, then ellipse with axes π and $\pi \sin b$.

Measurement difficult: $\pi \lesssim 0.76''$ (α Cen).

Define unit for distance:

Parsec: Distance where 1 AU has $\pi = 1''$.

$1\,\text{pc} = 206265\,\text{AU} = 3.08 \times 10^{18}\,\text{cm} = 3.26\,\text{ly}$

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Trigonometric Parallax, II

Best measurements to date: Hipparcos satellite (with Tübingen participation).

- systematic error of position: $\sim$0.1 mas
- effective distance limit: 1 kpc
- standard error of proper motion: $\sim$1 mas/yr
- broad band photometry
- narrow band: $B - V$, $V - J$
- magnitude limit: 12
- complete to mag: 7.3–9.0

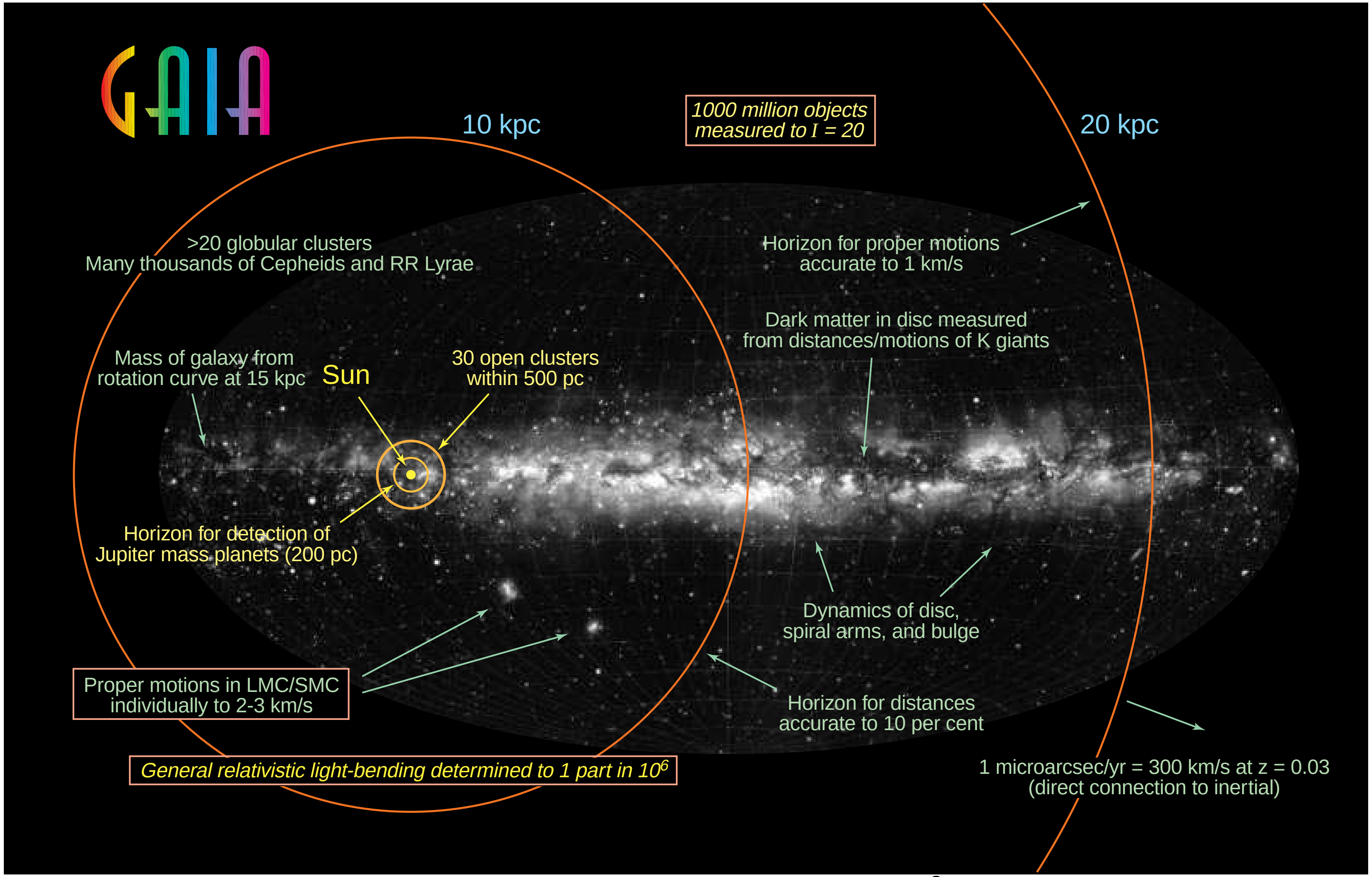
Results available at `http://astro.estec.esa.nl/Hipparcos/`:

Hipparcos catalogue: 120000 objects with milliarcsecond precision.

Tycho catalogue: 10^6 stars with 20–30 mas precision, two-band photometry

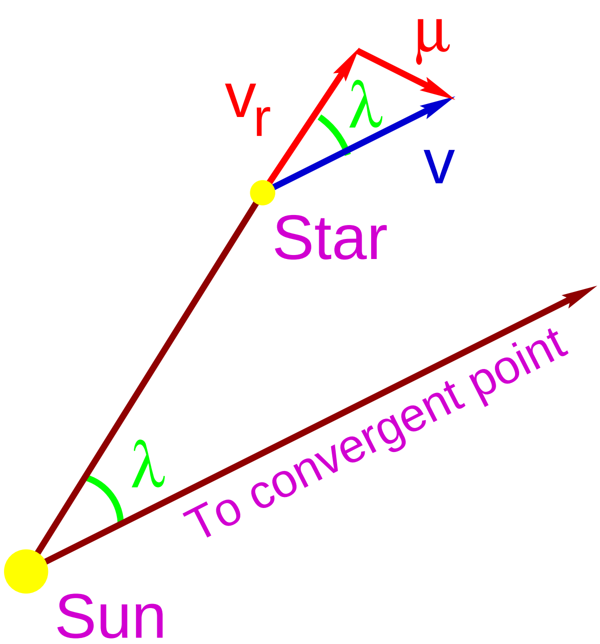
Trigonometric Parallax, III

Plans for the future: GAIA (ESA mission, ∼2010–2012):



GAIA: $\sim 4\mu$arcsec precision, 4 color to $V = 20$ mag, 10^9 objects.

Moving Cluster, I



Perspective effect of spatial motion towards convergent point:

$$\tan\lambda = \frac{v_{\mathrm{t}}}{v_{\mathrm{r}}} = \frac{\mu d}{v_{\mathrm{r}}} \qquad (5.2)$$

or

$$\frac{d}{1\,\mathrm{pc}} = \frac{v_{\mathrm{r}}/(1\,\mathrm{km/s})\tan\lambda}{4.74\,\mu/(1''/\mathrm{a})} \qquad (5.3)$$

Problem: determination of convergent point

Less error prone: moving cluster method = rate of variation of angular diameter of cluster:

$$\dot{\theta} d = \theta v_{\mathrm{r}} \qquad (5.4)$$

Observation of proper motions gives

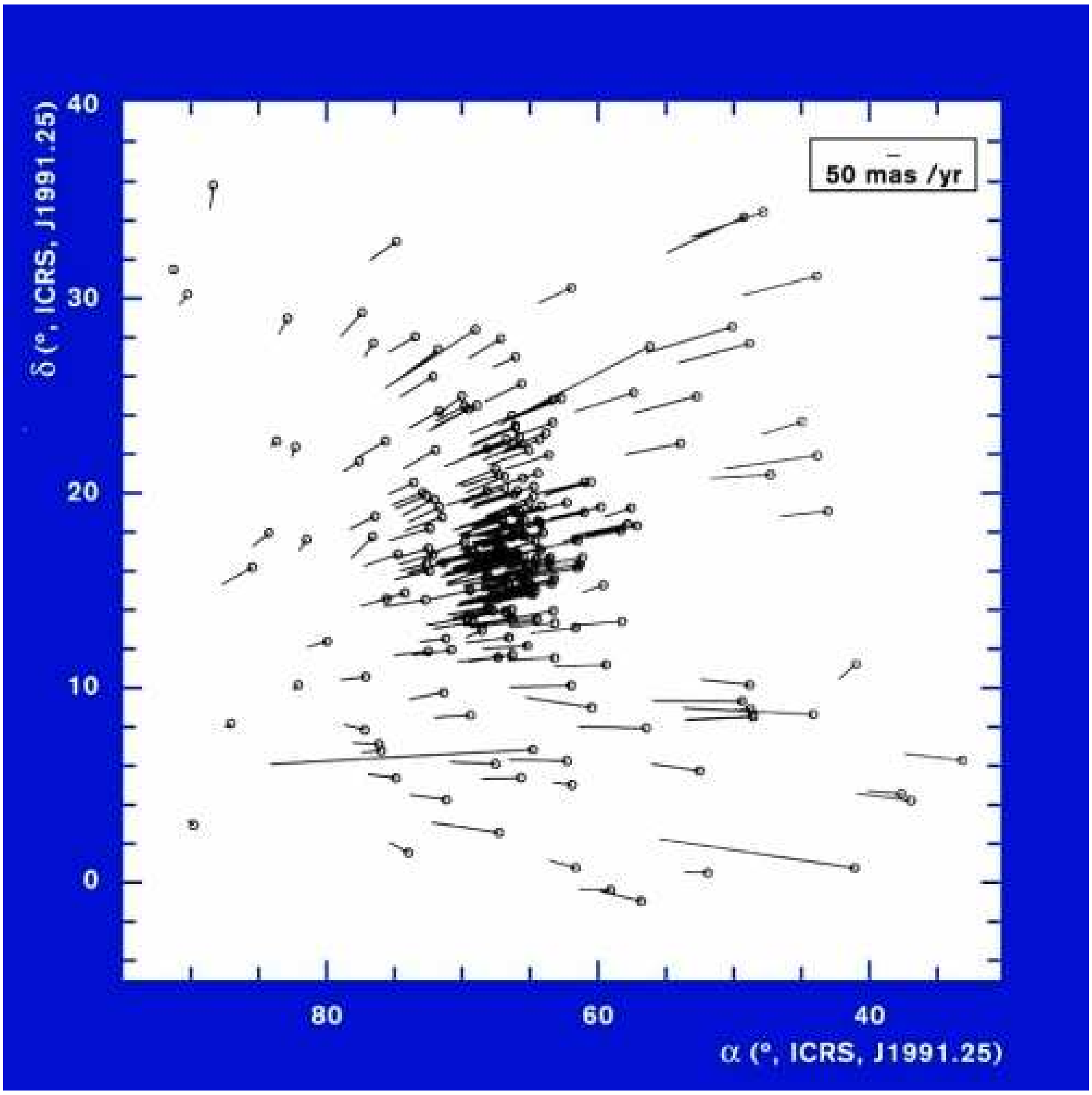
$$\frac{\dot{\theta}}{\theta} = \frac{\mathrm{d}\mu_\alpha}{\mathrm{d}\alpha} = \frac{\mathrm{d}\mu_\delta}{\mathrm{d}\delta} \qquad (5.5)$$

where $\mu_{\alpha,\delta}$ proper motion in α and δ, and from Eq. (5.4),

$$d = v_{\mathrm{r}}\frac{\dot{\theta}}{\theta} \qquad (5.6)$$

v_{r} from spectroscopical radial velocity measurements.

Moving Cluster, II



Source: ESA

Application: Distance to Hyades.

Tip of “arrow”: Position of stars in 100000 a.

Moving cluster (Hanson): DM $\sim$ 3.3.

Hipparcos: geometric distance to Hyades is

$d = 46.34 \pm 0.27$ pc, i.e., DM $= 3.33 \pm 0.01$ mag $\Longrightarrow$

Moving cluster method only of historic interest.

Interlude

Parallax and Moving Cluster: geometrical methods.

All other methods (exception: light echoes): standard candles.

Requirements for standard candles (Mould, Kennicutt, Jr. & Freedman, 2000):

1. Physical basis should be understood.
2. Parameters should be measurable objectively.
3. No corrections ("fudges") required.
4. Small intrinsic scatter ($\Longrightarrow$ requiring small number of measurements!).
5. Wide dynamic range in distance.

Magnitudes, I

Assuming isotropic emission, distance and luminosity are related ("inverse square law") $\Longrightarrow$ luminosity distance:

$$F = \frac{L}{4\pi d_{\mathrm{L}}^2} \tag{5.7}$$

where F is the measured flux ($\mathrm{erg\,cm^{-2}\,s^{-1}}$) and L the luminosity ($\mathrm{erg\,s^{-1}}$).

Definition also true for flux densities, I_ν ($\mathrm{erg\,cm^{-2}\,s^{-1}\,Å^{-1}}$).

The magnitude is defined by

$$m = A - 2.5\log_{10} F \tag{5.8}$$

where A is a constant used to define the zero point (defined by $m = 0$ for Vega).

For a filter with transmission function ϕ_ν,

$$m_i = A_i - 2.5\log \int \phi_\nu F_\nu \,\mathrm{d}\nu \tag{5.9}$$

where, e.g., $i = U, B, V$.

Magnitudes, II

To enable comparison of luminosities: define

absolute magnitude M = magnitude at distance 10 pc

Thus, since $m = A - 2.5 \log(L/4\pi d^2)$,

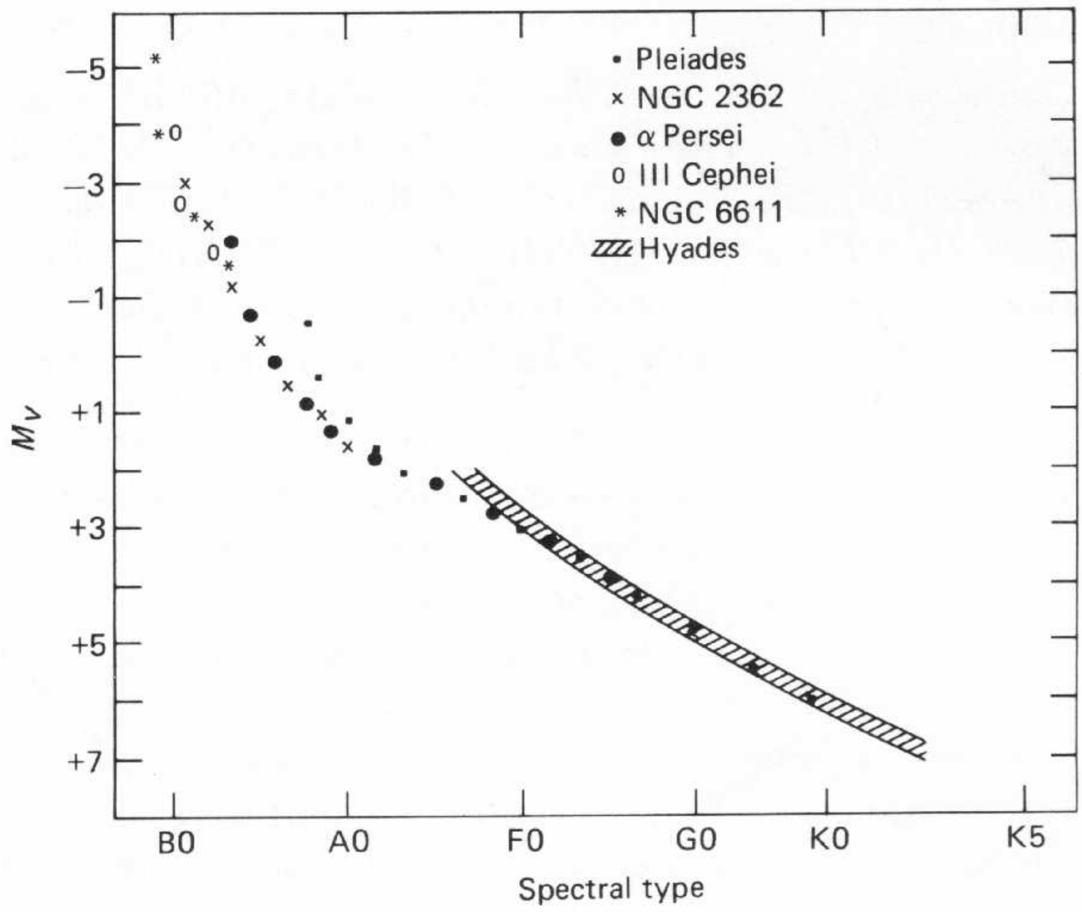
$$M = m - 5 \log \left(\frac{d_{\mathrm{L}}}{10\,\mathrm{pc}} \right) \tag{5.10}$$

The difference $m - M$ is called the distance modulus, μ_0:

$$\mu_0 = \mathrm{DM} = m - M = 5 \log \left(\frac{d_{\mathrm{L}}}{10\,\mathrm{pc}} \right) \tag{5.11}$$

Often, distances are given in terms of $m - M$, and not in pc.

Main Sequence Fitting, I



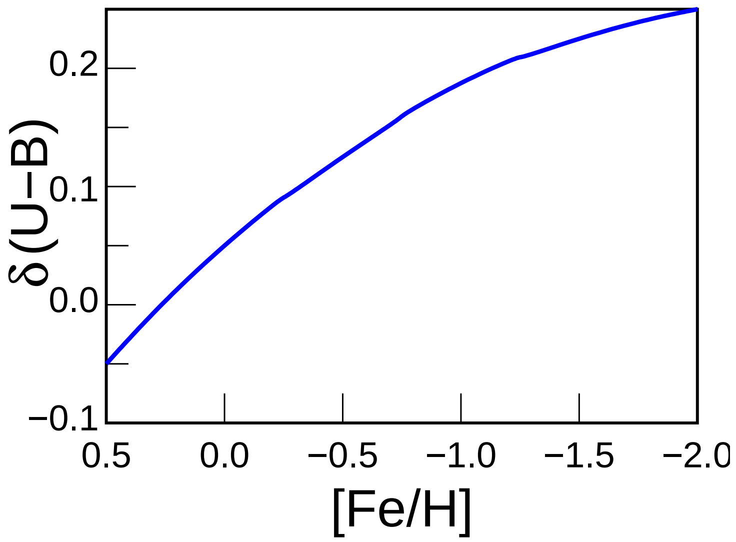
after Rowan-Robinson (1985, Fig. 2.11)

All open clusters are comparably young

$\Longrightarrow$ Hertzsprung Russell Diagram (HRD) dominated by Zero Age Main Sequence (ZAMS).

$\Longrightarrow$ Measure HRD (or Color Magnitude Diagram; CMD), shift magnitude scale until main sequence aligns $\Longrightarrow$ distance modulus.

Main Sequence Fitting, II



(after Rowan-Robinson, 1985, Fig. 2.12)

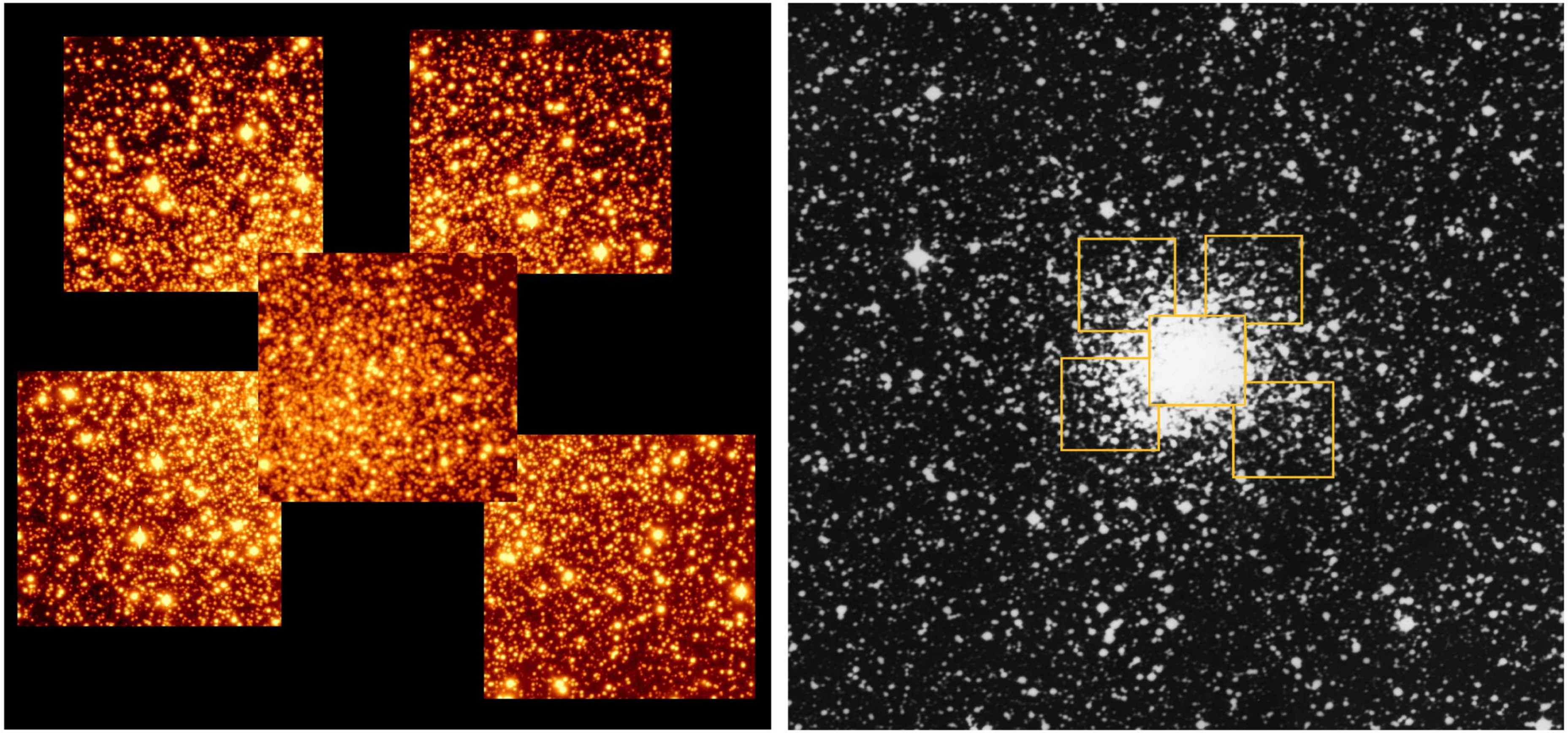
Caveats:

1. Location of ZAMS more age dependent than expected (van Leeuwen, 1999).
2. interstellar extinction $\Longrightarrow \mu_0 = \mu_\mathrm{V} - A_\mathrm{V}$, where μ_V, A_V DM/extinction measured in V-band.
3. metals: line blanketing (change in stellar continuum due to metal absorption lines, see figure) $\Longrightarrow$ Changes color $\Longrightarrow$ horizontal shift in CMD.

   van den Bergh (1977): $Z_\text{Hyades} \sim 1.6 Z_\odot$, while other open clusters have solar metallicity $\Longrightarrow$ Cepheid DM were overestimated by 0.15 mag.
4. identification of unevolved stars crucial (evolution to larger magnitudes on MS during stellar life).

Currently: distances to $\sim$200 open clusters known (Fenkart & Binggeli, 1979).

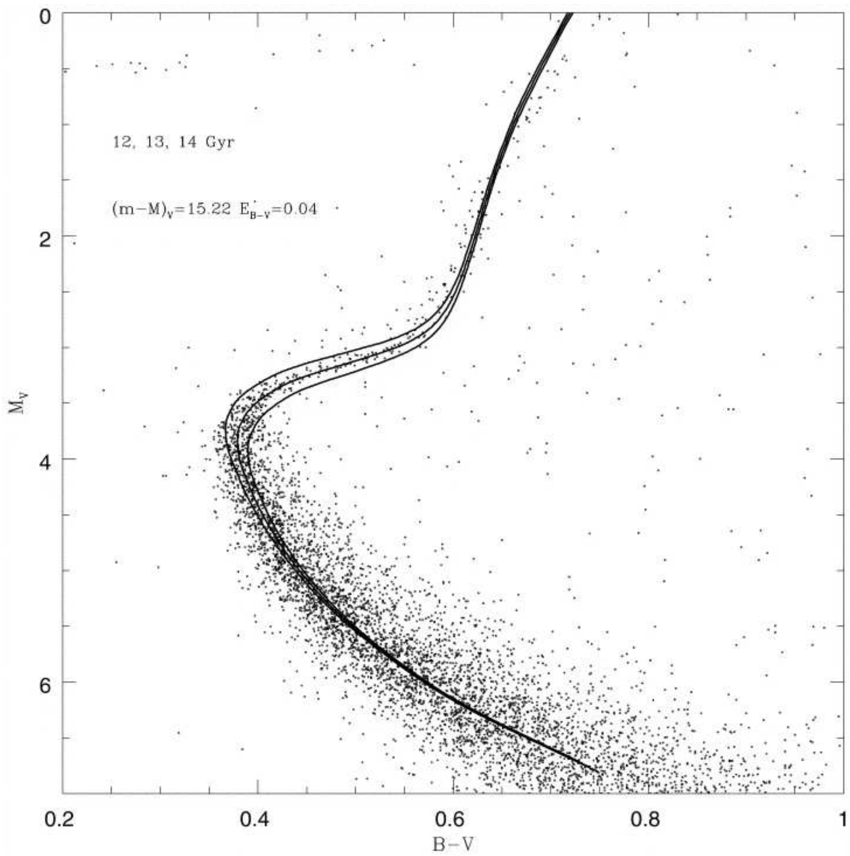
Distance limit $\sim$7 kpc.



Globular Cluster NGC 6712

ESO PR Photo 06a/99 (18 February 1999)

Main Sequence Fitting, IV



(M68, Straniero, Chieffi & Limongi, 1997, Fig. 11)

Globular clusters: HRD different from open clusters:

- population II $\Longrightarrow Z \ll Z_\odot$
- evolved

Use theoretical HRDs (isochrones) to obtain distance.

For distant clusters: MS unobservable $\Longrightarrow$ position of horizontal branch.

Baade-Wesselink

Basic principle (Baade, 1926): Assume black body $\Longrightarrow$ Use color/spectrum to get kT_{eff} $\Longrightarrow$ Emitted intensity is Planckian $\Longrightarrow$ Observed Intensity is $I_\nu \propto \pi r_*^2 B_\nu$.

Radius from integrating velocity profile of spectral lines:

$$R_2 - R_1 = p \int_1^2 v \, \mathrm{d}t \tag{5.12}$$

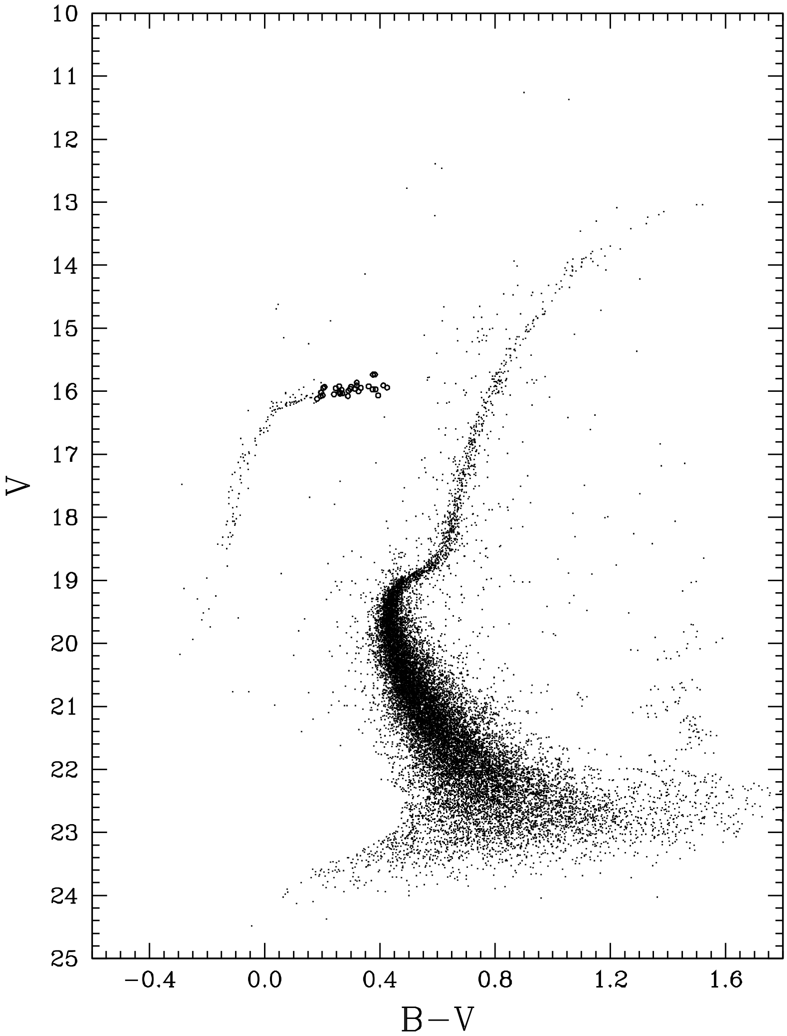
(p: projection factor between velocity vector and line of sight).

Wesselink (1947): Determine brightness for times of same color $\Longrightarrow$ rather independent of knowledge of stellar spectrum (deviations from B_ν).

Stars: Calibration using interferometric diameters of nearby giants.

Baade-Wesselink works for pulsating stars such as RR Lyr, Cepheids, Miras, and expanding supernova remnants.

RR Lyr, I



M2: Lee & Carney (1999, Fig. 2)

RR Lyrae variables: Stars crossing instability strip in HRD

$\Longrightarrow$ Variability ($P \sim 0.2\ldots1\,$d)

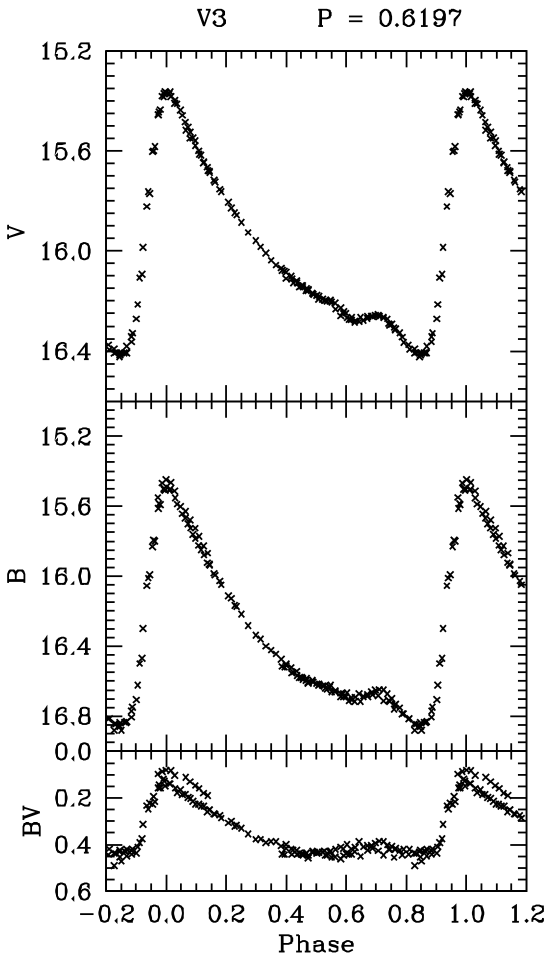
$\Longrightarrow$ RR Lyr gap (change in color!).

Absolute magnitude of RR Lyr gap:

$M_\mathrm{V} = 0.6$, $M_\mathrm{B} = 0.8$, i.e., $L_\mathrm{RR} \sim 50\,L_\odot$).

M determined from ZAMS fitting, statistical parallax, and Baade-Wesselink method.

RR Lyr, II



Lightcurve (here: Lee & Carney, 1999, Fig. 5) shows characteristic color variations over pulsation (temperature change!), and a fast rise, slow decay behavior.

RR Lyr in GCs show bimodal number distribution: RRab with $P > 0.5\,\mathrm{d}$ and most probable period of $P_{\mathrm{ab}} \sim 0.7\,\mathrm{d}$, and RRc, with $P < 0.5\,\mathrm{d}$ and $P_{\mathrm{c}} \sim 0.3\,\mathrm{d}$ (metallicity effect).

Caveat: M dependent on metallicity: larger for higher Z (i.e., metal-rich RR Lyr are *fainter*, i.e., difference in RR Lyr from population I and II).

Works out to LMC and other dwarf galaxies of local group, however, used mainly for globular clusters.

Interlude, I

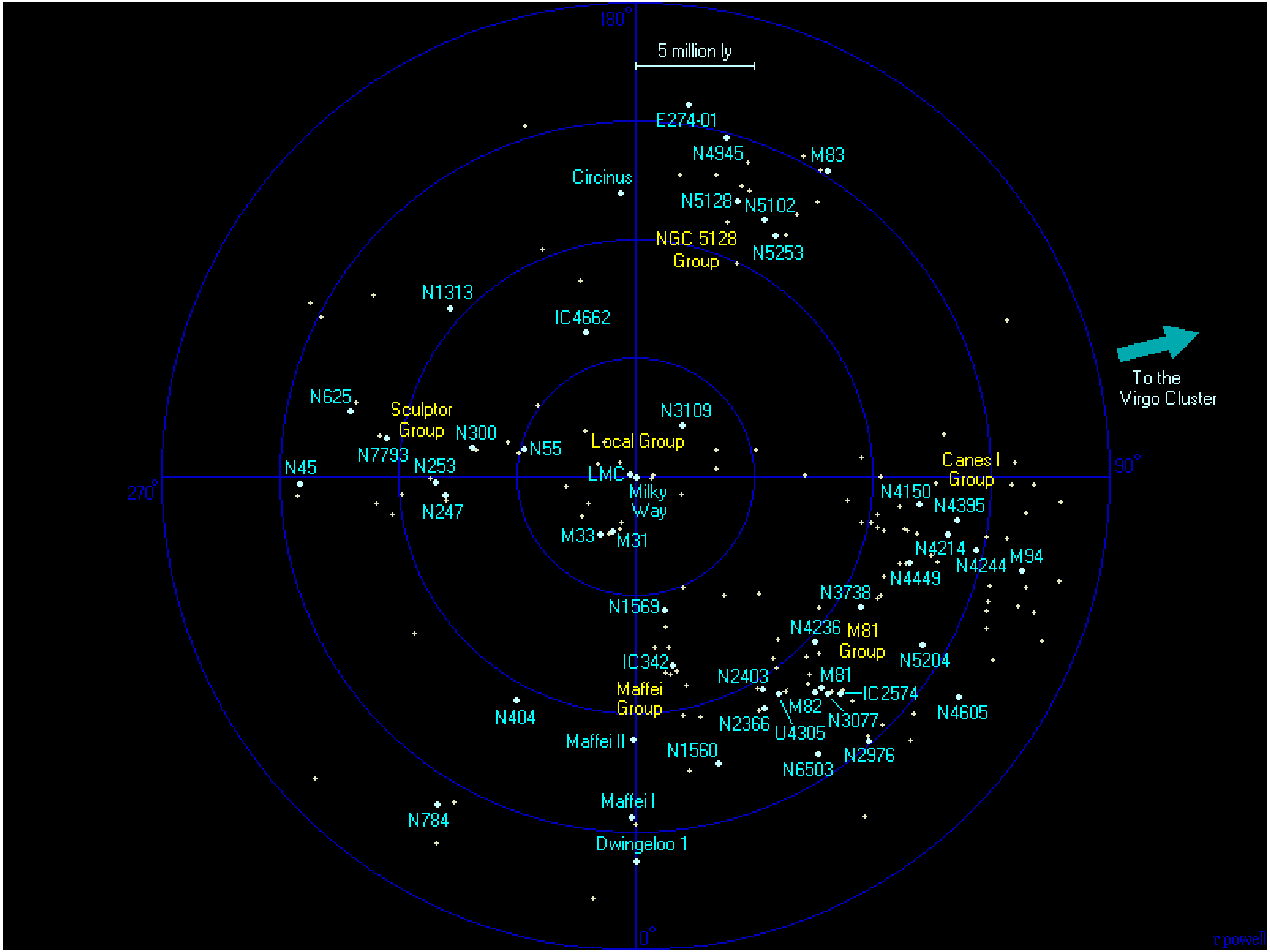
Previous methods: Selection of methods for distances within Milky Way (and Magellanic Clouds): Basis for extragalactic distance scale.

Primary extragalactic distance indicators:
Distance can be calibrated from observations *within* milky way or from theoretical grounds.

Primary indicators usually work within our neighborhood (i.e., out to Virgo cluster at 15–20 Mpc).
Examples: Cepheids, light echos,. . .

Secondary extragalactic distance indicators:
Distance calibrated from primary distance indicators.

Examples: Type Ia SNe, methods based on integral galaxy properties.



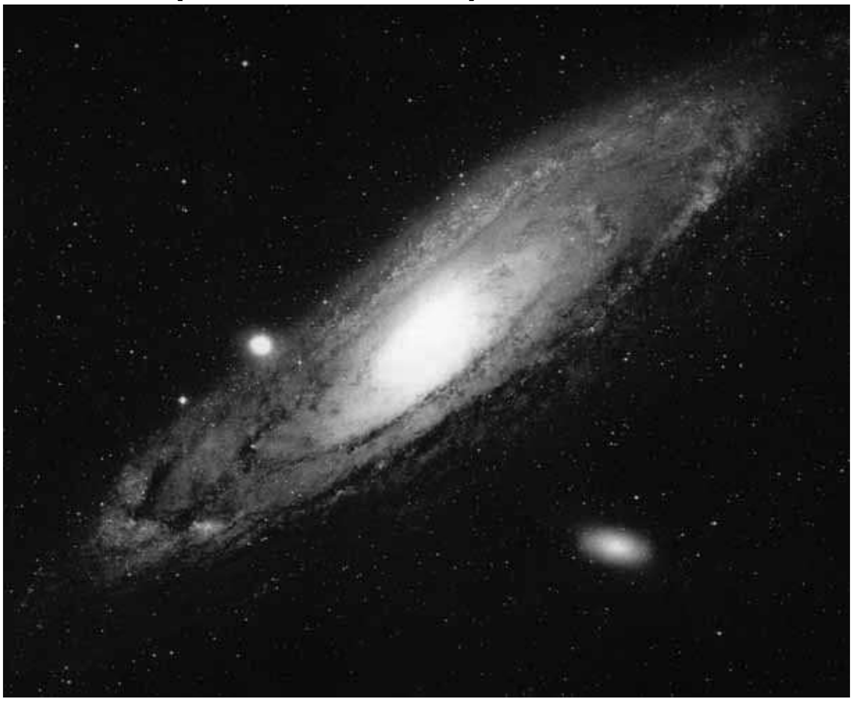
source: `http://anzwers.org/free/universe/galgrps.html`

Interlude, III

To get a feel for the distances in our "neighborhood":

50 kpc: LMC, SMC, some other dwarf galaxies

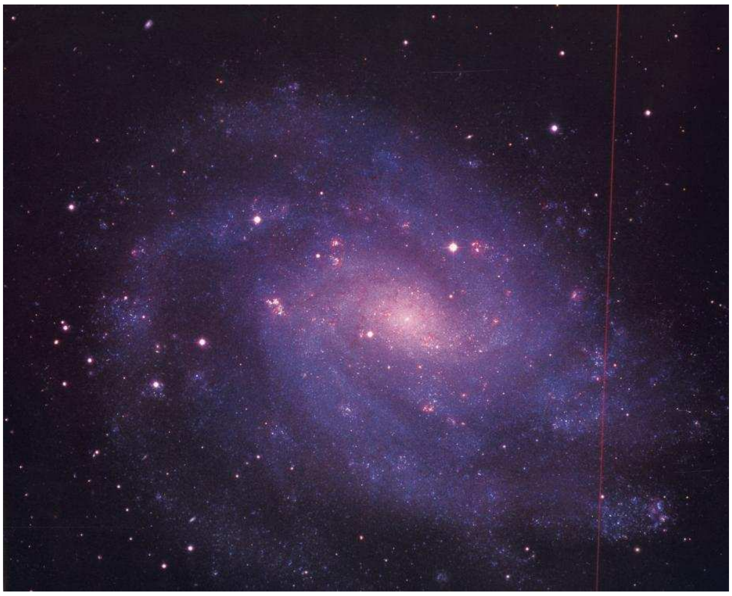
700 kpc: M31 (Andromeda)



Palomar Schmidt

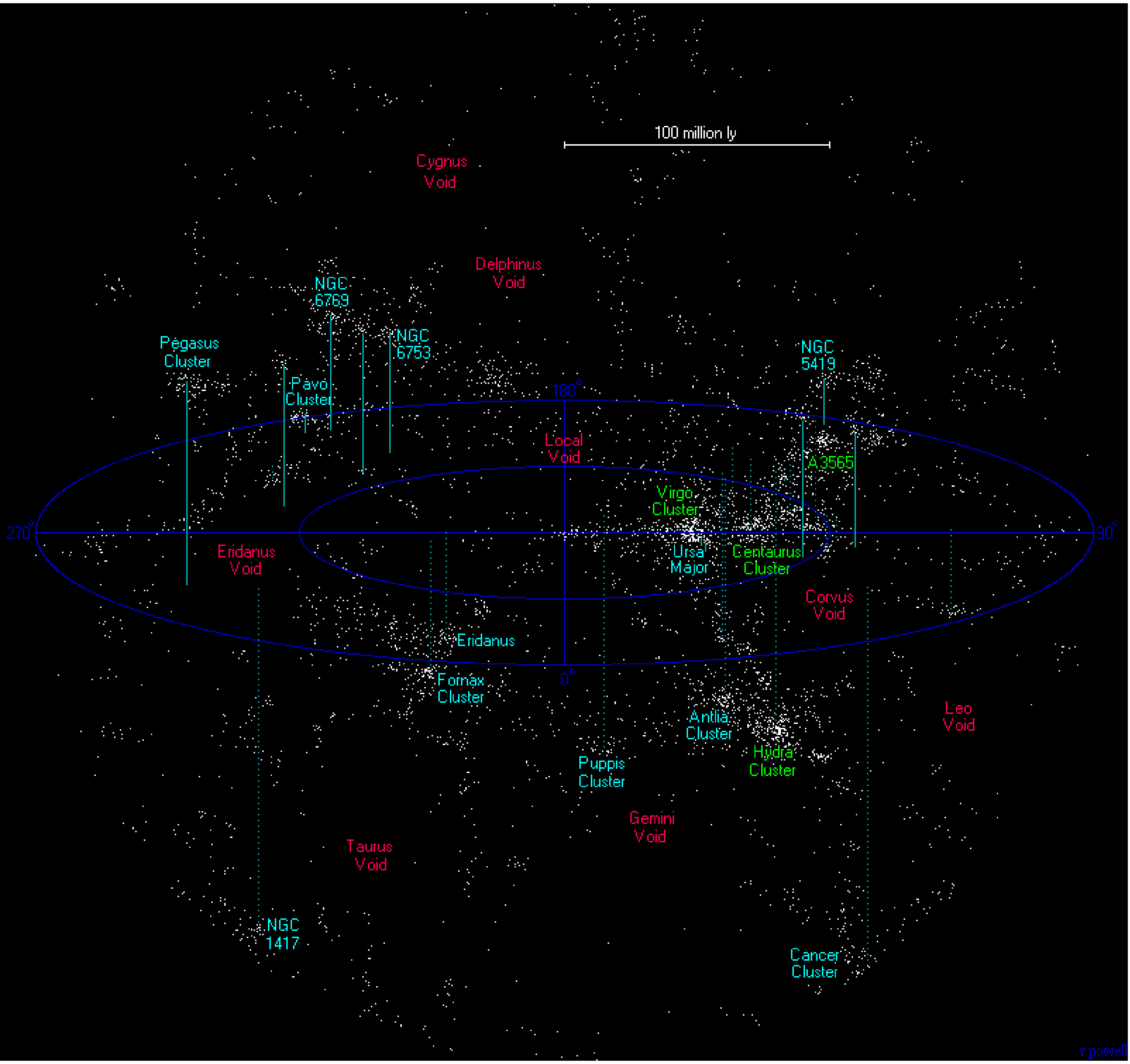
Interlude, IV

2–3 Mpc: Sculptor, M81 group (groups similar to local group: a few large spirals, plus smaller stuff).



NGC 300 (Sculptor; Laustsen, Madsen, West, 1991)

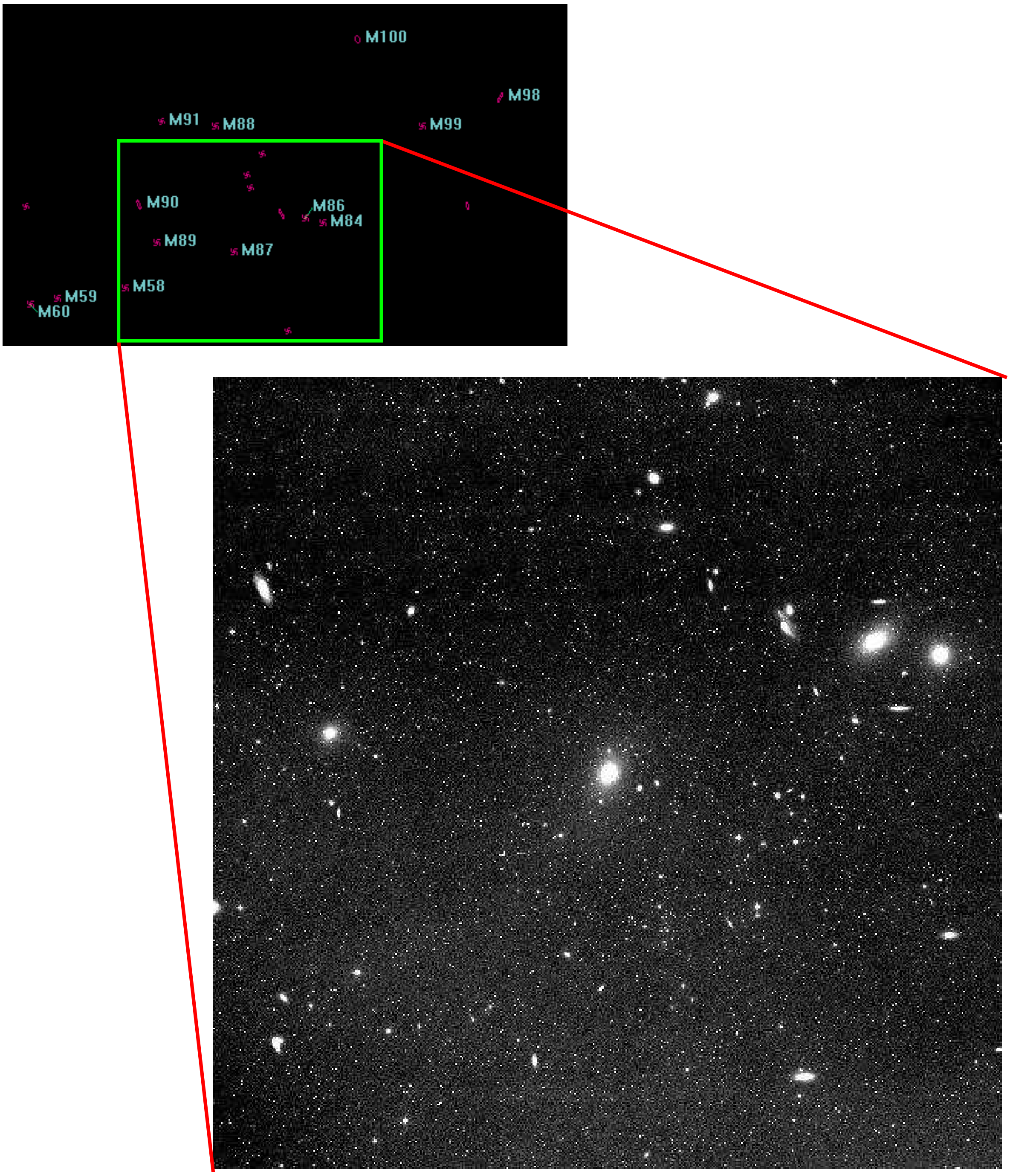
5–7 Mpc: M101 group ("pinwheel galaxy").
Important because of high L.



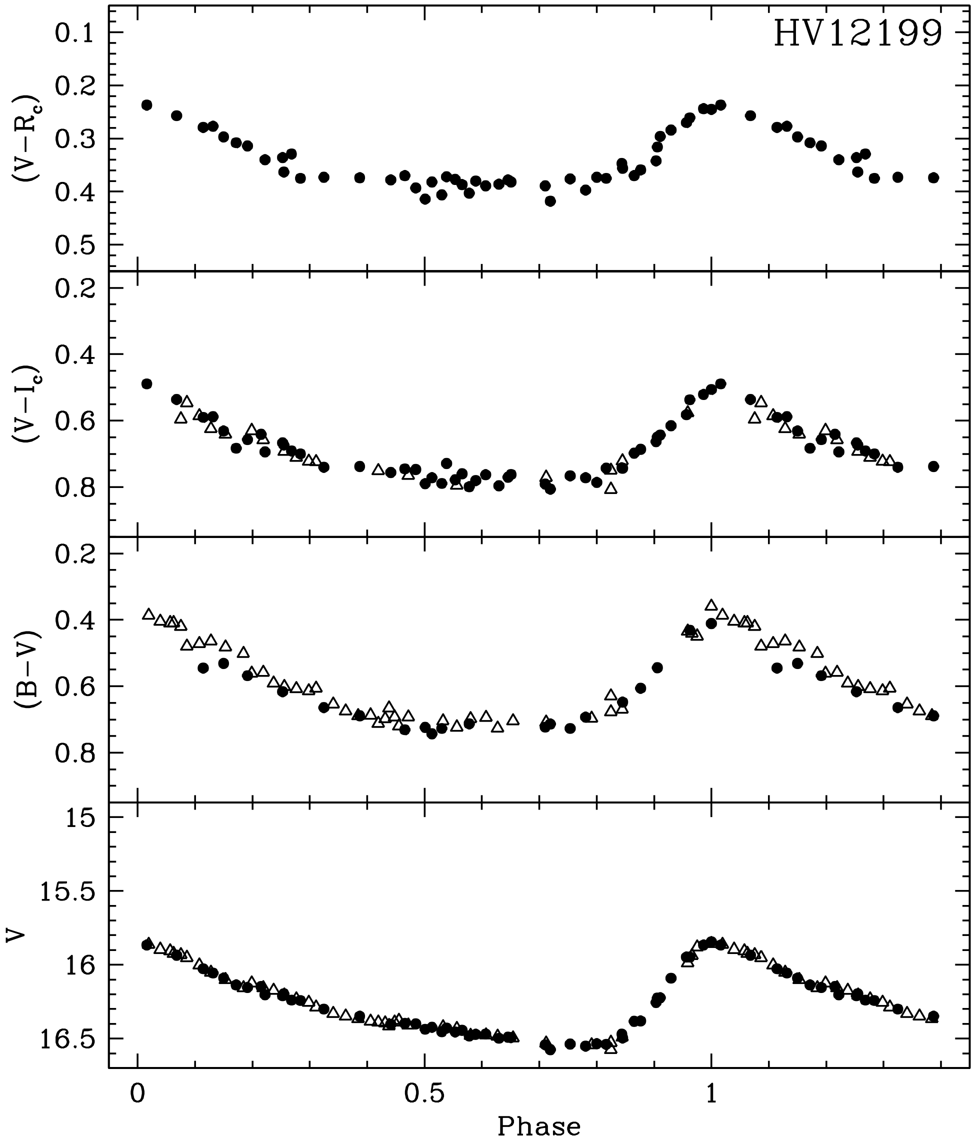
source: `http://anzwers.org/free/universe/200mill.html`

Interlude, VI

15–20 Mpc: Virgo cluster.

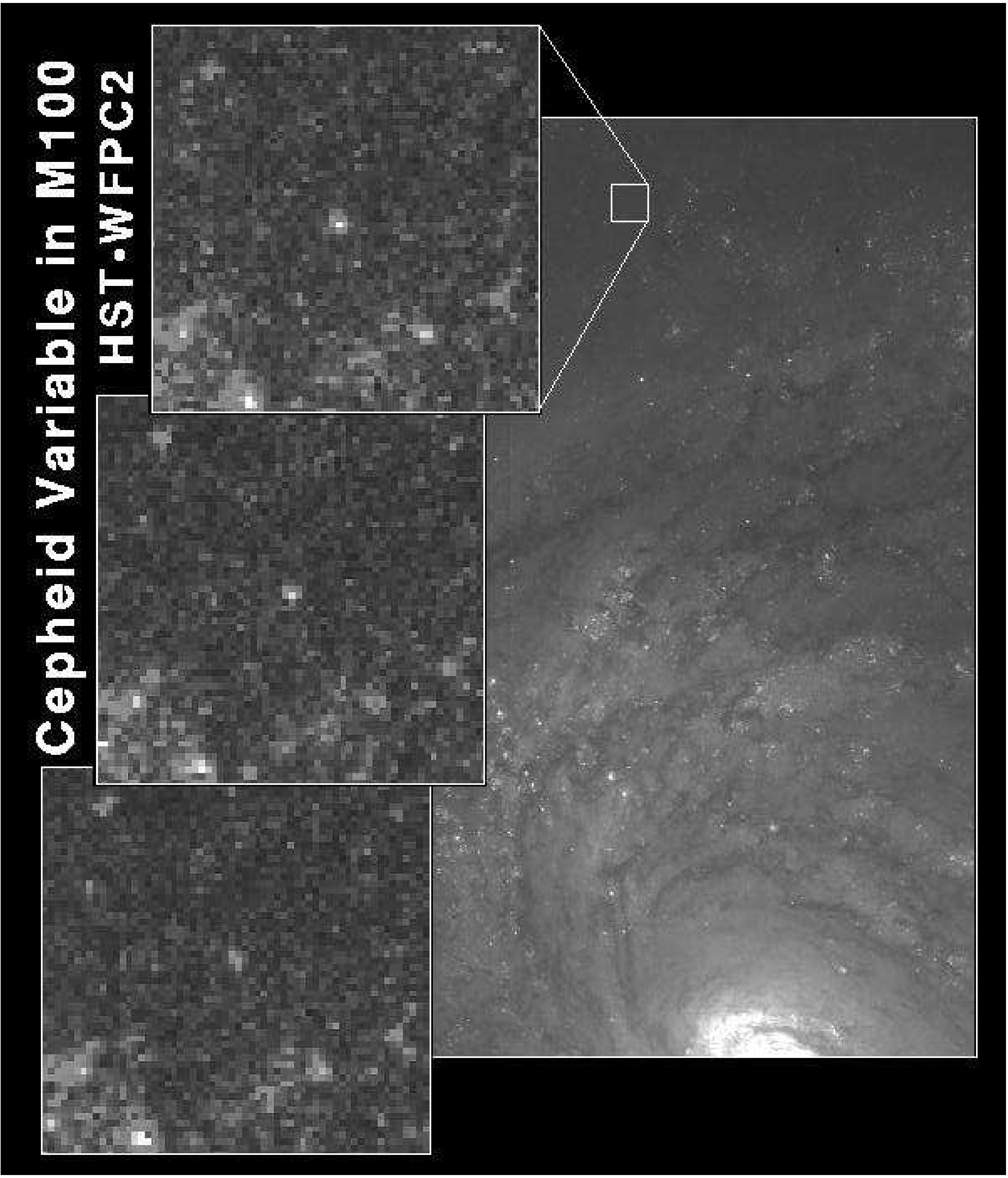


Cepheids, I



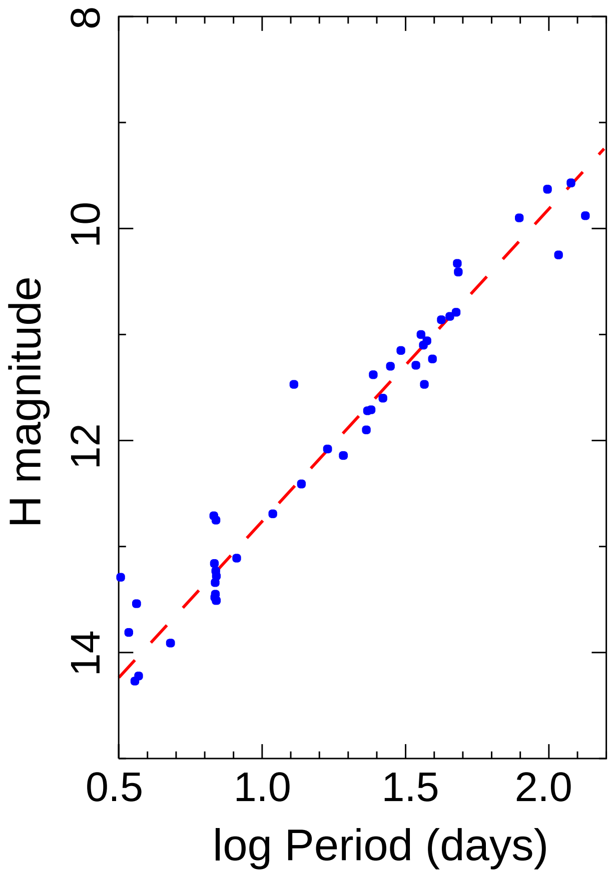
(Gieren et al., 2000, Fig. 3)

Cepheids: Luminous stars ($L \sim 1000\,L_\odot$) in instability strip (He II–He III ionization) with large amplitude variation, $P \sim 2 \ldots 150\,\mathrm{d}$ (easily measurable). Recent review: Feast (1999).



STScI PR94-49

Cepheids, III



PL relation for the LMC Cepheids (after Mould, Kennicutt, Jr. & Freedman, 2000, Fig. 2).

Henrietta Leavitt (1907): Period-Luminosity (PL) relation: $M_{\mathrm{V}} \propto -2.76 \log P$.

Low luminosity Cepheids have lower periods.

Good indications that also influence of color

$\Longrightarrow$ Period-Luminosity-Color (PLC) relation

Cepheids, IV

Physics of Period-Luminosity-Color relation:

Star pulsates such that outer parts remain bound:

$$\frac{1}{2}\left(\frac{R}{P}\right)^2 \lesssim \frac{GM}{R} \quad \Longrightarrow \quad \frac{M}{R^3} \propto P^{-2} \tag{5.13}$$

where P period. Therefore:

$$P \propto \rho^{-1/2} \quad \Longleftrightarrow \quad P\rho^{1/2} = Q \tag{5.14}$$

(Q: pulsational constant, $\rho \propto MR^{-3}$ mean density). But Radius R related to luminosity L:

$$L = 4\pi R^2 \sigma T^4 \quad \Longrightarrow \quad R \propto L^{1/2} T^{-2} \tag{5.15}$$

Inserting everything into Eq. (5.14) gives:

$$PL^{-3}T^3 = \text{const.} \tag{5.16}$$

$$\Longleftrightarrow \quad \log P - 3\log L + 3\log T = \text{const.} \tag{5.17}$$

But:

bolometric magnitude: $M_{\mathrm{bol}} \propto -\log L$;

colors: $\mathrm{B}-\mathrm{V} \propto \log T$

such that

$$c_1 \log P + c_2 M_{\mathrm{bol}} + c_3(\mathrm{B}-\mathrm{V}) = \text{const.} \tag{5.18}$$

where $c_{1,2,3}$ calibration constants.

Cepheids, V

Calibration: Need slope and zero point of PLC.

Slope is easy: Observations of nearby galaxies (e.g., open clusters in LMC, see previous slide).

Zero point is difficult:

- Cepheids in galactic clusters, distance to these via ZAMS fitting $\Longrightarrow$ problematic due to age dependency of ZAMS.
- Hipparcos: geometrical distances $\Longrightarrow$ problematic due to low SNR (resulting in 9% systematic error.
- Baade-Wesselink using IR info (low metallicity dependence).

Typical relations (Mould et al., 2000, 32 Cepheids):

$$\begin{aligned} M_{\mathrm{V}} &= -2.76 \log P - 1.40 + C(Z) \\ M_{\mathrm{I}} &= -3.06 \log P - 1.81 + C(Z) \end{aligned} \tag{5.19}$$

The metallicity (color) dependence is roughly

$$(m-M)_{\mathrm{true}} = (m-M)_{\mathrm{PL}} - \gamma \log Z/Z_{\mathrm{LMC}} \tag{5.20}$$

where $\gamma = -0.11 \pm 0.03\,\mathrm{mag/dex}$ (Z: metallicity) (=Cepheids with larger Z are fainter).

Cepheids, VI

Notes:

1. Pulsational constant $Q = Q(\rho, P)$? $\Longrightarrow$ possible deviation from PLC, especially at high luminosity $\Longrightarrow$ adds uncertainty at large distances.
2. M_{V} depends on metallicity (LMC Cepheids *are* bluer [$Z_{\mathrm{LMC}} < Z_{\odot}$]), but γ very uncertain.

   For V and I magnitudes, most probably $\delta(m - M)_0/\delta[\mathrm{O/H}] \lesssim -0.4\,\mathrm{mag\,dex}^{-1}$, however, others find $+0.75\,\mathrm{mag\,dex}^{-1}$, see Ferrarese et al. (2000) for details...
3. Stellar evolution unclear (multiple crossings of instability strip possible).

UWarwick

W Vir Stars

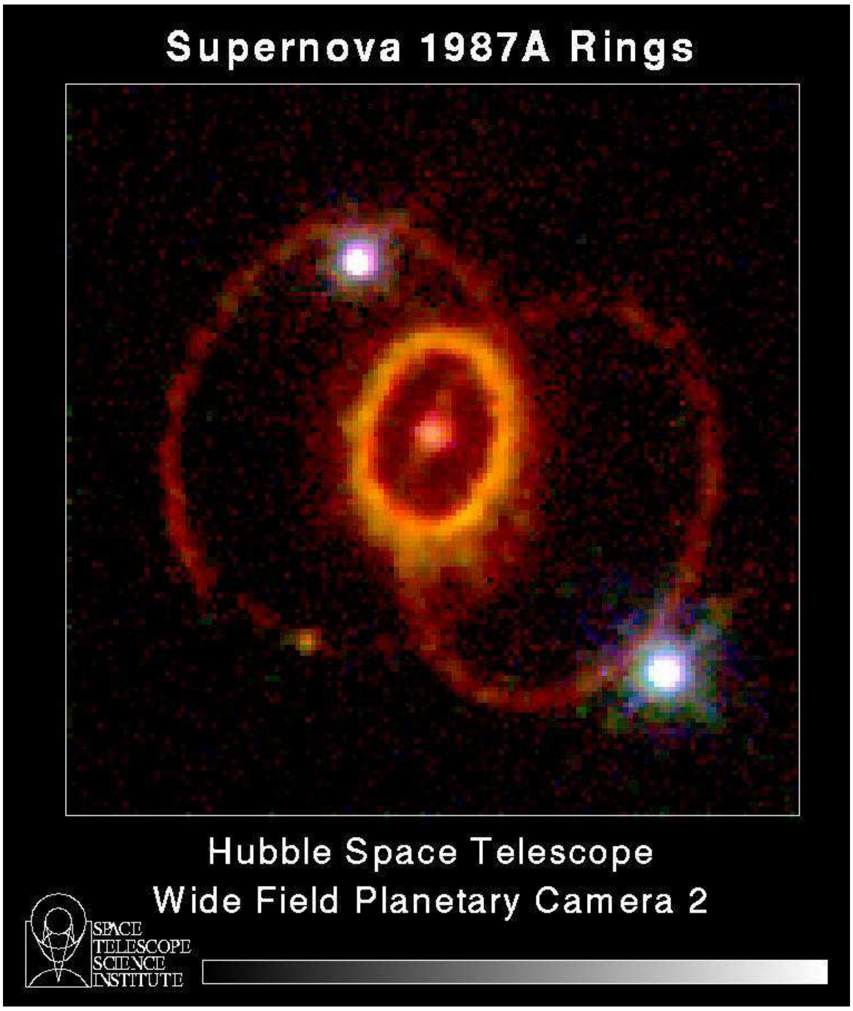
W Vir stars, also called type II Cepheids = “little brother of Cepheids” (present in globular clusters).

Less luminous than normal Cepheids, similar PLC relation, first confused with Cepheids $\Longrightarrow$ Cause for early thoughts of much smaller universe.

Cause for early confusion with Cepheids by Hubble (realization vastly increased assumed size of universe).

Light echos, I

Light echo: specialized way to determine distance to LMC using Supernova 1987A.



STScI PR94-22

February 1987: Supernova in Large Magellanic Cloud.

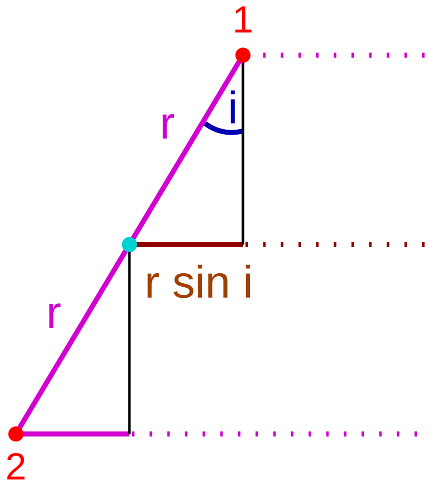
87 d after explosion: Ring of ionized C and N around SN

$\Longrightarrow$ Excitation of C, N in ring-like shell (ejecta from stars equator during red giant phase?).

Observed size: $1.66'' \times 1.21''$

Light echos, II

Assuming ring-geometry: direct geometrical determination of distance to LMC possible:



Time delay SN – close side of ring:

$$\begin{aligned} ct_1 &= r(1 - \sin i) \\ &= 86 \pm 6\,\mathrm{d} \end{aligned} \tag{5.21}$$

Time delay SN – far side of ring:

$$\begin{aligned} ct_2 &= r(1 + \sin i) \\ &= 413 \pm 24\,\mathrm{d} \end{aligned} \tag{5.22}$$

The radius is (Eq. 5.21+Eq. 5.22):

$$r = c\,\frac{t_1 + t_2}{2} = 250 \pm 12\,\mathrm{lt\,d} \tag{5.23}$$

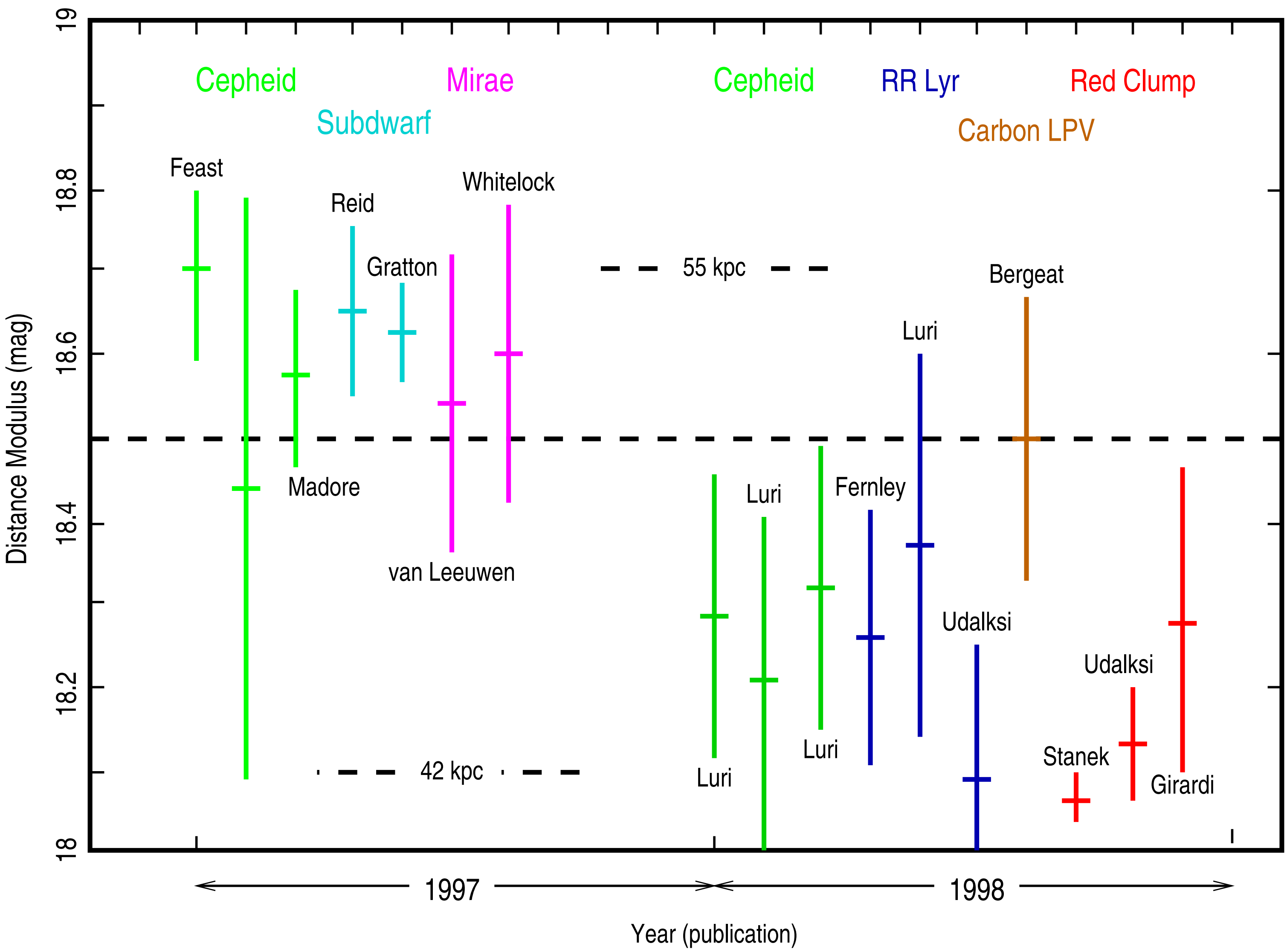
and the inclination is (Eq. 5.21+Eq. 5.22):

$$\sin i = \frac{t_2 - t_1}{t_1 + t_2} \quad \Longrightarrow \quad i \sim 41^\circ \tag{5.24}$$

From ring-geometry: $\cos i = 1.21''/1.66'' \Longrightarrow i \sim 43^\circ$).
Thus from angular size of ring:

$$1.66'' = \frac{r \cos i}{d} \quad \Longrightarrow \quad d = 52 \pm 3\,\mathrm{kpc} \tag{5.25}$$

Large Magellanic Cloud (LMC) distance: “anchor point” of extragalactic distance scale.



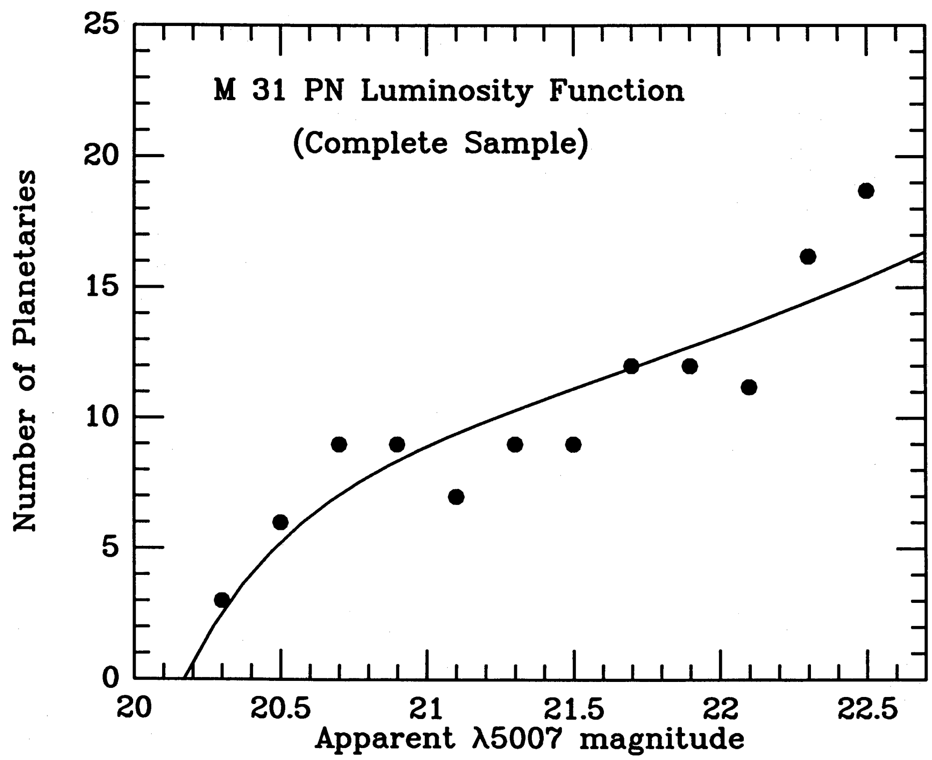
After Gaia Science Workgroup

Problems that are still not understood:

- Strong dependence on Hipparcos calibration. Values between 18.7 ± 0.1 (Feast & Catchpole) and 18.57 ± 0.11 (Madore & Freedman) obtained.
- Eclipsing binaries and red clump stars: $\mu_{\rm LMC} \sim 18.23$ (Mould, Kennicutt, Jr. & Freedman, 2000) $\Longrightarrow$ Inconsistent with other methods!?!

Currently, the distance to the LMC is less well known than desirable.

PN Luminosity Function, I



(Ciardullo et al., 1989, Fig. 4)

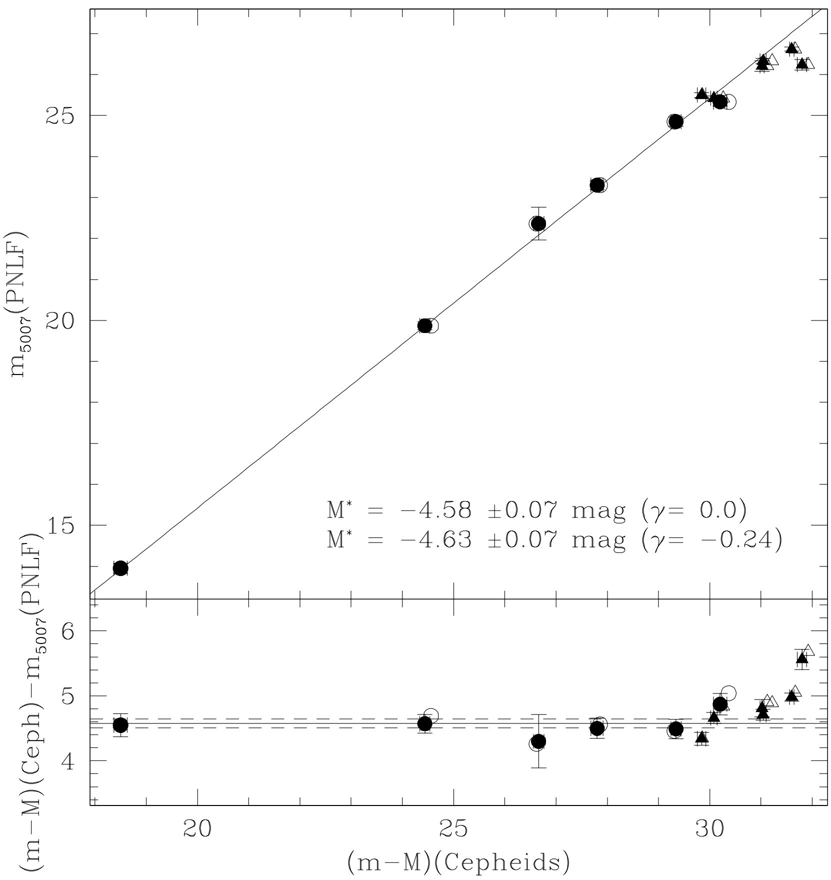
Planetary Nebulae have empirical universal luminosity function:

$$N(M) \propto \mathrm{e}^{0.307M}\left(1 - \mathrm{e}^{3(M_{\mathrm{PN}}-M)}\right) \qquad (5.26)$$

Measurement of "cutoff magnitude" $M_{\mathrm{PN}} \Longrightarrow$ Standard candle!

PN detection with narrow band filter of O[III] λ5007Å.

PN Luminosity Function, II



(Ferrarese et al., 2000, Fig. 3), left to right: LMC, M31, NGC 300, M81, M101, NGC 3368, and several galaxy groups.

Result of calibration using Cepheid distances (Ferrarese et al., 2000):

Cutoff of luminosity function:

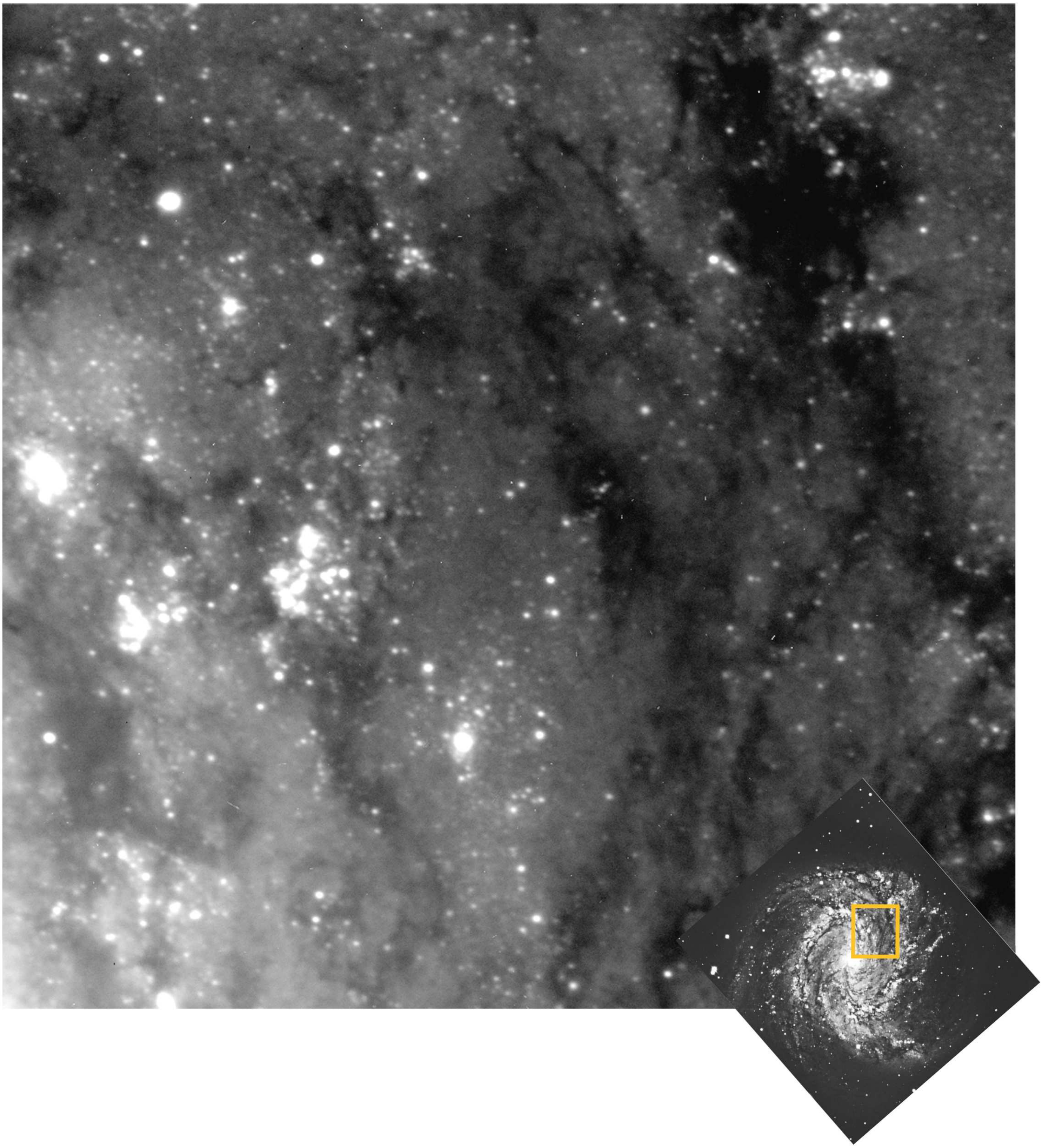
$$M_{\mathrm{PN}} = -4.58 \pm 0.13\,\mathrm{mag} \qquad (5.27)$$

Out to $\sim$40 Mpc with 8 m class telescope.

PN Luminosity Function, III

Caveats: Effects of metallicity, population age, parent galaxy most probably small, but

- Contamination by H II regions (but distinguish using Hα/[O III] ratio.
- Background emission-line galaxies at $z = 3.1$
- intracluster PNe (i.e., PNe outside galaxies)



The VLT Looks Deep into a Spiral Galaxy

ESO PR Photo 20/98 (23 June 1998)

M83

Brightest Stars, II

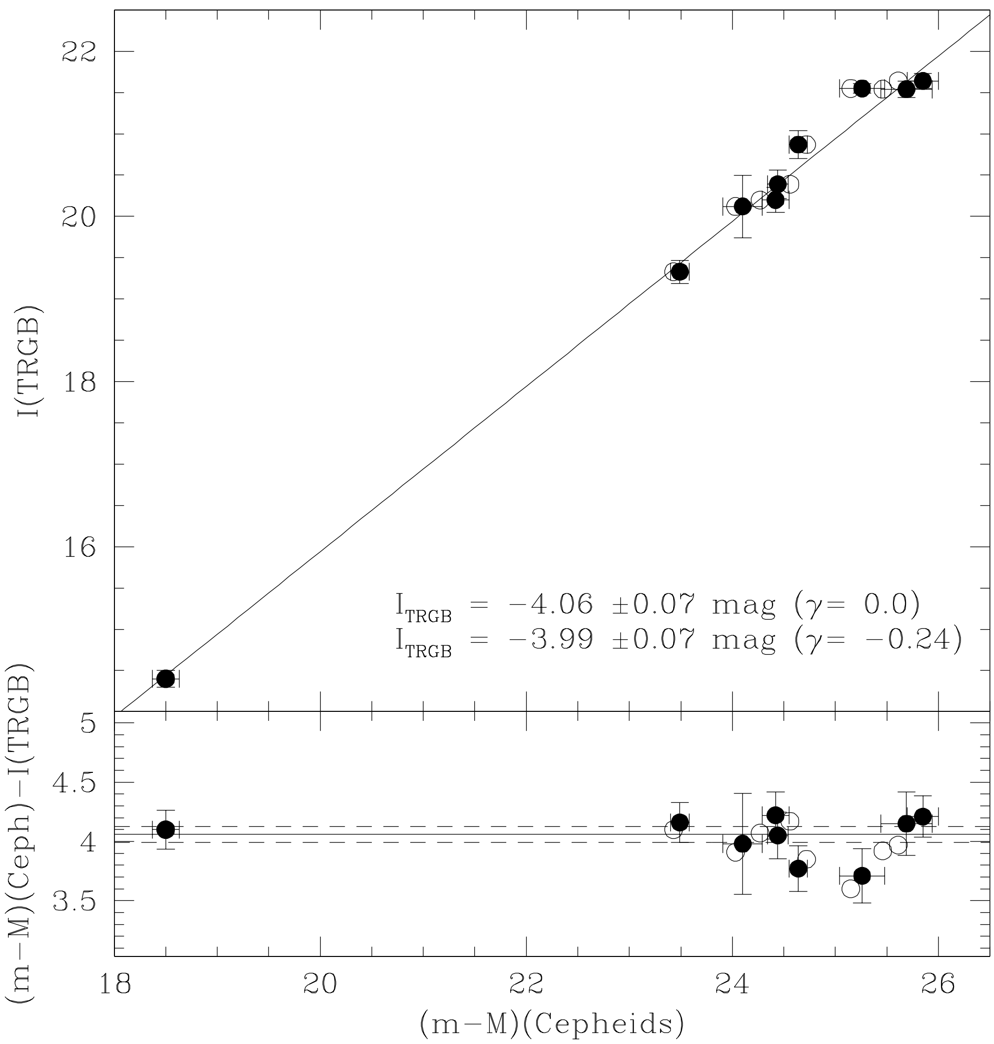
Brightest Stars= O, B, A supergiants, absolute magnitudes usable in local group, large scatter. Brightest stars possible: upper limit to stellar luminosity due to mass loss in supergiants

Possible Improvement: Strength of Balmer series lines. Hα and Hβ appear biased (class of supergiants with anomalously strong Balmer lines?).

Problems:

- Contamination by foreground halo stars $\Longrightarrow$ Choose stars with unusual color (rare, i.e. less foreground contamination): B $-$ V $<$ 0.4 or B $-$ V $>$ 2.0 $\Longrightarrow$ Tip of Red Giant Branch
- Internal extinction.
- Scatter in max. $L \Longrightarrow$ Average over brightest N stars (Sandage, Tammann: $N = 3$).
- Metallicity dependence.

Brightest Stars, III



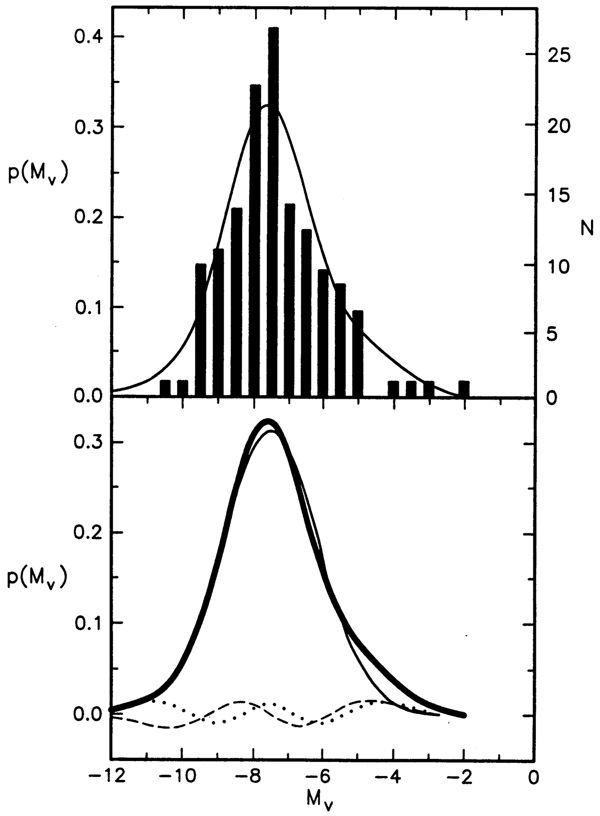
(Ferrarese et al., 2000, Fig. 1)

Tip of Red Giant Branch: Usable within local group, possibly out to Virgo.

Calibration:

$$M_{\mathrm{I}} = -4.06 \pm 0.13\,\mathrm{mag} \qquad (5.28)$$

Globular Cluster



Globular Cluster Luminosity Function very stable ≈Gaussian $\Longrightarrow$ Use maximum of distribution ("turnover magnitude", M_T) as standard candle.

(MW GCs, Abraham & van den Bergh, 1995, Fig. 1)

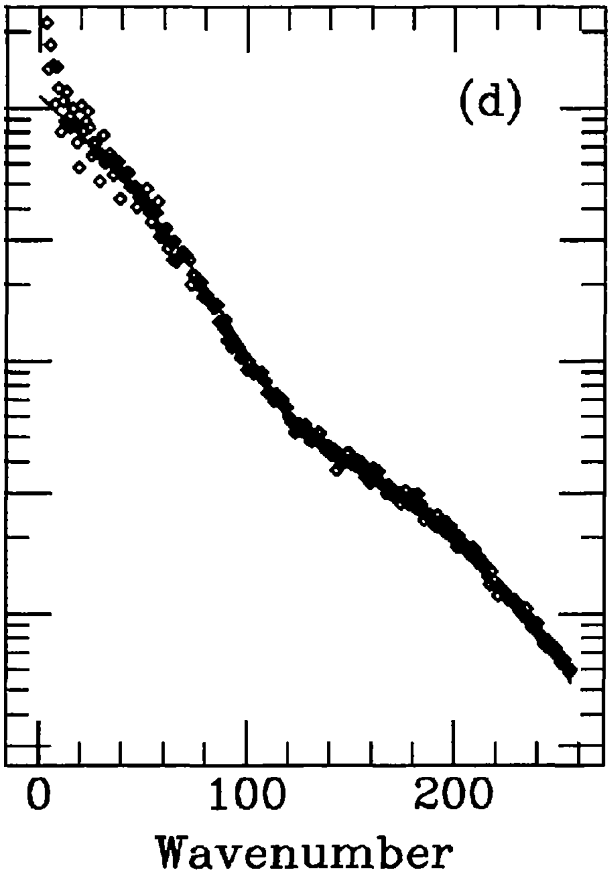
From Virgo and Fornax Cepheid distances (Ferrarese et al., 2000):

$$M_{\mathrm{T,V}} = -7.60 \pm 0.25\,\mathrm{mag} \qquad (5.29)$$

Caveats:

1. M_T depends on luminosity and type of host galaxy (GC of dwarf galaxies weaker by $\sim$ 0.3 in V).
2. Metallicity of galaxy cluster influences M_T.
3. Measurement difficult (need the weak GCs!).
4. Large scatter in data $\Longrightarrow$ Method rather unreliable.

Surface Brightness Fluctuations, I



(Ajhar et al., 1997, Fig. 3d)

For early type galaxies: Assume N stars in picture element (pixel), with average flux f.

$\Longrightarrow$ Mean pixel intensity:

$$\mu = Nf \tag{5.30}$$

μ independent of distance, since $N \propto r^2$ and $f \propto r^{-2}$.

Standard Deviation (Poisson):

$$\sigma = \sqrt{N} f \propto r^{-1} \tag{5.31}$$

Therefore:

$$f = \frac{\sigma^2}{\mu} = \frac{L}{4\pi r^2} \tag{5.32}$$

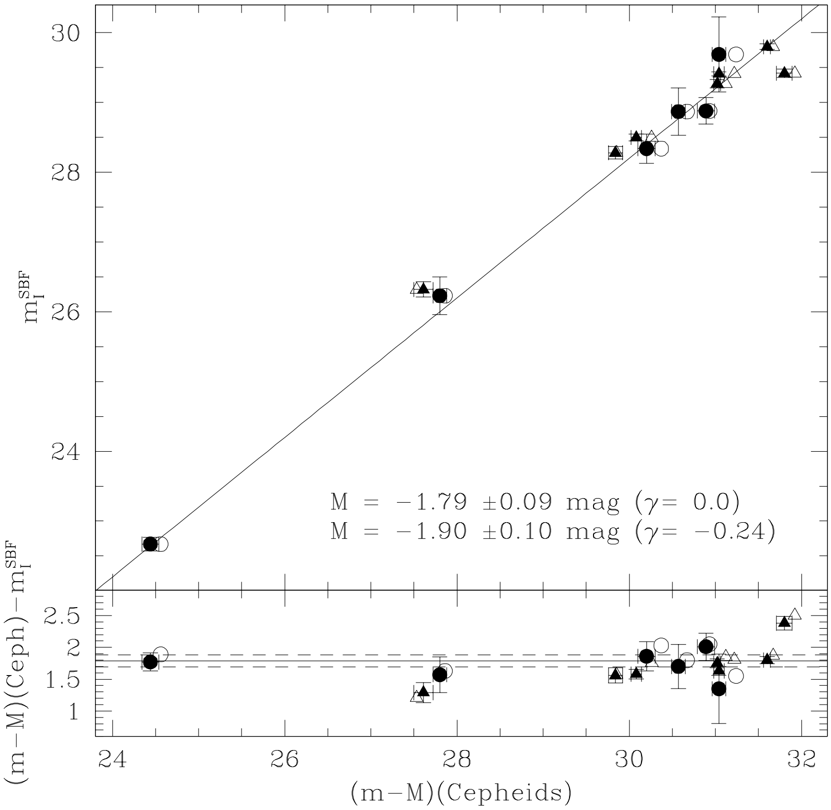
which gives the distance r.

Review: Blakeslee, Ajhar & Tonry (1999).

Complication: Adjacent pixels not independent (point spread function of telescope!)

$\Longrightarrow$ Use radial power spectrum to obtain σ^2 and μ.

Surface Brightness Fluctuations, II



(Ferrarese et al., 2000, Fig. 5)

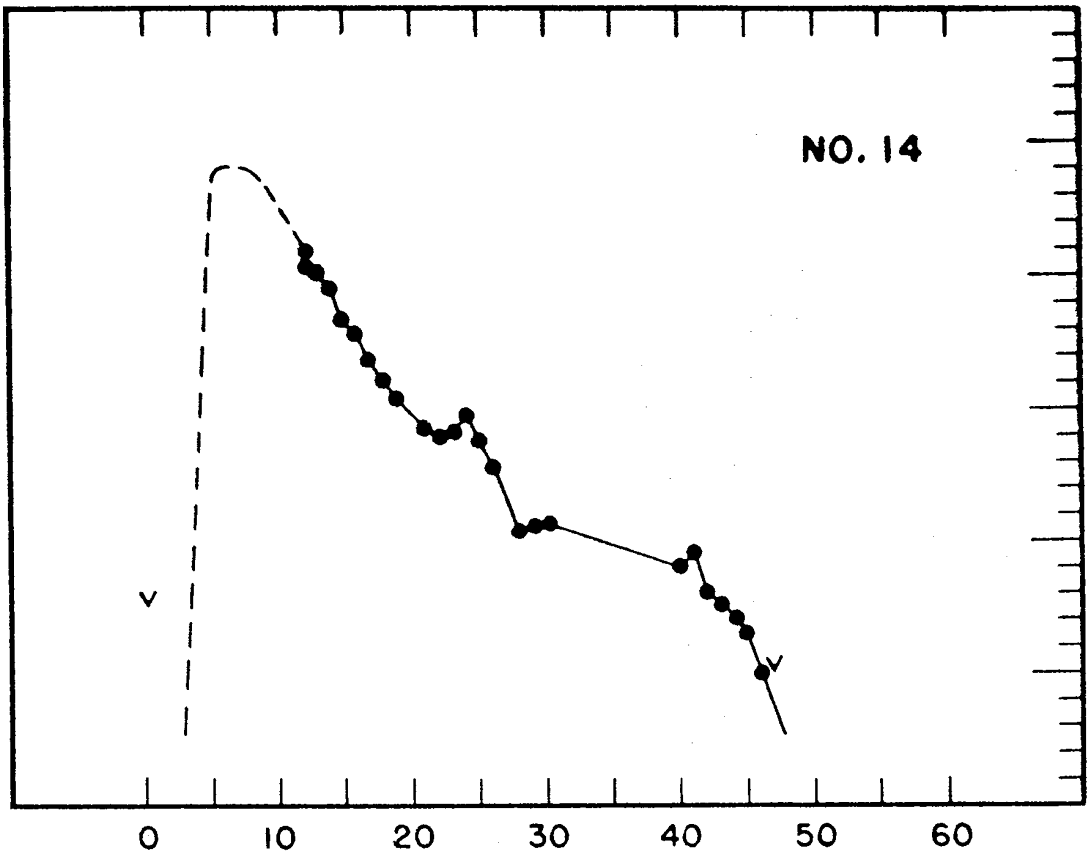
Luminosity of galaxy dominated by Red Giant Branch stars $\Longrightarrow$ Strong wavelength and color dependence $\Longrightarrow$ Primary calibration: I-band plus broad-band color dependency to give standard candle.

Often also used: HST WFPC2 plus F814W filter (close to I-band),

$$M_{\mathrm{F814W}} = (-1.70 \pm 0.16) + (4.5 \pm 0.3)\left[(\mathrm{V}-\mathrm{I})_0 - 1.15\right] \quad (5.33)$$

Works out to $\sim 70\,\mathrm{Mpc}$ with HST.

Novae, I



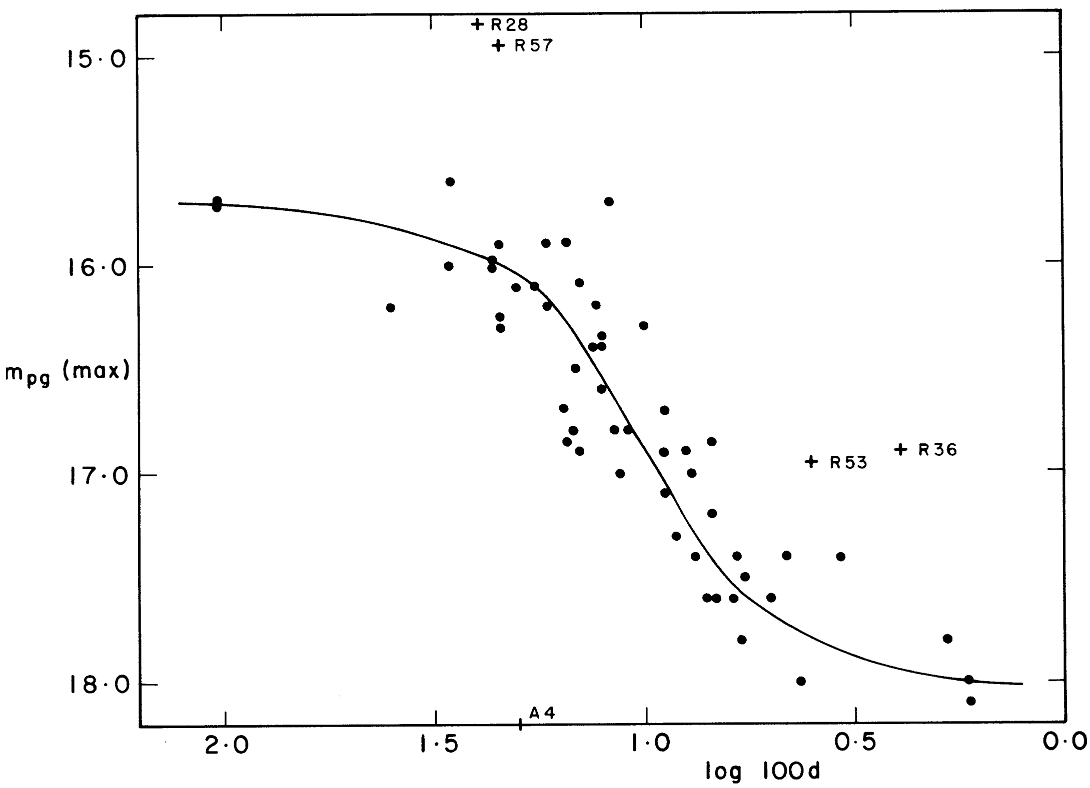
(Nova in M31, Arp, 1956, p. 18)

"classical nova"$=$ explosion on surface of white dwarf

Novae only in binary systems $\Longrightarrow$ slow accretion of material onto WD $\Longrightarrow$ outer skin reaches $M_{\rm crit}$ for fusion $\Longrightarrow$ explosion $\Longrightarrow$ ejection of $10^{-6}\dots10^{-4}\,M_\odot$ with $v\sim 500$ km/s

Explosion produces characteristic lightcurve.

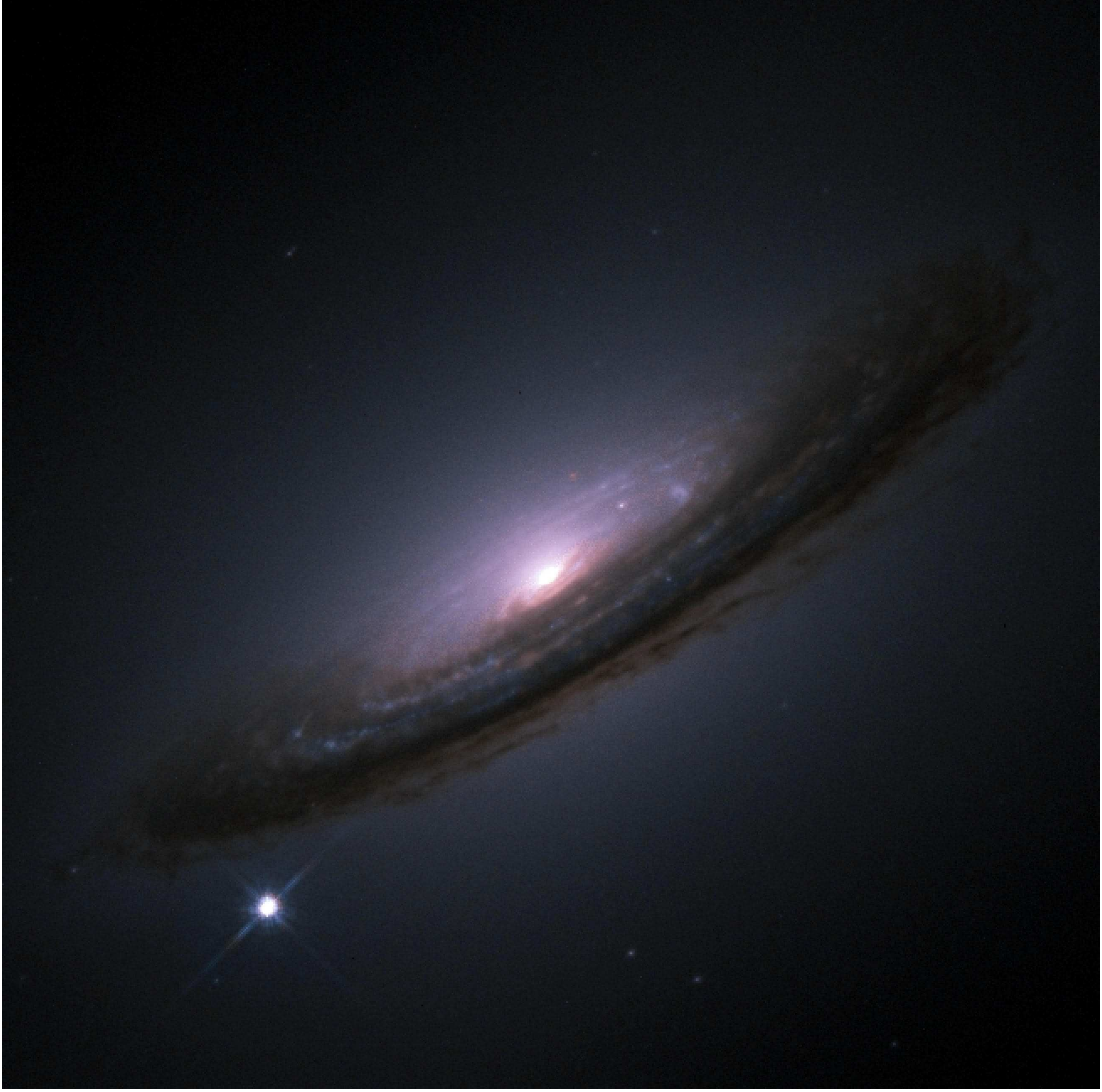
Novae, II



(van den Bergh & Pritchet, 1986, Fig. 1).

Strong scatter in lightcurves (higher $L_{\max} \Longrightarrow$ faster decline, but typically $\sim 3\times$ brighter than Cepheids), but good Correlation luminosity vs. decline timescale (t_i, time to reach $m(t_i) = m_{\max} + i$).

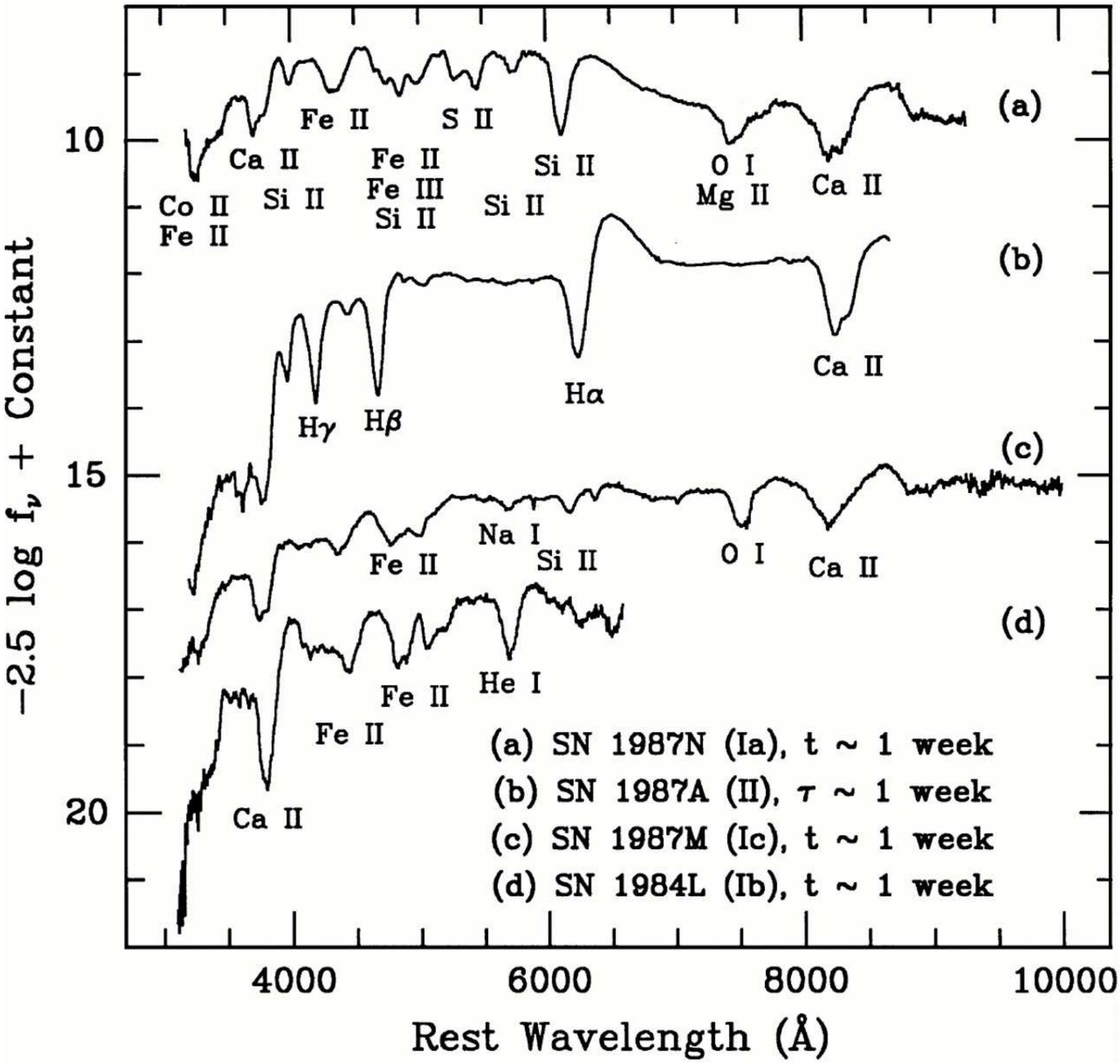
Calibration: galactic novae.



SN1994d (HST WFPC)

Supernovae have luminosities comparable to whole galaxies: $\sim 10^{51}$ erg/s in light, $100\times$ more in neutrinos.

Type Ia Supernovae, II



(Filippenko, 1997, Fig. 1); t: time after maximum light; τ: time after core collapse; P Cyg profiles give $v \sim 10000\,\mathrm{km\,s^{-1}}$

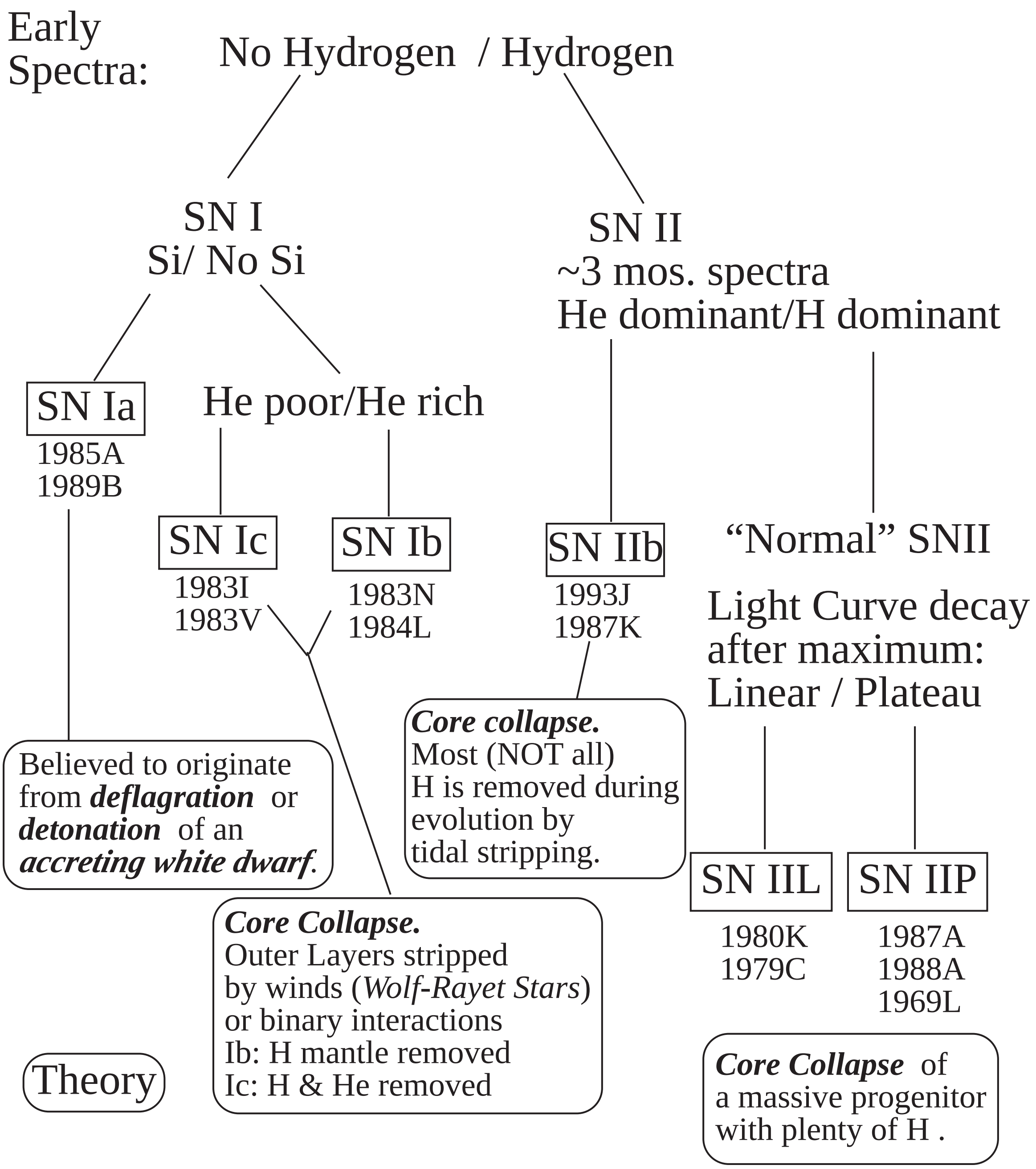
Rough classification (Minkowski, 1941):

Type I: no hydrogen in spectra; subtypes Ia, Ib, Ic

Type II: hydrogen present, subtypes II-L, II-P

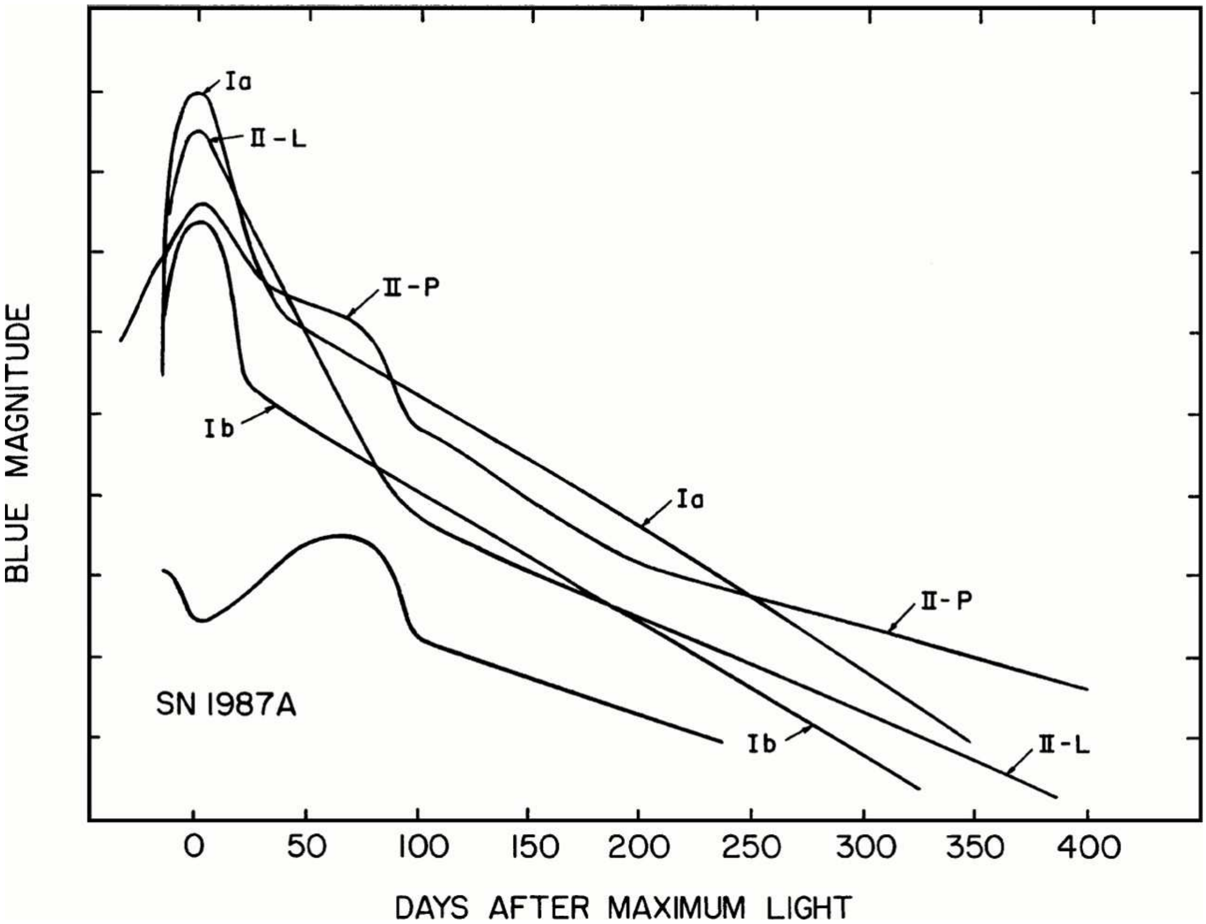
Note: pre 1985 subtypes Ia, Ib had different definition than today $\Longrightarrow$ beware when reading older texts.

Type Ia Supernovae, III



courtesy M.J. Montes

Type Ia Supernovae, IV

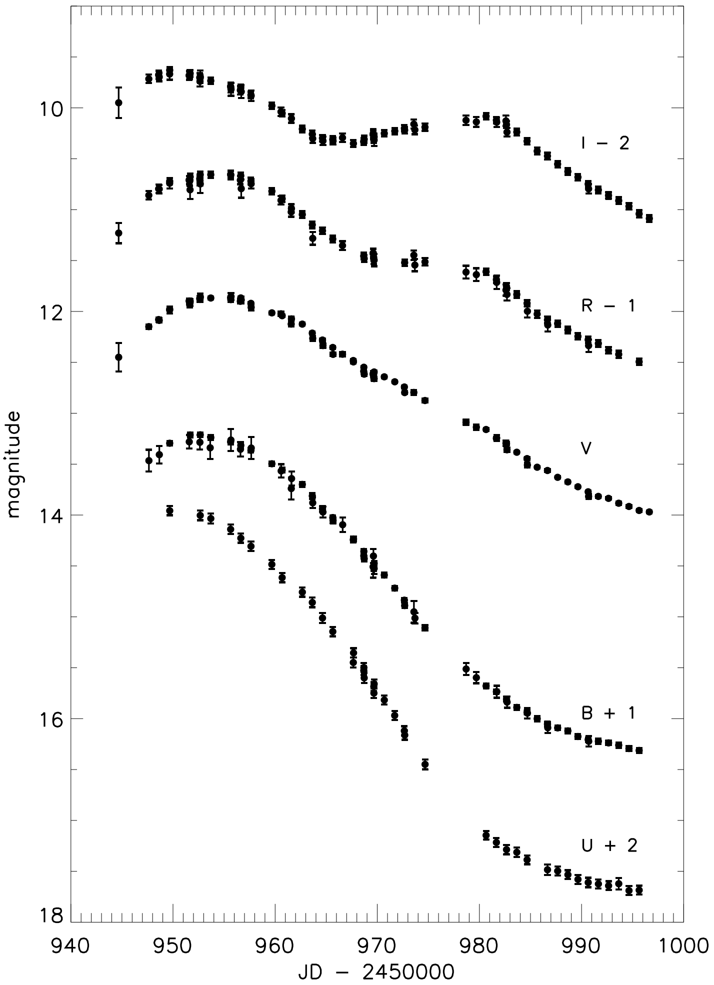


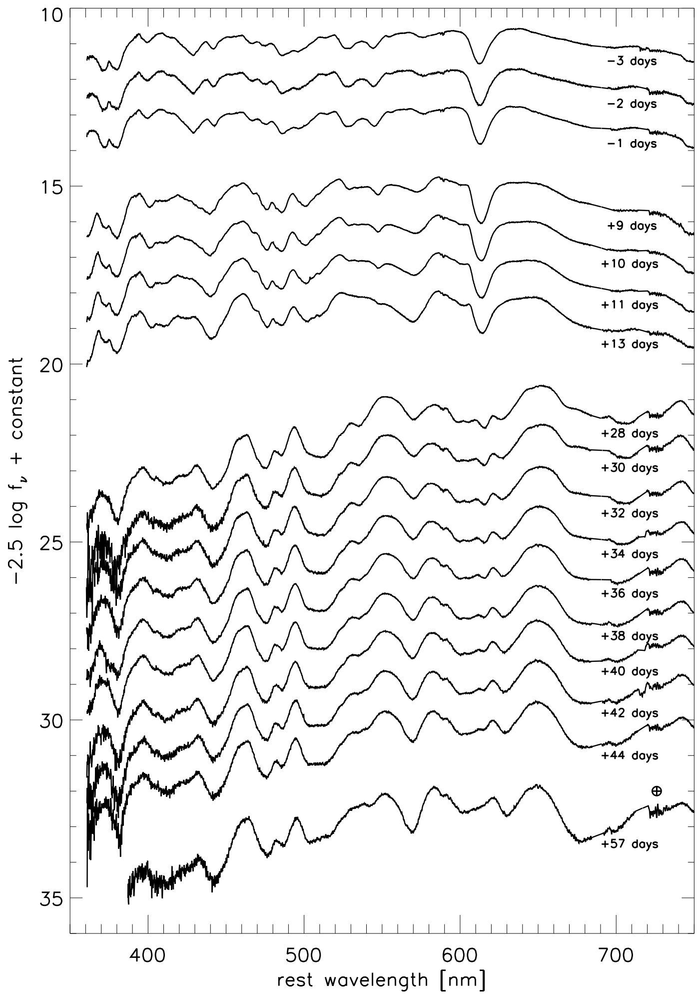
(Filippenko, 1997, Fig. 3)

Light curves of SNe I all very similar, SNe II have much more scatter.

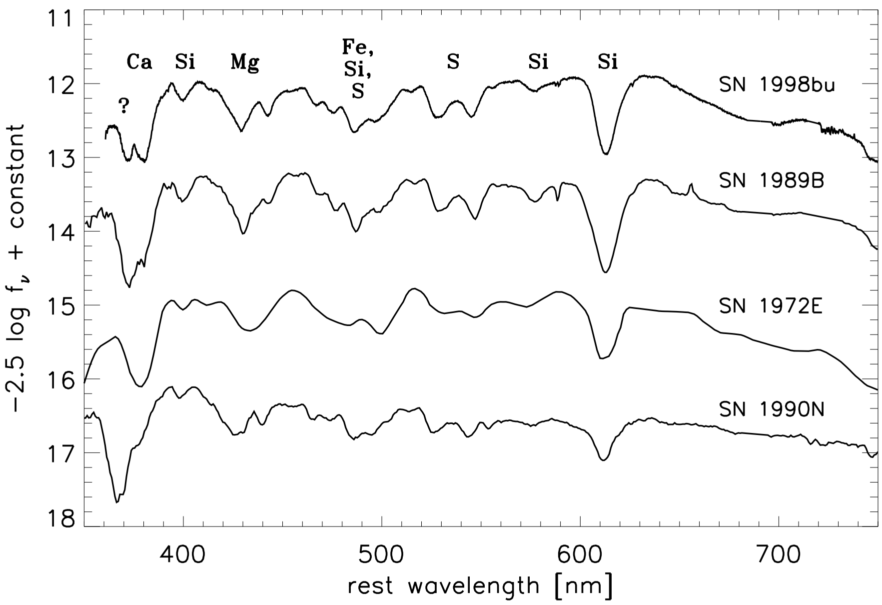
SNe II-L ("linear") resemble SNe I

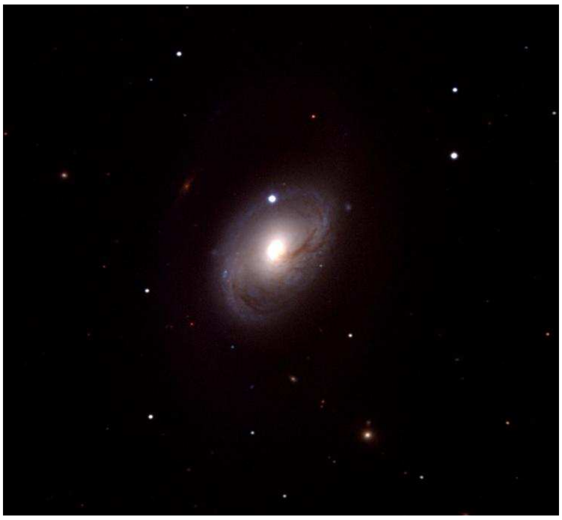
SNe II-P ("plateau") have const. brightness to within 1 mag for extended period of time.





(SN 1998bu in M96, Jha et al., 1999, Figs. 2 and 4)





(SN 1998bu, Jha et al., 1999, Fig. 6)

90 cm CTIO, N. Suntzeff

Type Ia Supernovae, VI

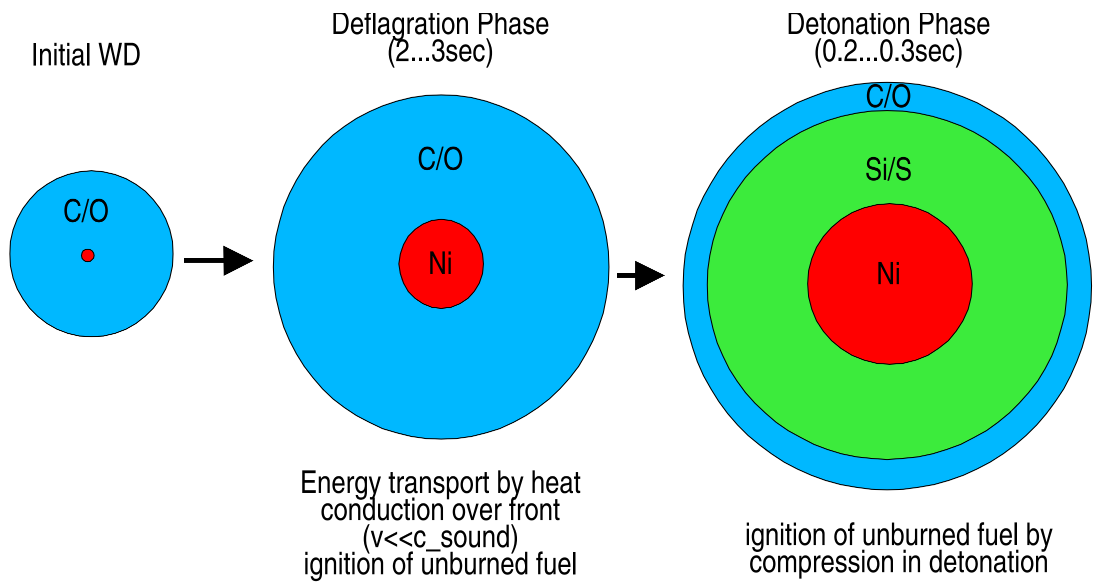
Clue on origin from supernova statistics:

- SNe II, Ib, Ic: never seen in ellipticals; rarely in S0; generally associated with spiral arms and H II regions.

$\Longrightarrow$ progenitor of SNe II, Ib, Ic: massive stars ($\gtrsim 8\,M_\odot$) $\Longrightarrow$ core collapse

- SNe Ia: all types of galaxies, no preference for arms.

$\Longrightarrow$ progenitor of SNe Ia: accreting carbon-oxygen white dwarfs, undergoing thermonuclear runaway



after P. Höflich

Type Ia Supernovae, VII

SN Ia = Explosion of CO white dwarf when pushed over Chandrasekhar limit (1.4 $M_\odot$) (via accretion?).

$\Longrightarrow$ Always similar process

$\Longrightarrow$ Very characteristic light curve: fast rise, rapid fall, exponential decay with half-time of 60 d.

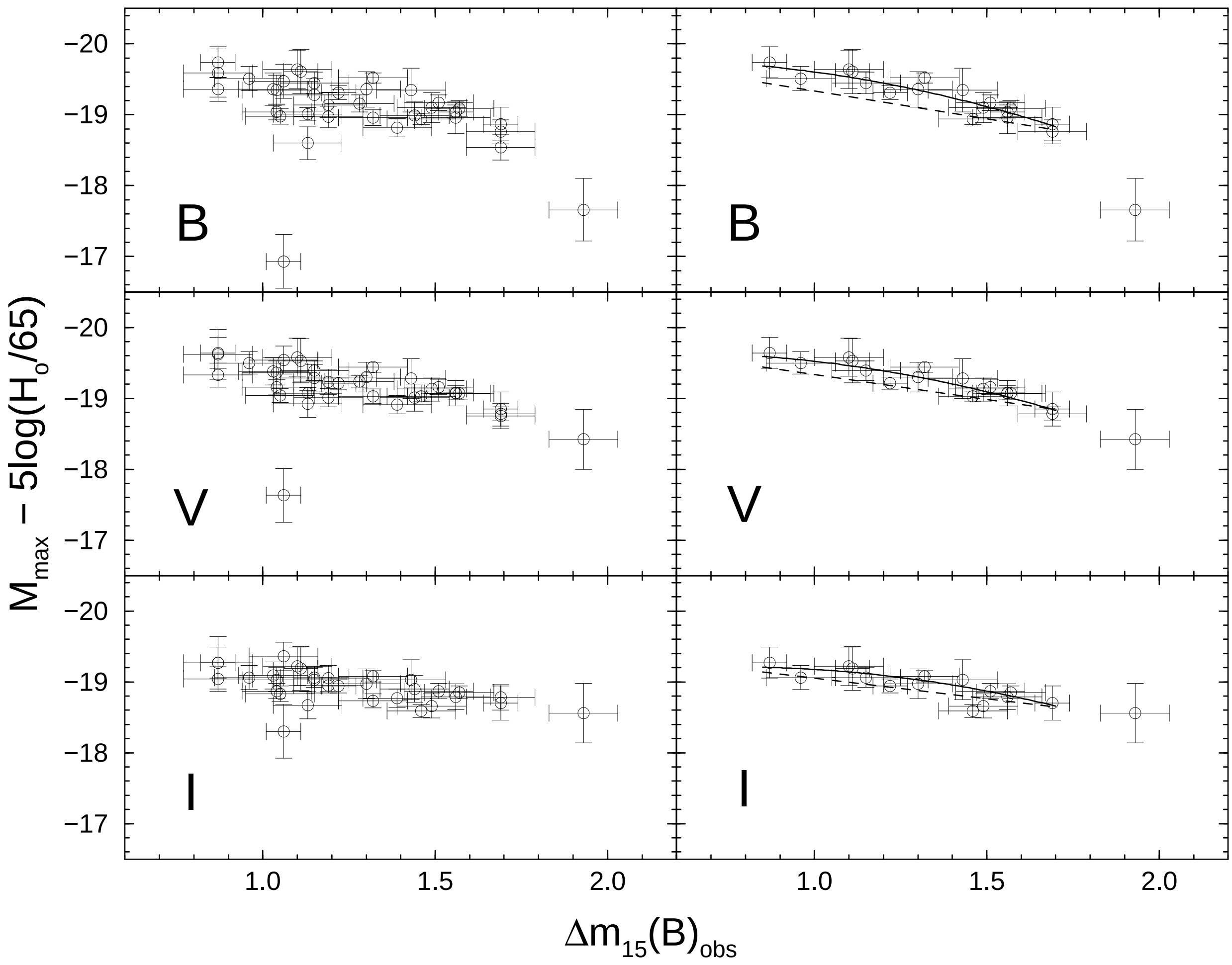
60 d time scale from radioactive decay $\mathrm{Ni}^{56} \rightarrow \mathrm{Co}^{56} \rightarrow \mathrm{Fe}^{56}$ ("self calibration" of lightcurve if same amount of Ni^{56} produced everywhere).

Calibration: SNe Ia in nearby galaxies where Cepheid distances known.
At maximum light:

$$M_\mathrm{B} = -18.33 \pm 0.11 + 5 \log h_{100} \qquad (5.34)$$

($L \sim 10^{9\ldots10}\,L_\odot$).
Intrinsic dispersion: $\lesssim$0.25 mag (possibly due to size of clusters analyzed?!?)
Observable out to 1000 Mpc

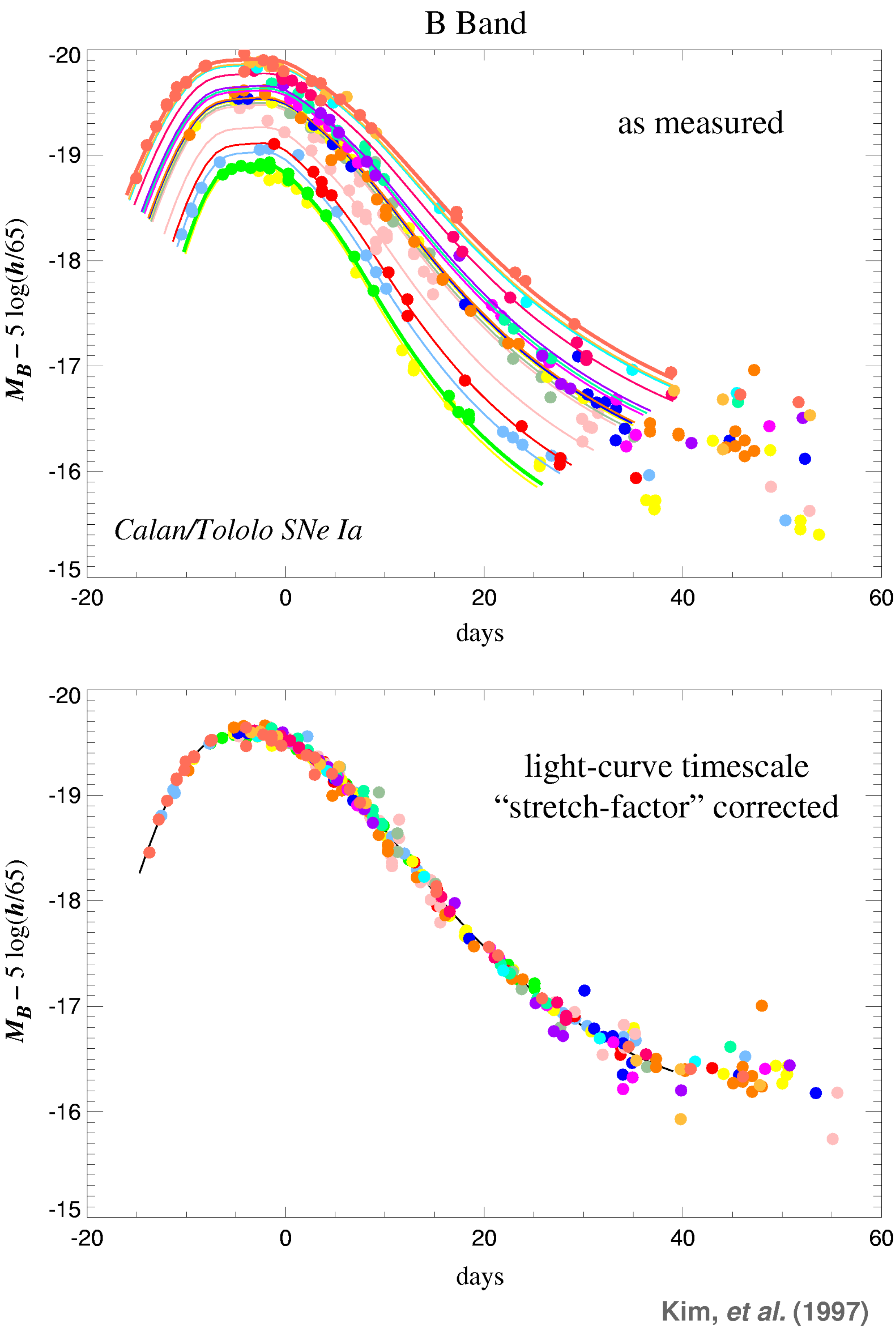
Type Ia Supernovae, VIII



(Phillips et al., 1999, Fig. 8)

Caveats:

1. Are they *really* identical? $\Longrightarrow$ history of pre-WD star?
2. Correction for extinction in parent galaxy difficult.
3. Baade-Wesselink for calibration Eq. (5.34) depends crucially on assumed $(\mathrm{B}-\mathrm{V})$-T_{eff} relation.
4. Some SN Iae spectroscopically peculiar $\Longrightarrow$ Do not use these!
5. Decline rate and color vary, but max. brightness and decline rate correlate (see figure).



Lightcurves of Hamuy et al. SN Ia sample (18 SNe discovered within 5 d past maximum, with $3.6 < \log cz < 4.5$, i.e., $z < 0.1$, after correction of systematic effects and time dilatation (Kim et al., 1997).

Type Ia Supernovae, X

Recalibration of SN Ia distances with Cepheids gives (Gibson et al., 2000):

$$\begin{aligned}\log H_0 = 0.2\Big\{ & M_{\mathrm{B}}^{\max} - 0.720(\pm 0.459) \\ & \cdot \left[\Delta m_{\mathrm{B},15,t} - 1.1\right] - 1.010(\pm 0.934) \\ & \cdot \left[\Delta m_{\mathrm{B},15,t} - 1.1\right]^2 + 28.653(\pm 0.042)\Big\} \end{aligned} \quad (5.35)$$

where

$$\Delta m_{\mathrm{B},15,t} = \Delta m_{\mathrm{B},15} + 0.1E(\mathrm{B}-\mathrm{V}) \quad (5.36)$$

where

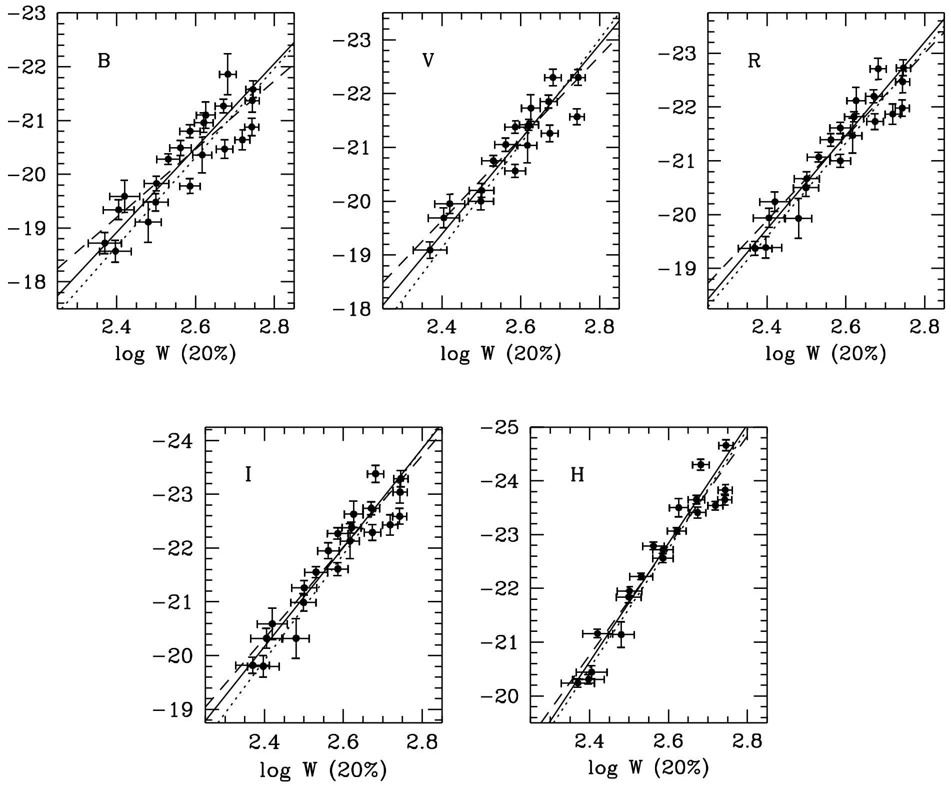
$\Delta m_{\mathrm{B},15}$: observed 15 d decline rate,

$E(\mathrm{B}-\mathrm{V})$: total extinction (galactic+intrinsic).

Eq. (5.35) valid for B-band, equivalent formulae exist for V and I.

Overall, the calibration is good to better than 0.2 mag in B.

Tully-Fisher, I



(Sakai et al., 2000, Fig. 1)

Tully-Fisher relation for spiral galaxies: Width of 21 cm line of H correlated with galaxy luminosity:

$$M = -a \log\left(\frac{W_{20}}{\sin i}\right) - b \qquad (5.37)$$

where W_{20}: 20% line width (km/s; typically $W_{20} \sim 300\,\text{km/s}$), i inclination angle.

For the B- and I-Bands (Sakai et al., 2000):

	B	I
a	7.97± 0.72	9.24± 0.75
b	19.80± 0.11	21.12± 0.12

Tully-Fisher, II

Qualitative Physics: Line width related to mass of galaxy: $W/2 \sim V_{\max}$, where $V_{\max}$ max. velocity of rotation curve

$\Longrightarrow$ Assume $M/L =$ const. (good assumption)

$\Longrightarrow$ width related to luminosity.

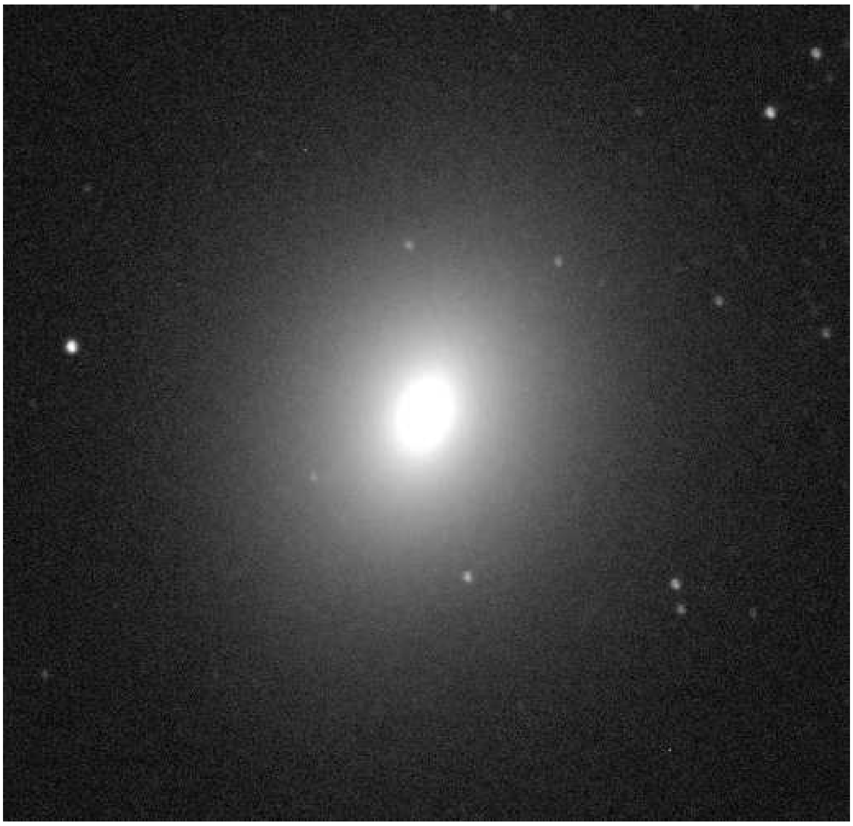
Detailed physical basis unknown. Might be related to galaxy formation in CDM models ("hierarchical clustering", see later).

I-band is better (less internal extinction).

Caveats:

1. Determination of inclination i.
2. Influence of turbulent motion within galaxy.
3. Constants dependent on galaxy type (Sa and Sb similar, Sc more luminous by factor of ∼2).
4. Optical extinction.
5. Intrinsic dispersion ∼0.2 mag.
6. Barred Galaxies problematic.

D_n-σ, I



M32 (companion of Andromeda), courtesy W. Keel

"Faber-Jackson" law for elliptical galaxies:

The luminosity L of an elliptical galaxy scales with its intrinsic velocity dispersion, σ, as $L \propto \sigma^4$.

Note that ellipticals have virtually no Hydrogen
$\Longrightarrow$ cannot use 21 cm.

Ellipticals:

$$M_{\mathrm{B}} = -19.38 \pm 0.07 - (9.0 \pm 0.7)(\log\sigma - 2.3) \quad (5.38)$$

Lenticulars:

$$M_{\mathrm{B}} = -19.65 \pm 0.08 - (8.4 \pm 0.8)(\log\sigma - 2.3) \quad (5.39)$$

D_n-σ, II

The Faber-Jackson law is a specialized case of the more general D_n–σ-relation:

The intensity profile of an elliptical galaxy is given by de Vaucouleurs' $r^{1/4}$ law:

$$I(r) = I_0 \exp\left(-(r/r_0)^{1/4}\right) \quad \Longrightarrow \quad L = \int I \propto I_0 r_0^2 \tag{5.40}$$

Because of the virial theorem ($E_\mathrm{kin} = -E_\mathrm{pot}/2$):

$$\frac{1}{2} m\sigma^2 = G\frac{mM}{r_0} \quad \Longleftrightarrow \quad \sigma^2 \propto \frac{M}{r_0} \tag{5.41}$$

where σ: velocity dispersion.
Assume mass-to-light ratio

$$M/L \propto M^\alpha \tag{5.42}$$

($\alpha \sim 0.25$). and use r_0 from Eq. (5.40) to obtain

$$L^{1+\alpha} \propto \sigma^{4-4\alpha} I_0^{\alpha-1} \tag{5.43}$$

This is called the "fundamental plane" relationship (Dressler et al., 1987).

D_n-σ, III

Observationally easier: Instead of inserting r_0, I_0, measure diameter D_n of aperture to reach some mean surface brightness (typically sky brightness, $20.75\,\mathrm{mag\,arcsec^{-2}}$ in B), and use calibration.

Note: Assumptions are

1. M/L same everywhere.
2. ellipticals have same stellar population everywhere

Calibration paper: Kelson et al. (2000).

Brightest Cluster Galaxies

For very large distances: use brightest cluster galaxies as indicators.

Assumption: Galaxy clusters are similar, brightest galaxy has similar brightness.

Calibration: Close clusters.

10 close galaxy clusters: brightest galaxy has

$$M_{\mathrm{V}} = -22.82 \pm 0.61 \tag{5.44}$$

Problems:

- Cosmological evolution (e.g., galaxy cannibalism)
- Scatter in brightest galaxy large $\Longrightarrow$ Use 2nd, 3rd brightest, or average brightest N galaxies.

$\Longrightarrow$ The method of brightest cluster galaxies should not be used anymore.

Path to H_0

To obtain H_0: need two things:

1. distances, and
2. redshifts

Distances:

Hubble Space Telescope Key Project on Extragalactic Distance Scale.

Summary paper: Freedman et al. (2001), there are a total of 29 papers on the HST key project!

Strategy:

1. Use high-quality standard candle: Cepheid variables as primary distance calibrator.
2. Calibrate secondary calibrators that work out to $cz = 10000\,\mathrm{km\,s^{-1}}$:
   - Tully-Fisher,
   - Type Ia Supernovae,
   - Surface Brightness Fluctuations,
   - Fundamental-plane for Ellipticals.
3. Combine uncertainties from these methods.

Redshift determination is obviously trivial compared to distance determination. . .

Velocity Field, I

Before determining H_0: correct for influence of velocity field (cluster motion wrt. comoving coordinates).

The observed redshift is given by

$$1 + z = (1 + z_\mathrm{R})\left(1 - \frac{v_0}{c} + \frac{v_\mathrm{G}}{c}\right) \tag{5.45}$$

where

v_0: observer's radial velocity in direction of galaxy

v_G: radial velocity of the galaxy, difficult to find

z_R: cosmological redshift

Older galaxy catalogues often attempt to correct the measured values of z to produce “corrected redshifts”, e.g., by setting $v_\mathrm{G} = 0$ and

$$1 + z = (1 + z_\mathrm{R})\left(1 + \frac{v_0}{c}\right) \sim 1 + z_\mathrm{R} - \frac{v_0}{c} \tag{5.46}$$

and thus

$$z_\mathrm{R} \sim z + \frac{v_0}{c} \tag{5.47}$$

since v_0 was up to COBE not well known $\Longrightarrow$ introduces unnecessary problems $\Longrightarrow$ correction not used anymore in recent redshift surveys!

see Harrison & Noonan (1979) for details

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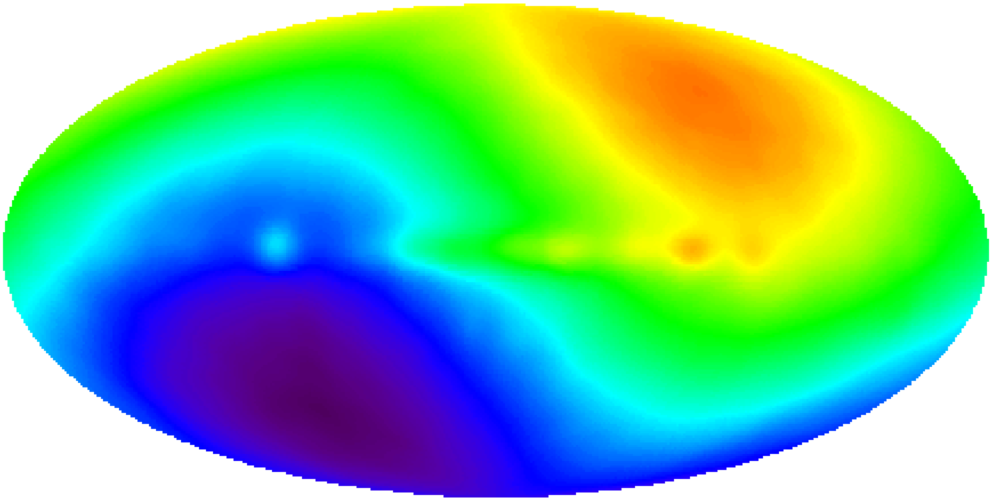
Velocity Field, II



(Bennett et al., 1996, COBE DMR;)

v_0 is easy to find $\Longrightarrow$ Measure velocity of Earth with respect to 3 K radiation. COBE finds speed of (369.1 ± 2.6) km/s, such that

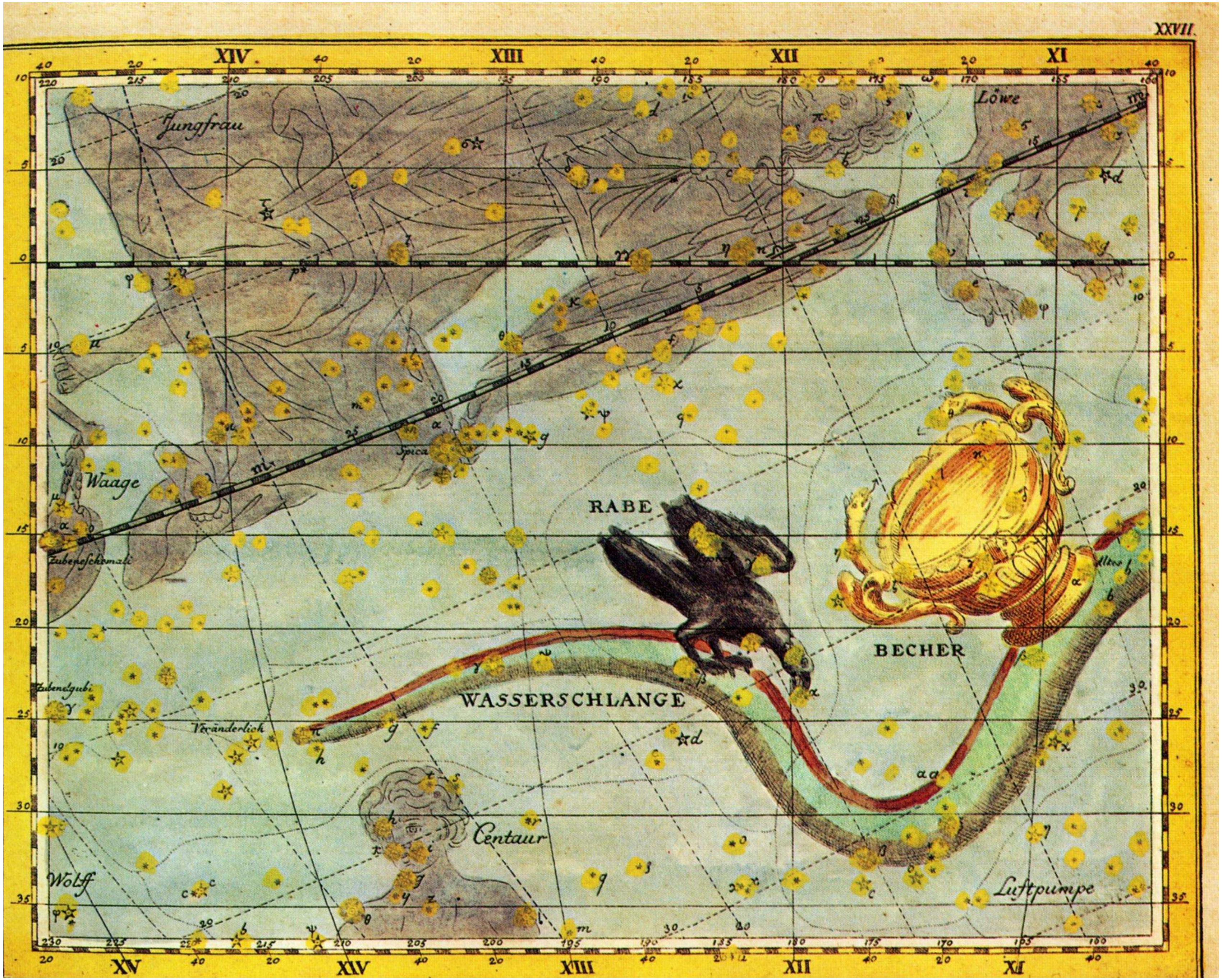
$$v_0 = 370\,\mathrm{km\,s^{-1}} \cdot \cos\theta_{\mathrm{CMB}} \tag{5.48}$$

where $\theta_{\mathrm{CMB}} = \angle(\mathbf{v}, \mathbf{v}_{\mathrm{CMB}})$, and $\mathbf{v}_{\mathrm{CMR}}$ points towards

$$(l, b) = (264.26^\circ \pm 0.33^\circ, 48.22^\circ \pm 0.13^\circ)$$
$$(\alpha, \delta)_{\mathrm{J2000.0}} = (11^{\mathrm{h}}12.2^{\mathrm{m}} \pm 0.8^{\mathrm{m}}, -7.06^\circ \pm 0.16^\circ)$$

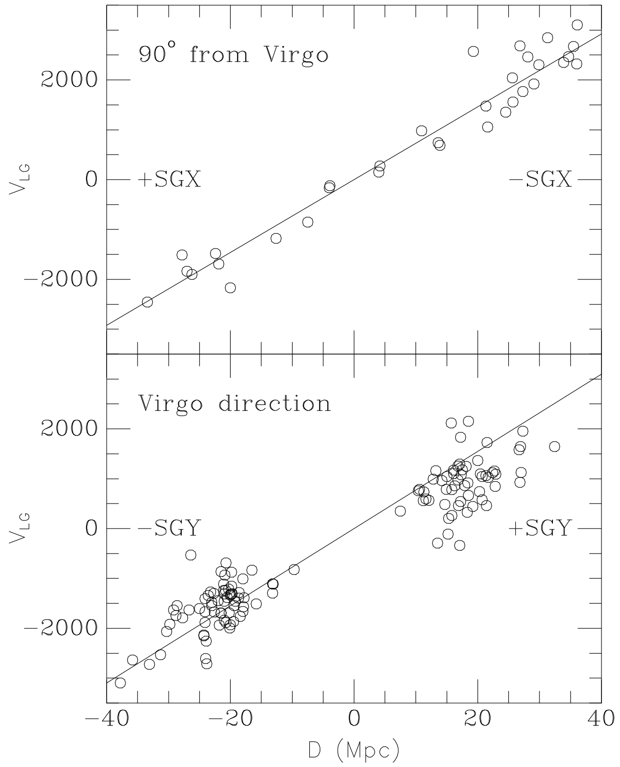
in constellation Crater.

Velocity comes from measured Dipole temperature anisotropy of $\Delta T = 3.353 \pm 0.024$ mK of 3K black-body spectrum of $T = 2.725 \pm 0.020$ K, using $\Delta T/T = v/c$.



The constellation Crater ("Becher") in Johan Elert Bode's Sternatlas
(after Slawik/Reichert, Atlas der Sternbilder, Spektrum, 2004)

Velocity Field, IV



To get feeling for v_G out to Virgo, need to study local velocity field surrounding local group and beyond.

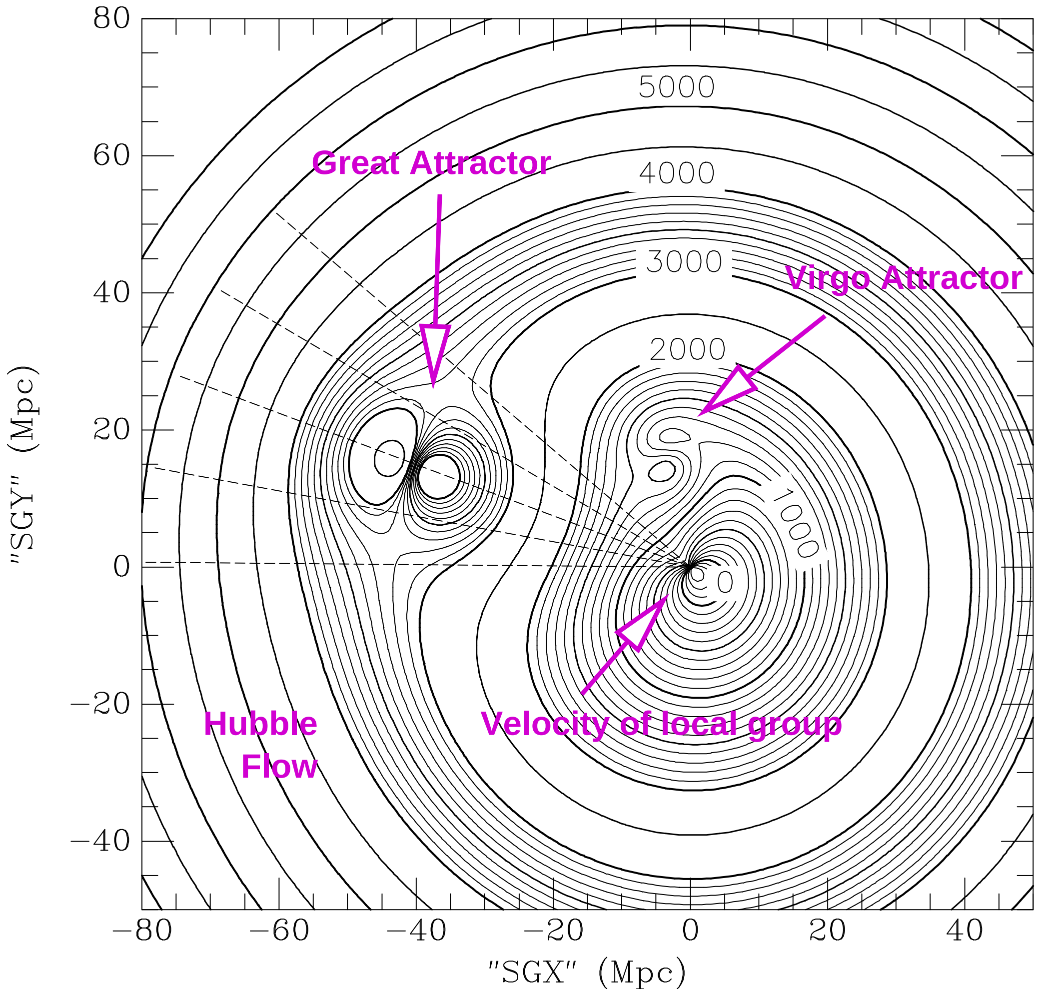
Two major velocity components:

1. Virgocentric infall (known since mid-1970s)
2. Motion towards great attractor ("Seven Samurai", 1980)

plus virialized galaxy motions within clusters.

General analysis: build maximum likelihood model of velocity field including above components *plus* Hubble flow. See Tonry et al. (2000) for details.

Velocity Field, V



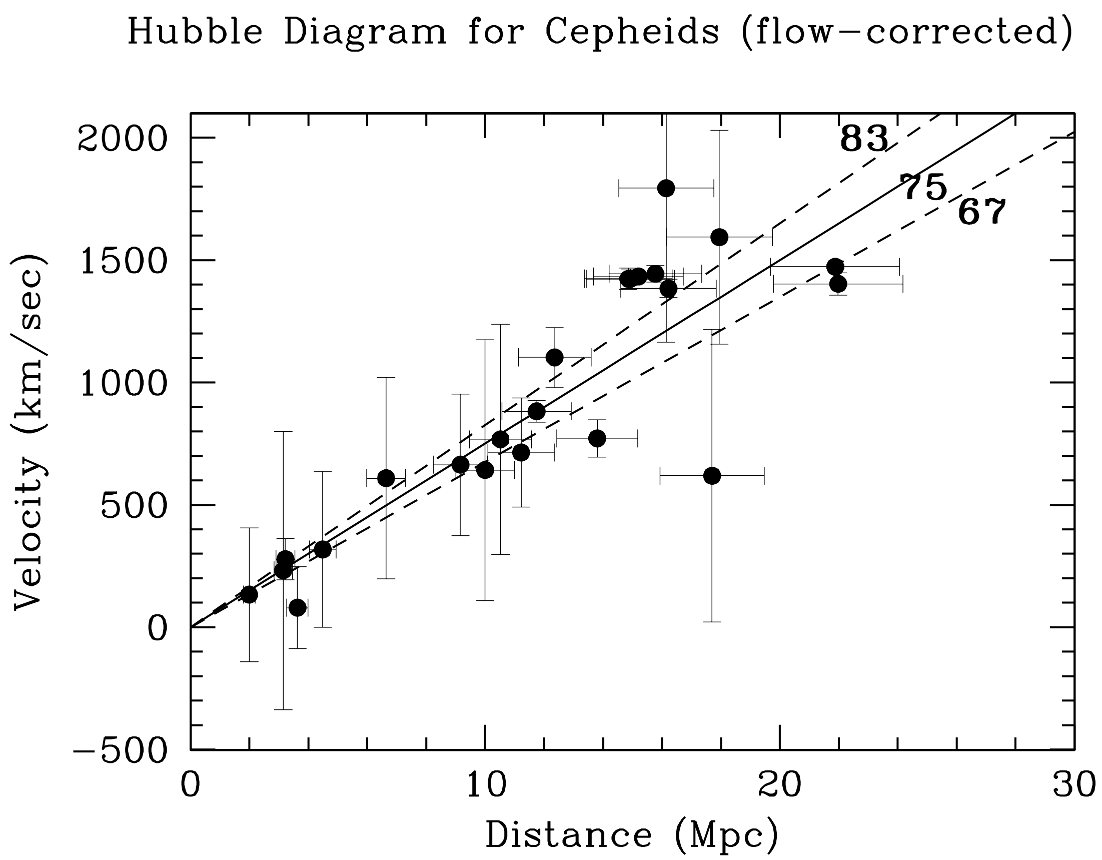
(Tonry et al., 2000, Fig. 20)

Decomposition of velocity field: (Mould et al., 2000, Tab. A1, note that Tonry et al. 2000 find slightly different values)

	$\alpha_{1950.0}$	$\delta_{1950.0}$	v (km s^{-1})
Virgo	$12^{\mathrm{h}}28^{\mathrm{m}}$	$+12^\circ 40'$	957
GA	$13^{\mathrm{h}}20^{\mathrm{m}}$	$+44^\circ 00'$	4380
Shapley	$13^{\mathrm{h}}30^{\mathrm{m}}$	$+31^\circ 00'$	13600

(v wrt. center of local group; *not* taking Hubble flow into account!).

H from HST



Freedman et al. (2001, Fig. 1)

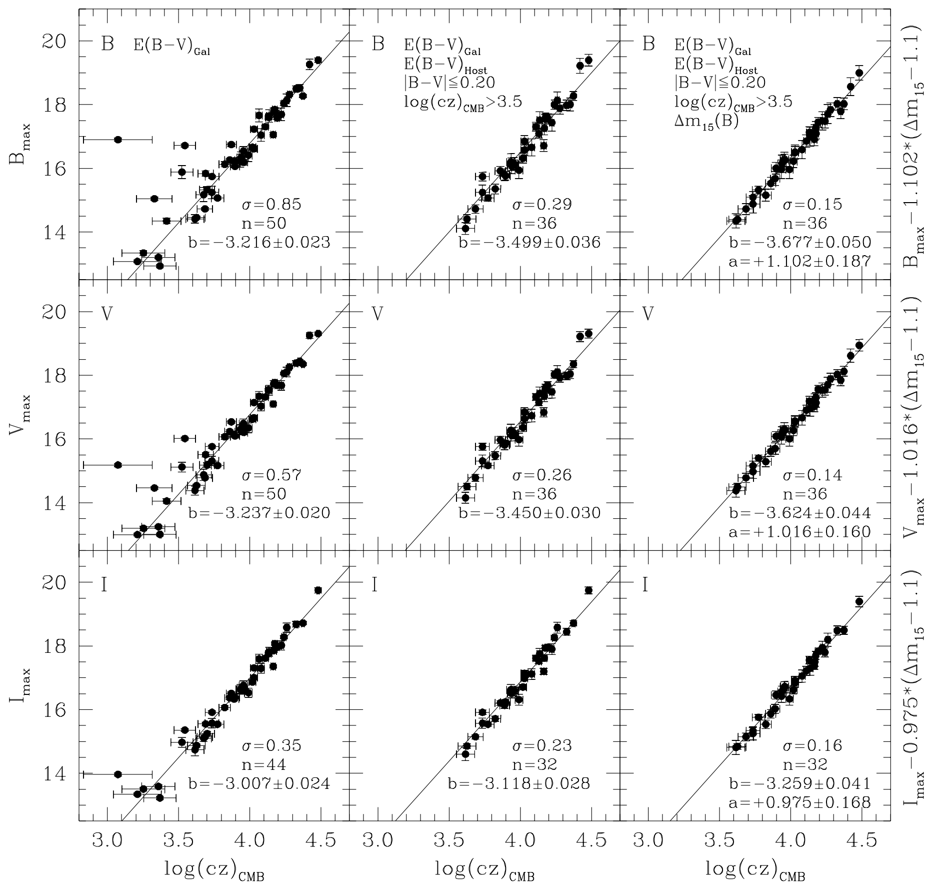
To obtain H_0:

1. Determine d with Cepheids and HST
2. Determine "v", corrected for local velocity field
3. Draw Hubble-diagram
4. Regression Analysis $\Longrightarrow$ H_0

Value from HST Key Project:

$$H_0 = 75 \pm 10\,\mathrm{km/s/Mpc} \tag{5.49}$$

H from HST

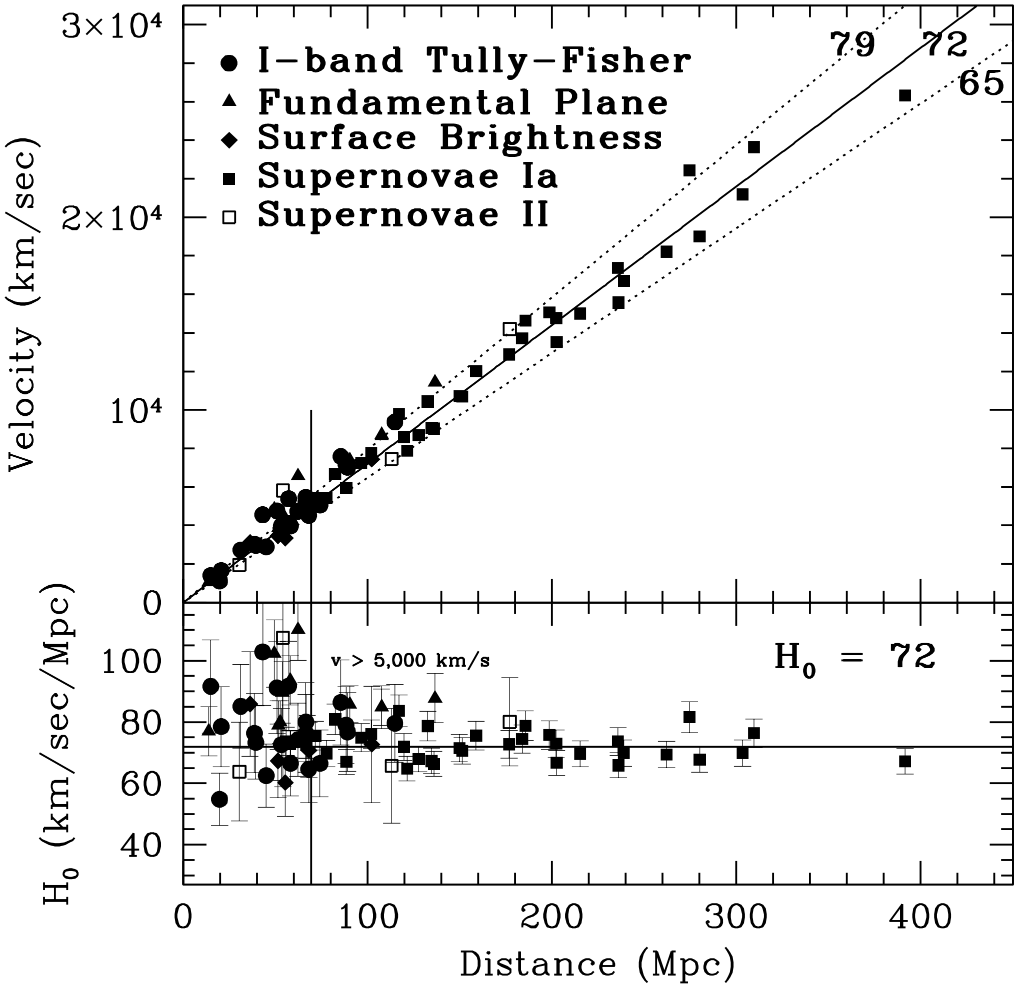


(SN Ia Hubble relations; left: full sample, middle: excluding strongly reddened SN Iae, right: same as middle, correcting for light-curve shape Freedman et al., 2001, Fig. 2)

Cepheids alone: nearby $\Longrightarrow$ systematic uncertainty due to local flow correction and small overall v $\Longrightarrow$ use secondary candles to get to larger distances.

Example above: magnitude-redshift diagram, analoguous to Hubble diagram ($m \propto -5 \log I$, and $I \propto 1/r^2 \propto 1/z^2$ because of Hubble $\Longrightarrow m \propto \log cz$).

H from HST

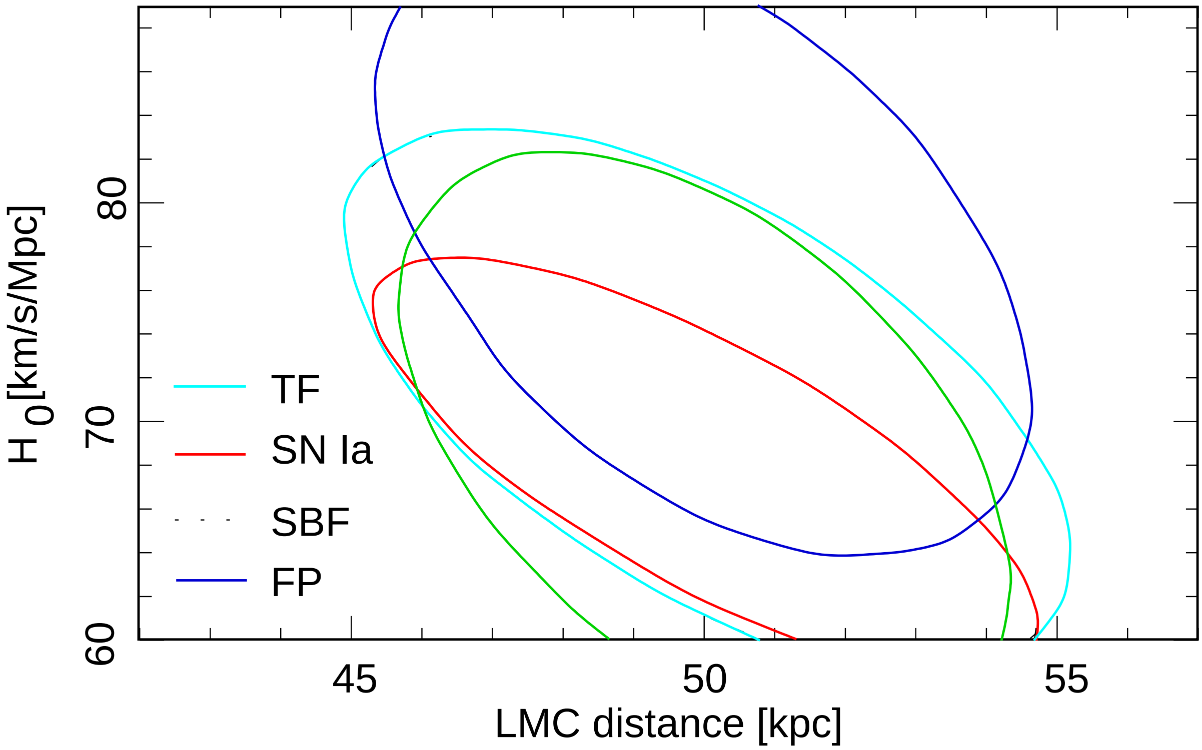


Freedman et al. (2001, Fig. 4)

Combining all secondary methods, best value found:

$$H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{5.50}$$

H from HST



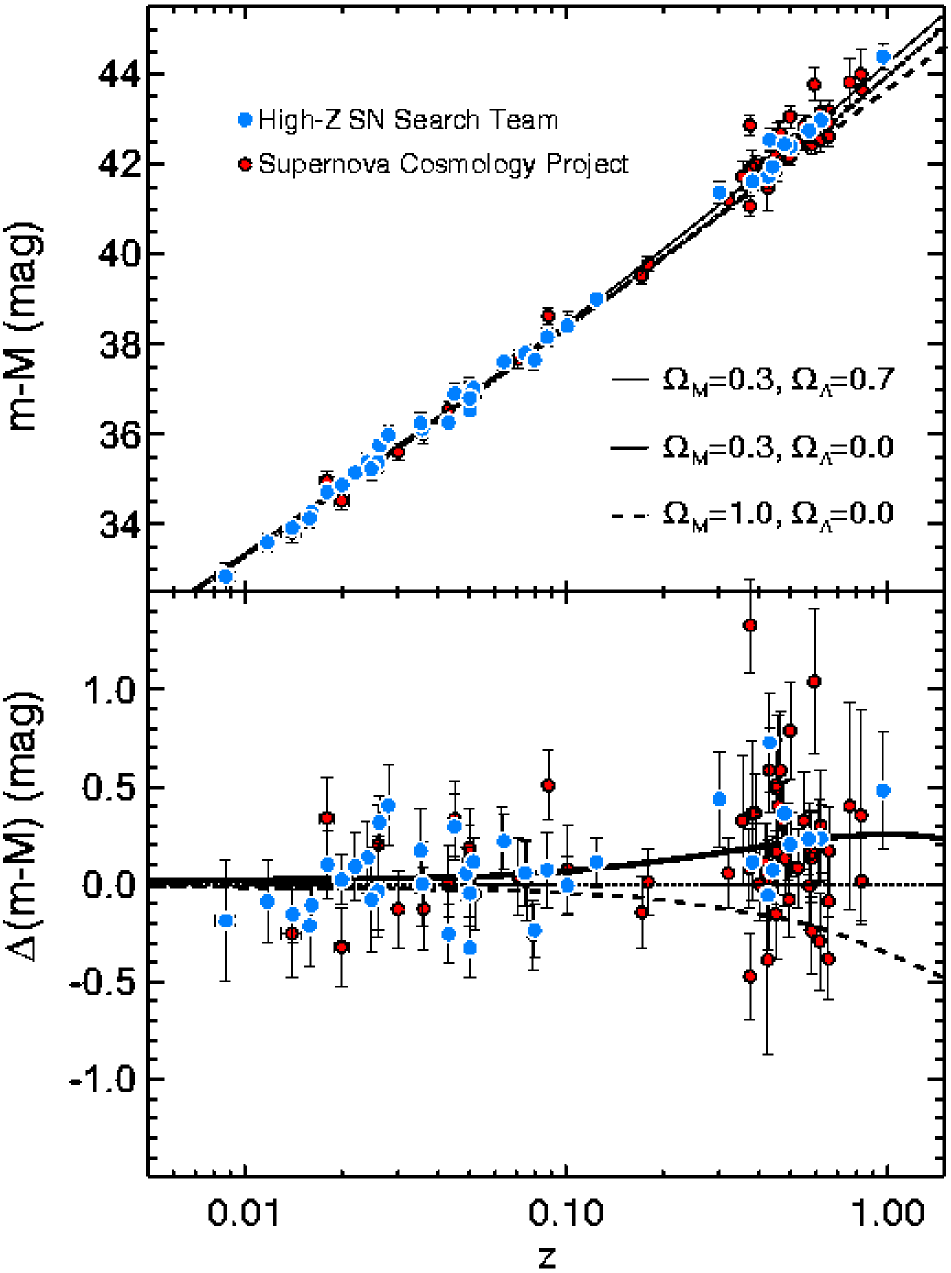
(Mould et al., 2000, Fig. 5)

Major systematic uncertainty in current H_0 value: zero-point of Cepheid scale, i.e., distance to Large Magellanic Cloud.

Despite these problems:
$\Longrightarrow$ All current values approach $\sim 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, with uncertainty $\sim$10%

H_0 controversy is over

H from HST



For larger distances: Deviations from Hubble-Relation!

Before we understand why: Understand Big-Bang itself!

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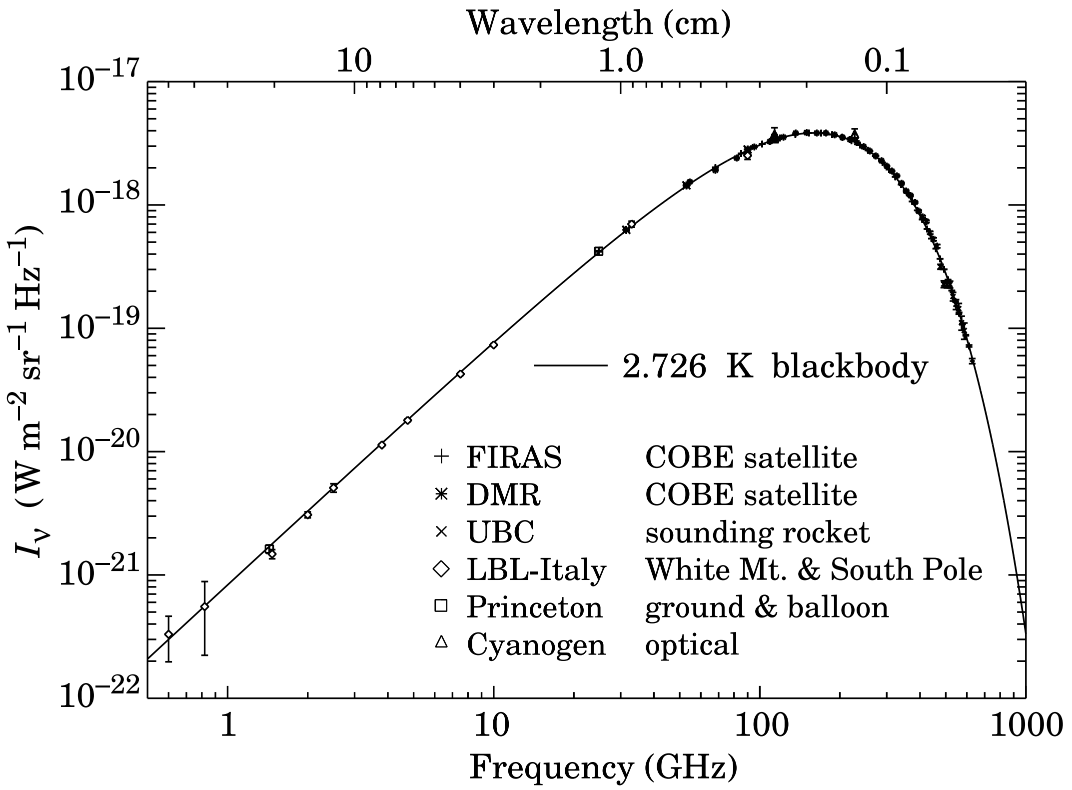
The Hot Big Bang

CMBR



(Smoot, 1997, Fig. 1)

Penzias & Wilson (1965): "Measurement of Excess Antenna Temperature at 4080 Mc/s"

$\Longrightarrow$ Cosmic Microwave Background Radiation (CMBR):

> The CMBR spectrum is fully consistent with a pure Planckian with temperature $T_{\text{CMBR}} = 2.728 \pm 0.004\,\text{K}$.

Now recognized as relict of hot big bang.

CMBR

Assumption: Early universe was hot and dense
$\Longrightarrow$ Equilibrium between matter and radiation.
Generation of radiation, e.g., in pair equilibrium,

$$\gamma + \gamma \longleftrightarrow \mathrm{e}^- + \mathrm{e}^+ \tag{6.1}$$

Equilibrium with electrons, e.g., via Compton scattering:

$$\mathrm{e}^- + \gamma \longrightarrow \mathrm{e}^- + \gamma \tag{6.2}$$

where the electrons linked to protons via Coulomb interaction.
Once density low and temperature below photoionization for Hydrogen,

$$\mathrm{H} + \gamma \longleftrightarrow \mathrm{p} + \mathrm{e}^- \tag{6.3}$$

Decoupling of radiation and matter $\Longrightarrow$ Adiabatic cooling of photon field.
Proof for these assumptions, and lots of gory details: this and the next few lectures!

CMBR

Reminder: Planck formula for energy density of photons:

$$B_\lambda = \frac{\mathrm{d}u}{\mathrm{d}\lambda} = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp(hc/k_\mathrm{B}T\lambda)-1} \tag{6.4}$$

(units: $\mathrm{erg\,cm^{-3}\,\AA^{-1}}$), where

$$k_\mathrm{B} = 1.38\times 10^{-16}\,\mathrm{erg\,K^{-1}} \quad \text{(Boltzmann)} \tag{6.5}$$

$$h = 6.625\times 10^{-27}\,\mathrm{erg\,s} \quad \text{(Planck)} \tag{6.6}$$

For $\lambda \gg hc/k_\mathrm{B}T$: Rayleigh-Jeans formula:

$$B_\lambda \sim \frac{8\pi k_\mathrm{B}T}{\lambda^4} \tag{6.7}$$

(classical case, diverges for $\lambda \longrightarrow 0$, “Jeans catastrophe”).

Maximum emission given by Wien’s displacement law:

$$\lambda_\mathrm{max} = 0.201\frac{hc}{k_\mathrm{B}T} \tag{6.8}$$

Total energy density by integration:

$$u = \int_0^\infty B_\lambda\,\mathrm{d}\lambda = \frac{8\pi^5(kT)^4}{15h^3c^3} = \frac{4\sigma_\mathrm{SB}}{c}T^4 = a_\mathrm{rad}T^4 \tag{6.9}$$

where

$$\sigma_\mathrm{SB} = 5.670\times 10^{-5}\,\mathrm{erg\,cm^{-3}\,K^{-4}} \quad \text{Stefan-Boltzmann} \tag{6.10}$$

$$a_\mathrm{rad} = 7.566\times 10^{-15}\,\mathrm{erg\,cm^{-2}\,K^{-4}\,s^{-1}} \quad \text{rad. dens. const.} \tag{6.11}$$

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CMBR

Since the energy of a photon is $E_\gamma = h\nu = hc/\lambda$, the number density of photons is

$$n = \int_0^\infty \frac{B_\lambda \,\mathrm{d}\lambda}{hc/\lambda} = 20.28\,T^3\,\text{photons}\,\text{cm}^{-3} \tag{6.12}$$

Thus, for the CMBR:

$$n_{\text{CMBR}} = 400\,\text{photons}\,\text{cm}^{-3} \tag{6.13}$$

Compare that to baryons: $\Longrightarrow$ critical density:

$$\rho_\text{c} = \frac{3H^2}{8\pi G} = 1.88\times 10^{-29} h^2\ \text{g}\,\text{cm}^{-3}$$
$$= 1.13\times 10^{-5}\,h^2\,\text{protons}\,\text{cm}^{-3} \tag{4.62}$$

since $m_\text{p} = 1.67\times 10^{-24}\,\text{g}$.

Therefore photons dominate the particle number:

$$\frac{n_{\text{CMBR}}}{n_{\text{baryons}}} = \frac{3.54\times 10^7}{\Omega h^2} \tag{6.14}$$

But, baryons dominate the energy density:

$$\frac{u_{\text{CMBR}}}{u_{\text{baryons}}} = \frac{a_{\text{rad}}T^4}{\Omega \rho_\text{c} c^2} = \frac{4.20\times 10^{-13}}{1.69\times 10^{-8}\Omega h^2} = \frac{1}{40260\Omega h^2} \tag{6.15}$$

That's why we talk about the matter dominated universe.

CMBR

Remember the scaling laws for the (energy) density of matter and radiation:

$$\rho_{\mathrm{m}} \propto R^{-3} \quad \text{and} \quad \rho_{\mathrm{r}} \propto R^{-4} \qquad (4.67, 4.68)$$

Therefore,

$$\frac{\rho_{\mathrm{r}}}{\rho_{\mathrm{m}}} \propto \frac{1}{R} \qquad (6.16)$$

$\Longrightarrow$ Photons dominate for large z, i.e., early in the universe!

Since $1 + z = R_0/R$ (Eq. 4.43), matter-radiation equality was at

$$1 + z_{\mathrm{eq}} = 40260\,\Omega h^2 \qquad (6.17)$$

(for $h = 0.75$, $1 + z_{\mathrm{eq}} = 22650$)

The above definition of z_{eq} is not entirely correct: neutrino background, which increases the background energy density, is ignored ($u_\nu \sim 68\% u_\gamma$, see later).

Formally, matter-radiation equality defined from $n_{\text{baryons}} = n_{\text{rel. particles}}$, $\Longrightarrow$

$$1 + z_{\mathrm{eq}} = 23900\,\Omega h^2 \qquad (6.18)$$

(for $h = 0.75$, $1 + z_{\mathrm{eq}} = 13440$).

CMBR

What happened to the temperature of the CMBR? Compare CMBR spectrum today with earlier times.
Differential Energy density:

$$\mathrm{d}u = B_\lambda \mathrm{d}\lambda \tag{6.19}$$

Cosmological redshift:

$$\frac{\lambda'}{\lambda} = \frac{R'}{R} = \frac{1}{1+z} = a \tag{4.50}$$

where $R(\text{today}) = 1$.
Taking the expansion into account:

$$\mathrm{d}u' = \frac{\mathrm{d}u}{a^4} = \frac{8\pi hc}{a^4\lambda^5}\frac{\mathrm{d}\lambda}{\exp(hc/kT\lambda)-1} \tag{6.20}$$

$$= \frac{8\pi hc}{a^5\lambda^5}\frac{a\mathrm{d}\lambda}{\exp(hc/kT\lambda)-1} \tag{6.21}$$

$$= \frac{8\pi hc}{\lambda'^5}\frac{\mathrm{d}\lambda'}{\exp(hca/kT\lambda')-1} \tag{6.22}$$

$$= B_{\lambda'}(T/a) \tag{6.23}$$

Therefore, the Planckian remains a Planckian, and the temperature of the CMBR scales as

$$T(z) = (1+z)T_0 \tag{6.24}$$

The early universe was hot $\Longrightarrow$ Hot Big Bang Model!

Overview

$a(t)$	t since BB	T[K] [K]	ρ_{matter} [g cm^{-3}]	Major Events
	10^{-42}	10^{30}		Planck era, "begin of physics"
	$10^{-40\ldots-30}$	10^{25}		Inflation?
10^{-13}	$\sim 10^{-5}$ s	$\sim 10^{13}$	$\sim 10^{9}$	generation of p-p^{-}, and baryon anti-baryon pairs from radiation background
3×10^{-9}	1 min	10^{10}	0.03	generation of e^{+}-e^{-} pairs out of radiation background
10^{-9}	10 min	3×10^{9}	10^{-3}	nucleosynthesis
$10^{-4} \ldots 10^{-3}$	$10^{6\ldots7}$ yr	$10^{3\ldots4}$	$10^{-21\ldots-18}$	End of radiation dominated epoch
7×10^{-4}	10^{7} yr	4000	10^{-20}	Hydrogen recombines, decoupling of matter and radiation
1	15×10^{9} yr	3	10^{-30}	now

Thermodynamics, I

Density in early universe is very high.

physical processes (e.g., photon-photon pair creation, electron-positron annihilation etc.) all have reaction rates

$$\Gamma \propto n\sigma v \tag{6.25}$$

where
n: number density (cm^{-3})
σ: interaction cross-section (cm^2)
v: velocity ($\mathrm{cm\,s}^{-1}$)
thermodynamic equilibrium reached if reaction rate much faster than "changes" in the system,

$$\Gamma \gg H \tag{6.26}$$

If thermodynamic equilibrium holds, then can assume evolution of universe as sequence of states of local thermodynamic equilibrium, and use standard thermodynamics.

Before looking at real universe, first need to derive certain useful formulae from relativistic thermodynamics.

Thermodynamics, II

For ideal gases, thermodynamics shows that number density $f(\mathbf{p})\,\mathrm{d}p$ of particles with momentum in $[p, p + \mathrm{d}p]$ is given by

$$f(\mathbf{p}) = \frac{1}{\exp\left((E-\mu)/k_\mathrm{B}T\right) + a} \tag{6.27}$$

where

$$a = \begin{cases} +1 & : \text{Fermions (spin=1/2, 3/2,...)} \\ -1 & : \text{Bosons (spin=1, 2,...)} \\ 0 & : \text{Maxwell-Boltzmann} \end{cases}$$

and where the energy needs to take the rest-mass into account:

$$E^2 = |\mathbf{p}|^2 c^2 + m^2 c^4 \tag{6.28}$$

μ is called the "chemical potential". It is preserved in chemical equilibrium:

$$i + j \leftrightarrow k + l \quad \Longrightarrow \quad \mu_i + \mu_j = \mu_k + \mu_l \tag{6.29}$$

photons: multi-photon processes exist $\Longrightarrow \mu_\gamma = 0$.
particles in thermal equilibrium: $\mu = 0$ as well because of the first law of thermodynamics,

$$\mathrm{d}E = T\,\mathrm{d}S - P\,\mathrm{d}V + \mu\,\mathrm{d}N \tag{6.30}$$

and in equilibrium system stationary wrt changes in particle number N.

Thermodynamics, III

In addition to number density: different particles have internal degrees of freedom, abbreviated with g.

Examples:

photons: two polarization states $\Longrightarrow g = 2$

neutrinos: one polarization state $\Longrightarrow g = 1$

electrons, positrons: spin=$1/2 \Longrightarrow g = 2$

Knowing g and $f(p)$, it is possible to compute interesting quantities:

particle number density:

$$n = \frac{g}{(2\pi\hbar)^3} \int f(\mathbf{p})\, \mathrm{d}^3p \tag{6.31}$$

energy density:

$$u = \rho c^2 = \frac{g}{(2\pi\hbar)^3} \int E(\mathbf{p})\, f(\mathbf{p})\, \mathrm{d}^3p \tag{6.32}$$

pressure: from kinetic theory we know

$$P = n\langle pv\rangle/3 = n\langle p^2c^2/E\rangle/3 \tag{6.33}$$

such that

$$P = \frac{g}{(2\pi\hbar)^3} \int \frac{p^2c^2}{3E} f(\mathbf{p})\, \mathrm{d}^3p \tag{6.34}$$

Thermodynamics, IV

Generally, we are interested in knowing n, u, and P in two limiting cases:

1. the ultra-relativistic limit, where $k_{\mathrm{B}}T \gg mc^2$, i.e., kinetic energy dominates the rest-mass
2. the non-relativistic limit, where $k_{\mathrm{B}}T \ll mc^2$

Transitions between these limits (i.e., what happens during “cooling”) are usually much more complicated $\Longrightarrow$ ignore...

To derive the number density, the energy density, and the equation of state, note that Eq. (6.28) shows

$$E = \sqrt{p^2c^2 + m^2c^4} \tag{6.28}$$

such that

$$p = \sqrt{E^2 - m^2c^4}/c \tag{6.35}$$

Therefore

$$\frac{\mathrm{d}E}{\mathrm{d}p} = \frac{pc^2}{\sqrt{p^2c^2 + m^2c^4}} \tag{6.36}$$

from which it follows that

$$E\,\mathrm{d}E = pc^2\,\mathrm{d}p \tag{6.37}$$

Thus the following holds

$$\iiint_{-\infty}^{+\infty} \mathrm{d}^3p = \int_0^\infty 4\pi p^2\,\mathrm{d}p = \int_{mc^2}^\infty \frac{4\pi}{c^3}\left(E^2 - m^2c^4\right)^{1/2} E\,\mathrm{d}E \tag{6.38}$$

Going to a system of units where

$$c = k_\mathrm{B} = \hbar = 1 \tag{6.39}$$

to save me some typing, substitute these equations into Eqs. (6.31)–(6.34) to find

$$n = \frac{g}{2\pi^2}\int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E\,\mathrm{d}E}{\exp\left((E-\mu)/T\right) \pm 1} \tag{6.40}$$

$$\rho = \frac{g}{2\pi^2}\int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E^2\,\mathrm{d}E}{\exp\left((E-\mu)/T\right) \pm 1} \tag{6.41}$$

$$P = \frac{g}{6\pi^2}\int_m^\infty \frac{\left(E^2 - m^2\right)^{3/2}\,\mathrm{d}E}{\exp\left((E-\mu)/T\right) \pm 1} \tag{6.42}$$

which can in some limiting cases be expressed in a closed form (Kolb & Turner, 1990, eq. 3.52 ff.) (see following viewgraphs).

Thermodynamics, V

In the ultra-relativistic limit, $k_{\mathrm{B}}T \gg mc^2$, and assuming $\mu = 0$,

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2}\, g \left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3 & \text{Bosons} \\ \frac{3}{4}\frac{\zeta(3)}{\pi^2}\, g \left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3 & \text{Fermions} \end{cases} \tag{6.43}$$

$$u = \begin{cases} \frac{\pi^2}{30}\, g\, k_{\mathrm{B}}T \left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3 & \text{Bosons} \\ \frac{7}{8}\frac{\pi^2}{30}\, g\, k_{\mathrm{B}}T \left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3 & \text{Fermions} \end{cases} \tag{6.44}$$

$$P = \rho c^2/3 = u/3 \tag{6.45}$$

where $\zeta(3) = 1.202\ldots$, and $\zeta(s)$ is Riemann's zeta-function (see handout, Eq. 6.53).

Eq. (6.45) is a simple result of the fact that in the relativistic limit, $E \sim pc$. Inserting this and $v = c$ into Eq. (6.33) gives the desired result.

As expected, T^4 proportionality well known from Stefan Boltzmann law!

UWarwick

Obtaining the previous formulae is an exercise in special functions. For example, the $T \gg m$, $T \gg \mu$ case for ρ for Bosons (Eq. 6.44) is obtained as follows (setting $c = k_\mathrm{B} = \hbar = 1$):

$$\rho_\text{Boson} = \frac{g}{2\pi^2} \int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E^2 \,\mathrm{d}E}{\exp\left((E-\mu)/T\right) \pm 1} \tag{6.46}$$

because of $T \gg \mu$

$$\approx \frac{g}{2\pi^2} \int_m^\infty \frac{\left(E^2 - m^2\right)^{1/2} E^2 \,\mathrm{d}E}{\exp(E/T) \pm 1} \tag{6.47}$$

for *Bosons*, choose -1, and substitute $x = E/T$:

$$= \frac{g}{2\pi^2} \int_{m/T}^\infty \frac{\left(x^2T^2 - m^2\right)^{1/2} x^2T^3 \,\mathrm{d}x}{\exp(x) - 1} \tag{6.48}$$

Since $T \gg m$,

$$\approx \frac{g}{2\pi^2} \int_0^\infty \frac{x^3T^4 \,\mathrm{d}x}{\exp(x) - 1} \tag{6.49}$$

$$= \frac{gT^4}{2\pi^2} \int_0^\infty \frac{x^3 \,\mathrm{d}x}{\exp(x) - 1} \tag{6.50}$$

$$= \frac{gT^4}{2\pi^2} \cdot 6\zeta(4) \tag{6.51}$$

$$= \frac{\pi^2}{30} gT^4 \tag{6.52}$$

where $\zeta(s)$ is *Riemann's zeta-function*, which is defined by (Abramowitz & Stegun, 1964)

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{\exp(x) - 1} \,\mathrm{d}x \quad \text{for } \mathscr{R}e\, s > 1 \tag{6.53}$$

where $\Gamma(x)$ is the Gamma-function. Note that $\zeta(4) = \pi^4/90$.

For *Fermions*, everything is the same except for that we now have to choose the $+$ sign. The equivalent of Eq. (6.50) is then

$$\rho_\text{Fermi} = \frac{gT^4}{2\pi^2} \int_0^\infty \frac{x^3 \,\mathrm{d}x}{\exp(x) + 1} \tag{6.54}$$

Now we can make use of formula 3.411.3 of Gradstein & Ryshik (1981),

$$\int_0^\infty \frac{x^{\nu-1} \,\mathrm{d}x}{\exp(\mu x) + 1} = \frac{1}{\mu^\nu}(1 - 2^{1-\nu})\Gamma(\nu)\zeta(\nu) \quad \text{for } \mathscr{R}e\, \mu, \nu > 1 \tag{6.55}$$

to see where the additional factor of $7/8$ in Eq. (6.44) comes from.

Thermodynamics, VI

In the non-relativistic limit: $k_\mathrm{B}T \ll mc^2$

$\Longrightarrow$ can ignore the ± 1 term in the denominator

$\Longrightarrow$ Same formulae for Bosons and Fermions!

$$n = \frac{2g}{(2\pi\hbar)^3}(2\pi m k_\mathrm{B} T)^{3/2} \mathrm{e}^{-mc^2/k_\mathrm{B}T} \tag{6.56}$$

$$u = nmc^2 \tag{6.57}$$

$$P = nk_\mathrm{B}T \tag{6.58}$$

Therefore:

- density dominated by rest-mass ($\rho = u/c^2 = mn$)
- $P \ll \rho c^2/3$, i.e., *much* smaller than for relativistic particles.
- Particle pressure only important if particles are relativistic.

Obviously, relativistic particles with $m = 0$ (or very close to 0) will never get nonrelativistic. Still, they can “decouple” from the rest of the universe when the interaction rates go to 0.

Equation of State

Pressure of ultra-relativistic particles $\gg$ Pressure of nonrelativistic particles $\Longrightarrow$ Nonrelativistic particles unimportant for equation of state.

For relativistic particles:

$$u_{\text{boson}} = \frac{\pi^2}{30} g\, k_{\text{B}} T \left(\frac{k_{\text{B}} T}{\hbar c}\right)^3 \qquad (6.44)$$

$$u_{\text{fermion}} = \frac{7}{8} u_{\text{boson}} \qquad (6.44)$$

$\Longrightarrow$ Total energy density for mixture of particles:

$$u = g_* \cdot \frac{\pi^2}{30} k_{\text{B}} T \left(\frac{k_{\text{B}} T}{\hbar c}\right)^3 \qquad (6.59)$$

where the effective degeneracy factor

$$g_* = \sum_{\text{bosons}} g_{\text{B}} \left(\frac{T_{\text{B}}}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_{\text{F}} \left(\frac{T_{\text{F}}}{T}\right)^4 \qquad (6.60)$$

g_* counts total number of internal degrees of freedom of *all* relativistic bosonic and fermionic species, i.e., all relativistic particles which are in thermodynamic equilibrium

Pressure obtained from Eq. (6.59) via $P = u/3$.

Early Expansion, I

Knowing the equation of state (EOS), we can now use Friedmann equations to determine the early evolution of the universe.

Friedmann:

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - kc^2 \tag{4.59}$$

or, dividing by R^2

$$\frac{\dot{R}^2}{R^2} = H(t)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} \tag{4.60}$$

But: Early universe dominated by relativistic particles

$\Longrightarrow \rho \propto R^{-4}$

$\Longrightarrow$ Density-term dominates

$\Longrightarrow$ can set $k = 0$.

Early universe is asymptotically flat!

This will prove to be one of the most crucial problems of modern cosmology...

Early Expansion, II

To obtain evolution, insert EOS (Eq. 6.59) into Eq. (4.60):

$$H(t)^2 = \frac{8\pi G}{3} g_* \frac{\pi^2}{30} \frac{(k_\mathrm{B}T)^4}{(\hbar c)^3} \tag{6.61}$$

$$= \frac{4\pi^3 G}{45(\hbar c)^3} g_* (k_\mathrm{B}T)^4 \tag{6.62}$$

such that

$$H(t) = \left(\frac{4\pi^3 G}{45(\hbar c)^3}\right)^{1/2} g_*^{1/2} (k_\mathrm{B}T)^2 \tag{6.63}$$

On the other hand, since $\rho \propto R^{-4}$ (relativistic background),

$$\rho = \rho_0 \left(\frac{R_0}{R}\right)^4 \tag{6.64}$$

Friedmann:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \sqrt{\frac{8\pi G\rho_0}{3} \frac{R_0^2}{R}} \tag{6.65}$$

Introducing the dimensionless scale factor, $a = R/R_0$ (Eq. 4.30),

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \sqrt{\frac{8\pi G\rho_0}{3} \frac{1}{a}} =: \xi a^{-1} \tag{6.66}$$

Early Expansion, III

Separation of variables gives

$$\int_0^{a(t)} a \, \mathrm{d}a = \int_0^t \xi \, \mathrm{d}t \tag{6.67}$$

such that finally

$$a(t) = \xi^{1/2} \cdot t^{1/2} \tag{6.68}$$

Therefore, the Hubble constant is

$$H(t) = \frac{\dot{a}}{a} = \frac{1}{2t} \tag{6.69}$$

Equating Eqs. (6.63) and (6.69) gives the time-temperature relationship:

$$t = \left(\frac{45(\hbar c)^3}{16\pi^3 G}\right)^{1/2} \frac{1}{g_*^{1/2}} \frac{1}{(k_\mathrm{B}T)^2} \tag{6.70}$$

Inserting all constants and converting to more useful units gives

$$t = \frac{2.4\,\mathrm{sec}}{g_*^{1/2}} \cdot \left(\frac{k_\mathrm{B}T}{1\,\mathrm{MeV}}\right)^{-2} \tag{6.71}$$

. . . one of the most useful equations for the early universe.

Elementary Particles, I

Precise behavior of universe depends on g_*
$\Longrightarrow$ Strong dependency on elementary particle physics.

Generally, particles present when energy in other particles allows generation of particle–antiparticle pairs, i.e., when $k_B T \gtrsim mc^2$ (threshold temperature)

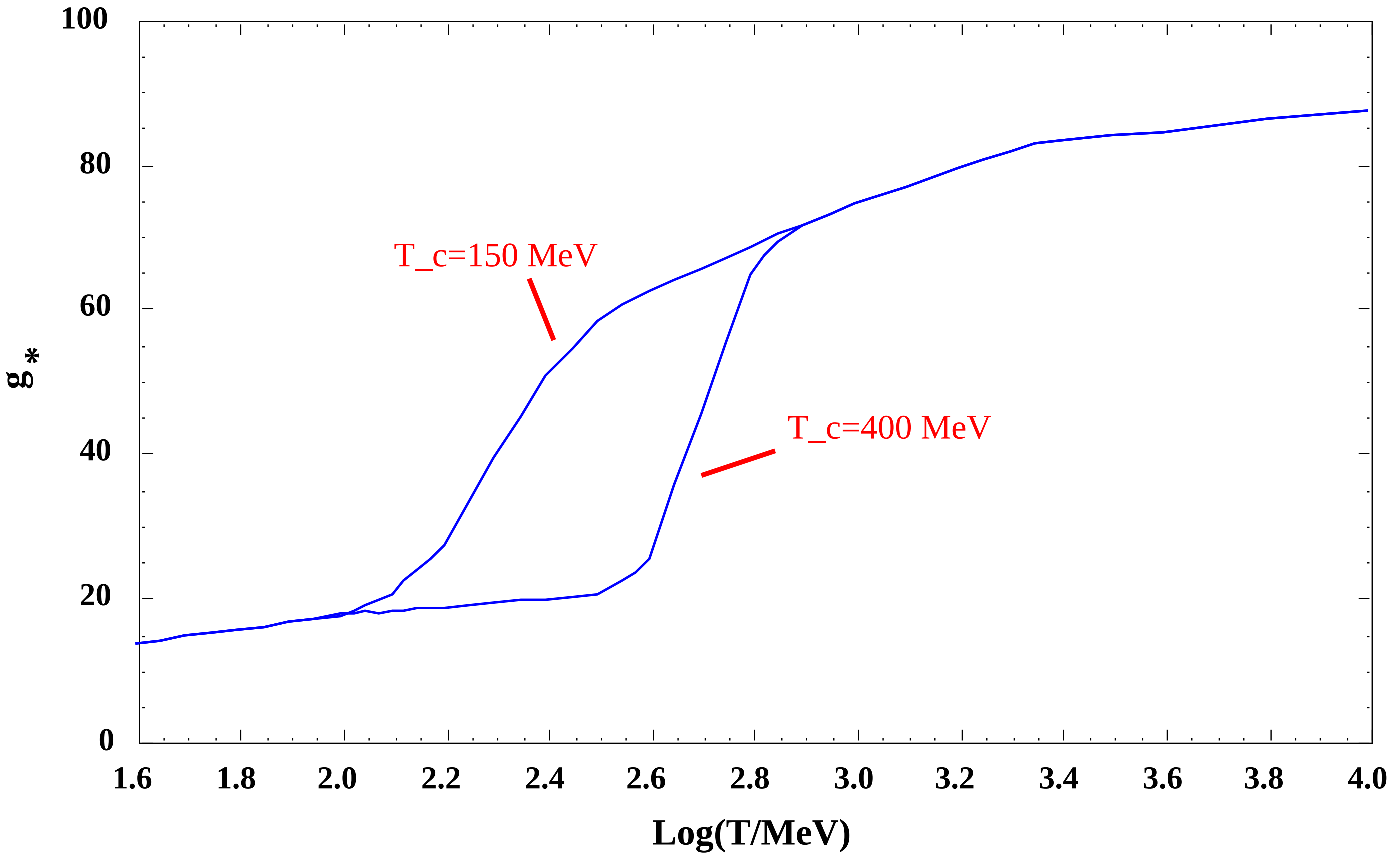
Current particle physics provides following picture (Olive, 1999, Tab. 1):

Temp.	New Particles	$4g_*(T)$
$k_B T < m_e c^2$	γ's and ν's	29
$m_e c^2 < k_B T < m_\mu$	$e^\pm$	43
$m_\mu c^2 < k_B T < m_\pi$	$\mu^\pm$	57
$m_\pi c^2 < k_B T < k_B T_c$	π's	69
$k_B T_c < k_B T < m_{\text{strange}} c^2$	$-\pi$'s$+$u, $\bar{\text{u}}$, d, $\bar{\text{d}}$, gluons	205
$m_s c^2 < k_B T < m_{\text{charm}} c^2$	s, $\bar{\text{s}}$	247
$m_c c^2 < k_B T < m_\tau c^2$	c, $\bar{\text{c}}$	289
$m_\tau c^2 < k_B T < m_{\text{bottom}} c^2$	$\tau^\pm$	303
$m_b c^2 < k_B T < m_{W,Z} c^2$	b, $\bar{\text{b}}$	345
$m_{W,Z} c^2 < k_B T < m_{\text{top}} c^2$	$W^\pm$, Z	381
$m_t c^2 < k_B T < m_{\text{Higgs}} c^2$	t, $\bar{\text{t}}$	423
$m_H c^2 < k_B T$	H^0	427

T_c: energy of confinement-deconfinement for transitions quarks $\Longrightarrow$ hadrons, somewhere between 150 MeV and 400 MeV.

Example: photons (2 polarization states, i.e., $g = 2$) and three species of neutrinos ($g = 1$, but with distinguishable anti-particles)
$\Longrightarrow g_* = 2 + (7/8) \cdot 2 \cdot 3 = 58/8 = 29/4$.

Elementary Particles, II



(Olive, 1999, Fig. 1)

Will now consider times when only Neutrinos and Electron/Positrons present (after baryogenesis, see next lecture for that).

Interlude

Previous (abstract) formulae allow to estimate quantities like

1. The existence and energy of primordial neutrinos,
2. The formation of neutrons,
3. The formation of heavier elements.

Detailed computations require solving nonlinear differential equations $\Longrightarrow$ difficult, only numerically possible.

Essentially, need to self-consistently solve Boltzmann equation in expanding universe for evolution of phase space density with time, using the correct QCD/QED reaction rates $\Longrightarrow$ too complicated (at least for me...).

Will use approximate analytical way here, which gives surprisingly exact answers.

Neutrinos, I

Neutrino equilibrium caused by weak interactions such as

$$\begin{aligned} \mathrm{e}^- + \mathrm{e}^+ &\longleftrightarrow \nu + \bar{\nu} \\ \mathrm{e}^- + \nu &\longleftrightarrow \mathrm{e}^- + \nu \end{aligned} \tag{6.72}$$

etc.

Reaction rate for these processes:

$$\Gamma = n \langle \sigma v \rangle \tag{6.73}$$

where the thermally averaged interaction cross-section is

$$\langle \sigma v \rangle \approx \left\langle \frac{\alpha^2 p}{m_{\mathrm{W}}^4} \cdot p \right\rangle \sim 10^{-2} \frac{(k_{\mathrm{B}} T)^2}{m_{\mathrm{W}}^4} \tag{6.74}$$

m_{W}: mass of W-boson (exchange particle of weak interaction),
$\alpha \approx 1/137$: fine structure constant.

But in the ultra-relativistic limit,

$$n \propto T^3 \tag{6.43}$$

such that

$$\Gamma_{\text{weak}} \propto \frac{\alpha^2 T^5}{m_{\mathrm{W}}^4} \tag{6.75}$$

Neutrinos, II

Because of Eqs. (6.69) and (6.70), the temperature dependence of the Hubble constant is

$$H(T) = 1.66 g_*^{1/2} \cdot \frac{T^2}{m_\mathrm{P}} \tag{6.76}$$

where m_P is the Planck mass, $m_\mathrm{P} c^2 = 1.22 \times 10^{19}\,\mathrm{GeV}$ (see later, Eq. 6.130).

Neutrino equilibrium possible as long as $\Gamma_\mathrm{weak} > H$, i.e., (inserting exact numbers)

$$k_\mathrm{B} T_\mathrm{dec} \gtrsim \left(\frac{500\, c^6\, m_\mathrm{W}^4}{m_\mathrm{P}} \right)^{1/3} \sim 1\,\mathrm{MeV} \tag{6.77}$$

Neutrinos decouple $\sim 1\,\mathrm{s}$ after the big bang.

This follows from Eq. (6.71), remembering that for this phase, $g_* \sim 10$.

Since decoupling, primordial neutrinos just follow expansion of universe, virtually no interaction with “us” anymore.

Entropy, I

The entropy of particles is defined through

$$S = \frac{E + PV}{T} \tag{6.78}$$

Important for cosmology: relativistic limit.

Define entropy density,

$$s = \frac{S}{V} = \frac{E/V + P}{T} = \frac{u + P}{T} \approx \frac{4}{3}\frac{u}{T} \tag{6.79}$$

(last step for relativistic limit; Eq. 6.45)

Inserting Eq. (6.44) gives

$$s = \frac{7}{8}\frac{2\pi^2}{45} g k_{\mathrm{B}} \left(\frac{k_{\mathrm{B}}T}{\hbar c}\right)^3 = \frac{7}{6}\frac{2\pi^4}{45\,\zeta(3)} k_{\mathrm{B}}\, n \tag{6.80}$$

(violet: only for Fermions).

$\Longrightarrow$ In the relativistic limit

$$\frac{s}{k_{\mathrm{B}}} = \begin{cases} 3.602n & \text{Bosons} \\ 4.202n & \text{Fermions} \end{cases} \tag{6.81}$$

Important for later:

Since $s \propto n$ for backgrounds, $\eta = n_{\mathrm{CMBR}}/n_{\mathrm{baryons}}$ is often called "entropy per baryon".

Entropy, II

For a mixture of backgrounds, Eq. (6.80) gives

$$\frac{s}{k_\mathrm{B}} = g_{*,S} \cdot \frac{2\pi^2}{45} \left(\frac{k_\mathrm{B}T}{\hbar c}\right)^3 \tag{6.82}$$

where $g_{*,S}$ is the analogue to g_* (Eq. 6.60),

$$g_{*,S} = \sum_{\text{bosons}} g_\mathrm{B} \left(\frac{T_\mathrm{B}}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_\mathrm{F} \left(\frac{T_\mathrm{F}}{T}\right)^3 \tag{6.83}$$

Note that if the species are not at the same temperature, $g_* \neq g_{*,S}$.

Entropy per mass today:

$$\frac{S}{M} = \frac{10^{16}}{\Omega h^2}\,\mathrm{erg\,K^{-1}\,g^{-1}} \tag{6.84}$$

while the entropy gain of heating water at 300 K by 1 K is $\sim 1.4 \times 10^5\,\mathrm{erg\,K^{-1}\,g^{-1}}$.

$\Longrightarrow$ “Human attempts to obey 2nd law . . . are swamped by . . . microwave background” (Peacock, 1999, p. 277).

$\Longrightarrow$ $S =$ const. for universe to very good approximation.

$\Longrightarrow$ Universe expansion is adiabatic!

Reheating

After decoupling of neutrinos, neutrino distribution just gets redshifted (similar to CMBR, Eq. 6.24):

$$\frac{T_\nu}{T_{\text{dec}}} = \frac{R_{\text{dec}}}{R(t)} \quad \Longrightarrow \quad T_\nu \propto R^{-1} \tag{6.85}$$

On the other hand, the temperature of the universe is

$$T \propto g_{*,S}^{1/3} R^{-1} \tag{6.86}$$

This follows from $S/V \propto T^3$ (Eq. 6.82), $V \propto R^3$, and $S = \text{const.}$ (adiabatic expansion of the universe).

$\Longrightarrow$ as long as $g_{*,S} = \text{const.}$ we have $T_\nu = T$

$\Longrightarrow$ Immediately after decoupling, neutrino background appears as if it is still in equilibrium.

However: Temperature for neutrino decoupling $\sim 2m_{\text{e}}c^2$
But, for $kT_{\text{BB}} < 2\,m_{\text{e}}c^2$, pair creation,

$$\gamma + \gamma \longleftrightarrow \text{e}^- + \text{e}^+ \tag{6.87}$$

kinematically impossible

$\Longrightarrow$ Shortly after neutrino decoupling: $\text{e}^\pm$ annihilation

$\Longrightarrow$ $g_{*,S}$ changes!

$\Longrightarrow$ Would expect $T_{\text{CMBR}} \neq T_\nu$.

Reheating

Difference in $g_{*,S}$:

- before annihilation:
  e^-, e^+, $\gamma \Longrightarrow g_{*,S} = 2 + 2 \cdot 2 \cdot (7/8) = 11/2$.
- after annihilation:
  $\gamma \Longrightarrow g_{*,S} = 2$

But: total entropy for particles in equilibrium conserved ("expansion is adiabatic"):

$$g_{*,S}(T_\text{before}) \cdot T_\text{before}^3 = g_{*,S}(T_\text{after}) \cdot T_\text{after}^3 \qquad (6.88)$$

such that

$$T_\text{after} = \left(\frac{11}{4}\right)^{1/3} T_\text{before} \sim 1.4 \cdot T_\text{before} \qquad (6.89)$$

Since $T_\text{after} > T_\text{before}$: "reheating".

Note that in reality the annihilation is not instantaneous and T decreases (albeit less rapidly) during "reheating"...

$\Longrightarrow$ Since neutrino-background does not "see" annihilation

$\Longrightarrow$ just continues to cool

$\Longrightarrow$ current temperature of neutrinos is

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\text{CMBR} \sim 1.95\,\text{K} \qquad (6.90)$$

History

After reheating: universe consists of p, n, γ (and e^- to preserve charge neutrality)
$\Longrightarrow$ Ingredients for Big Bang Nucleosynthesis (BBN).

Historical perspective:

Cross section to make Deuterium:

$$\langle\sigma v\rangle(\mathrm{p}+\mathrm{n}\longrightarrow \mathrm{D}+\gamma)\sim 5\times 10^{-20}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \qquad (6.91)$$

Furthermore, need temperatures of $T_{\mathrm{BBN}}\sim 100\,\mathrm{keV}$, i.e., $t_{\mathrm{BBN}}\sim 200\,\mathrm{s}$ (Eq. 6.71).
This implies density

$$n\sim\frac{1}{\langle\sigma v\rangle\cdot t_{\mathrm{BBN}}}\sim 10^{17}\,\mathrm{cm}^{-3} \qquad (6.92)$$

Today: Baryon density $n_{\mathrm{B}}\sim 10^{-7}\,\mathrm{cm}^{-3}$
Since $n\propto R^{-3}$, $\Longrightarrow$

$$T(\text{today})=\left(\frac{n_{\mathrm{B}}}{n}\right)^{1/3}\cdot T_{\mathrm{BBN}}\sim 10\,\mathrm{K} \qquad (6.93)$$

pretty close to the truth. . .

The above discussion was first used by Gamov and coworkers in 1948, and was the first prediction of the cosmic microwave background radiation!

Observations:

BBN required by observations, since no other production region for Deuterium known, and since He-abundance $\sim$ 25% by mass everywhere.

Proton/Neutron

Initial conditions: Set by Proton-Neutron-Ratio.

For $t \ll 1\,\mathrm{s}$, equilibrium via weak interactions:

$$\begin{aligned} \mathrm{n} &\longleftrightarrow p + \mathrm{e}^- + \bar{\nu}_\mathrm{e} \\ \nu_\mathrm{e} + \mathrm{n} &\longleftrightarrow p + \mathrm{e}^- \\ \mathrm{e}^+ + \mathrm{n} &\longleftrightarrow p + \bar{\nu}_\mathrm{e} \end{aligned} \tag{6.94}$$

Reactions fast as long as particles relativistic.

But, once $T \sim 1\,\mathrm{MeV}$, n, p non-relativistic

$\Longrightarrow$ Boltzmann statistics applies (or us Eq. (6.56)):

$$\frac{n_\mathrm{n}}{n_\mathrm{p}} = \mathrm{e}^{-\Delta mc^2/k_\mathrm{B}T} = \mathrm{e}^{-1.3\,\mathrm{MeV}/k_\mathrm{B}T} \tag{6.95}$$

$\Longrightarrow$ Suppression of n with respect to p because of larger mass ($m_\mathrm{n}c^2 = 939.57\,\mathrm{MeV}$, $m_\mathrm{p}c^2 = 938.27\,\mathrm{MeV}$)

Abundance freezes out when $\Gamma > H$, where reaction rate

$$\Gamma(\nu_\mathrm{e} + \mathrm{n} \leftrightarrow \mathrm{p} + \mathrm{e}^-) \sim 2.1 \left(\frac{T}{1\,\mathrm{MeV}}\right)^5 \mathrm{s}^{-1} \tag{6.96}$$

Neutron abundance freezes out at $k_\mathrm{B}T \sim 0.8\,\mathrm{MeV}$ ($t = 1.7\,\mathrm{s}$), such that $n_\mathrm{n}/n_\mathrm{p} = 0.2$

After that: Neutron decay ($\tau_\mathrm{n} = 886.7 \pm 1.2\,\mathrm{s}$).

$\Longrightarrow$ Nucleosynthesis has to be over before neutrons are gone!

Deuterium

The next step in nucleosynthesis is formation of deuterium (binding energy $E_\mathrm{B} = 2.225\,\mathrm{MeV}$, i.e., $1.7(m_\mathrm{n} - m_\mathrm{p})c^2$:

$$\mathrm{p} + \mathrm{n} \longleftrightarrow \mathrm{D} + \gamma \tag{6.97}$$

Note: Both reactions possible:
fusion and photodisintegration:

$$\Gamma_\text{fusion} = n_\mathrm{B}\sigma v \tag{6.98}$$

$$\Gamma_\text{photo} = n_\gamma \sigma v \mathrm{e}^{-E_\mathrm{B}/k_\mathrm{B}T} \tag{6.99}$$

At first: photodisintegration dominates ($\eta^{-1} = n_\gamma/n_\mathrm{B} \sim 10^{10}$).
Build up of D only possible once $\Gamma_\text{fusion} > \Gamma_\text{photo}$, i.e., when

$$\frac{n_\gamma}{n_\mathrm{B}} \mathrm{e}^{-E_\mathrm{B}/k_\mathrm{B}T} \sim 1 \tag{6.100}$$

Inserting numbers shows that

Deuterium production starts at $k_\mathrm{B}T \sim 100\,\mathrm{keV}$, $t \sim 100\,\mathrm{s}$.

Heavier Elements, I

Once deuterium present:

nucleosynthesis of lighter elements:

$$
\begin{aligned}
\mathrm{D} + \mathrm{D} &\longrightarrow \mathrm{T} + \mathrm{p} \\
\mathrm{D} + \mathrm{n} &\longrightarrow \mathrm{T} + \gamma \\
\mathrm{D} + \mathrm{p} &\longrightarrow {}^{3}\mathrm{He} + \gamma \\
\mathrm{D} + \mathrm{D} &\longrightarrow {}^{3}\mathrm{He} + \mathrm{n} \\
{}^{3}\mathrm{He} + \mathrm{n} &\longrightarrow \mathrm{T} + \mathrm{p}
\end{aligned}
\tag{6.101}
$$

production of ^{4}He:

$$
\begin{aligned}
\mathrm{D} + \mathrm{D} &\longrightarrow {}^{4}\mathrm{He} + \gamma \\
\mathrm{D} + {}^{3}\mathrm{He} &\longrightarrow {}^{4}\mathrm{He} + \mathrm{p} \\
\mathrm{T} + \mathrm{D} &\longrightarrow {}^{4}\mathrm{He} + \mathrm{n} \\
{}^{3}\mathrm{He} + {}^{3}\mathrm{He} &\longrightarrow {}^{4}\mathrm{He} + 2\mathrm{p} \\
\mathrm{T} + \mathrm{p} &\longrightarrow {}^{4}\mathrm{He} + \gamma \\
{}^{3}\mathrm{He} + \mathrm{n} &\longrightarrow {}^{4}\mathrm{He} + \gamma
\end{aligned}
\tag{6.102}
$$

Element gap at $A = 5$ can be overcome to produce Lithium:

$$
\begin{aligned}
{}^{3}\mathrm{He} + {}^{4}\mathrm{He} &\longrightarrow {}^{7}\mathrm{Be} + \gamma \\
{}^{7}\mathrm{Be} &\longrightarrow {}^{7}\mathrm{Li} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
\mathrm{T} + {}^{4}\mathrm{He} &\longrightarrow {}^{7}\mathrm{Li} + \mathrm{e}^{+} + \nu_{\mathrm{e}}
\end{aligned}
\tag{6.103}
$$

Gap at $A = 8$ prohibits production of heavier isotopes.

Heavier Elements, II

Major product of BBN: ^{4}He.

Mass fraction of ^{4}He assuming all neutrons incorporated into ^{4}He

$\Longrightarrow$ number density of H$=$number of remaining protons, i.e., mass fraction

$$X = \frac{n_\mathrm{p} - n_\mathrm{n}}{n_\mathrm{p} + n_\mathrm{n}} \tag{6.104}$$

and

$$Y = 1 - \frac{n_\mathrm{p} - n_\mathrm{n}}{n_\mathrm{p} + n_\mathrm{n}} = 2\left(1 + \frac{n_\mathrm{p}}{n_\mathrm{n}}\right)^{-1} \tag{6.105}$$

At $k_\mathrm{B}T = 0.8\,\mathrm{MeV}$, because of neutron decay, $n_\mathrm{n}/n_\mathrm{p} = 1/7$, therefore

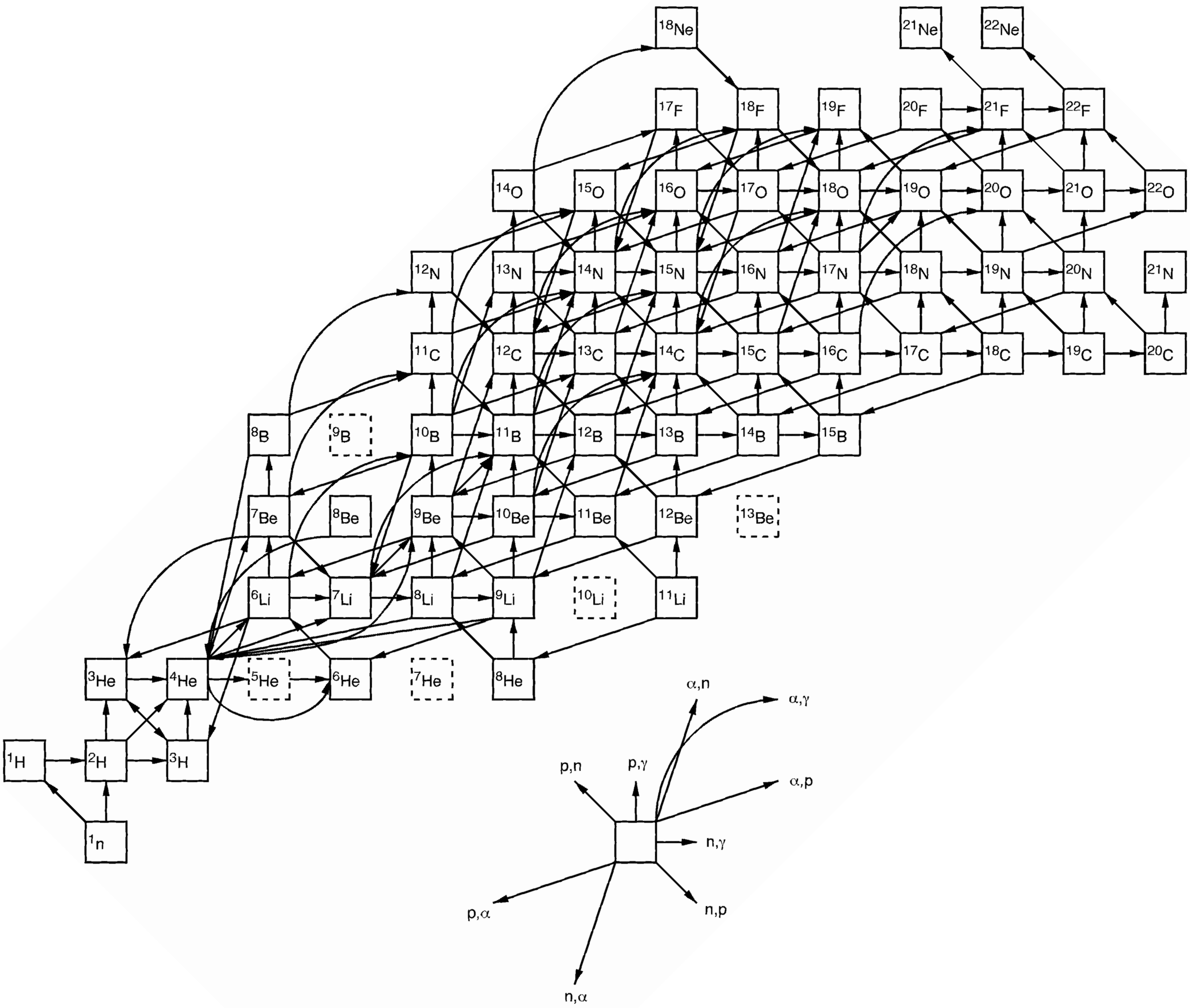
BBN predicts primordial He-abundance of $Y = 0.25$.

1. Generally, BBN function of entropy per baryon, η, i.e., of Ω_B:

$$\Omega_\mathrm{B} = 3.67 \times 10^7 \cdot \eta \tag{6.106}$$

   (since η, Ω determine expansion behavior) $\Longrightarrow$ Perform computations as function of η!

2. Since Y set by $n_\mathrm{p}/n_\mathrm{n}$ $\Longrightarrow$ Relatively independent on η (except for extreme values).



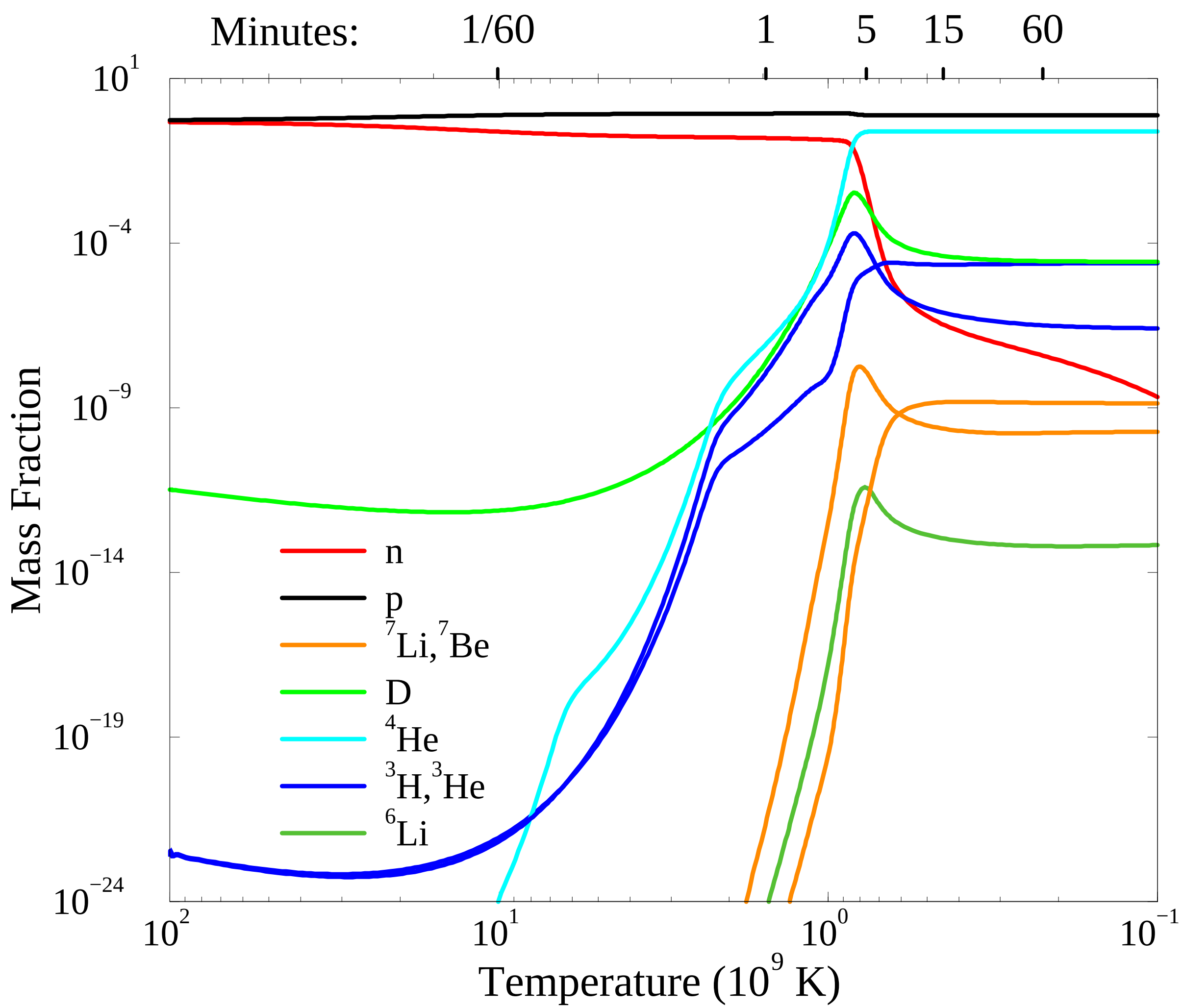
(Olive, 1999, Fig. 3)

Detailed Computations: Solution of rate-equations in expanding universe.

Recent computations: Thomas et al. (1993).

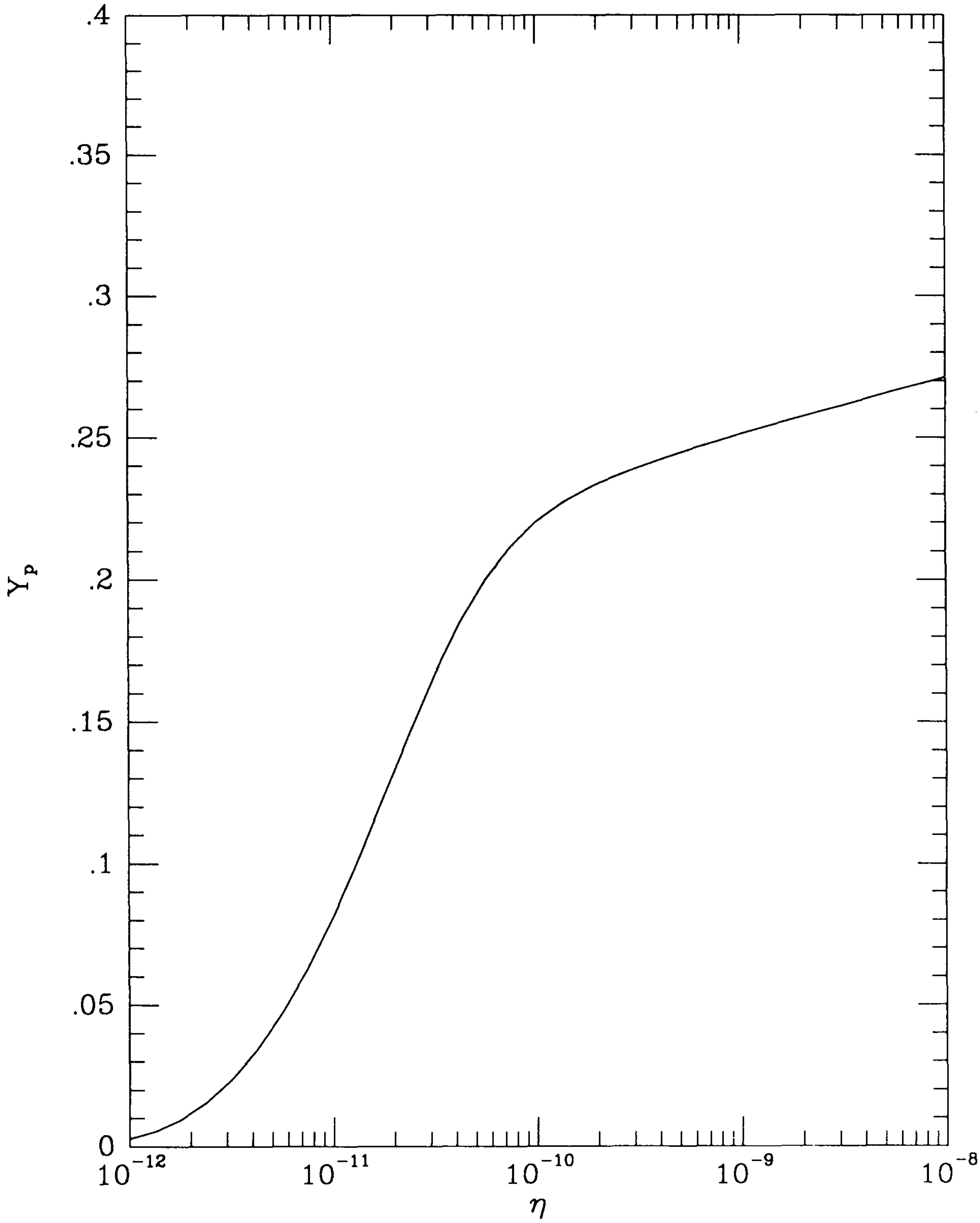
Recent reviews: Olive (1999), Tytler et al. (2000).

Detailed Computations, II



Build-up of abundances as function of time for $\eta = 5.1 \times 10^{-10}$ (Burles, Nollett & Turner, 1999, Fig. 3) [remember: $\eta = n_{\text{CMBR}}/n_{\text{baryons}}$]

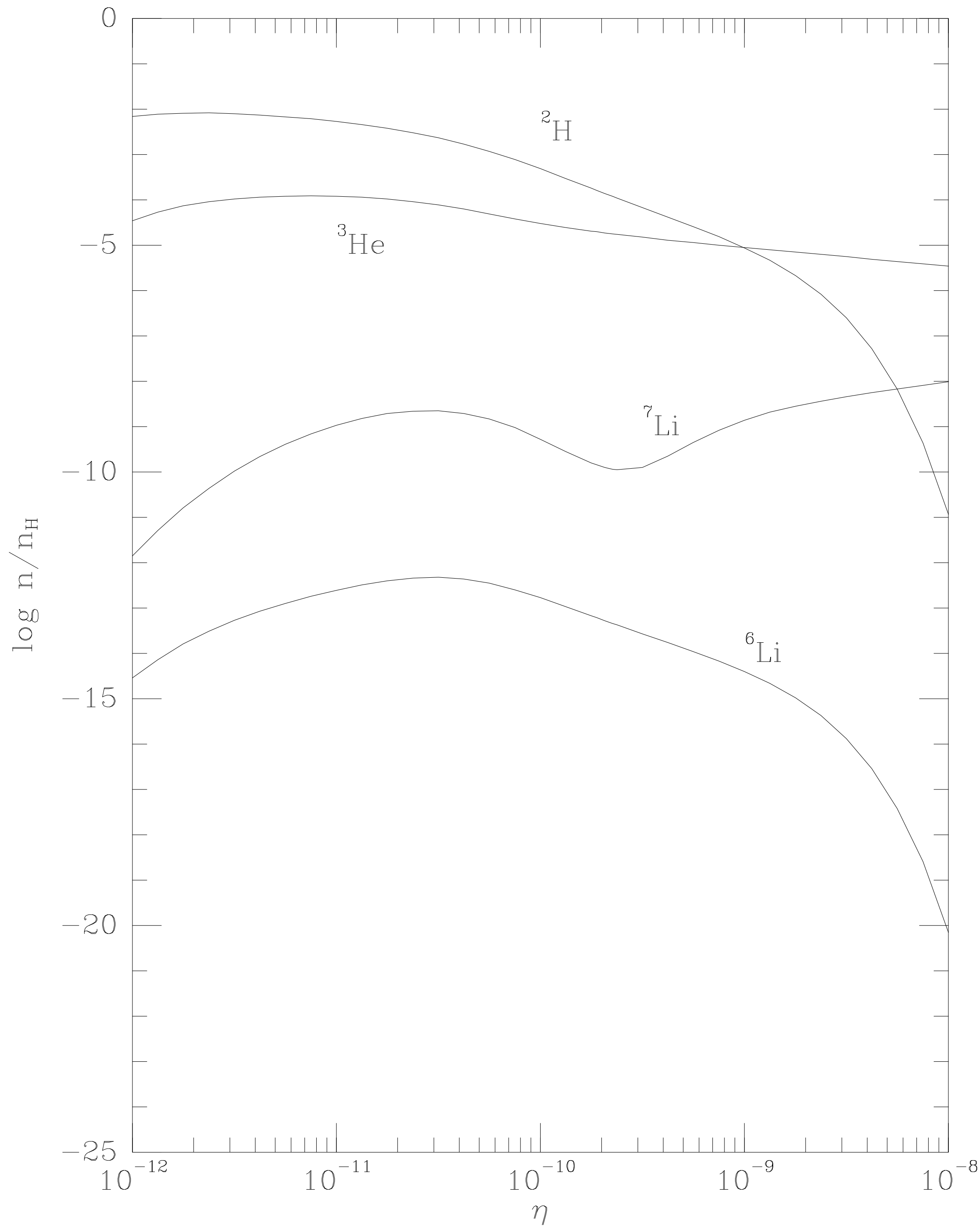
Detailed Computations, III



He abundance as function of η (Thomas et al., 1993, Fig. 3a).

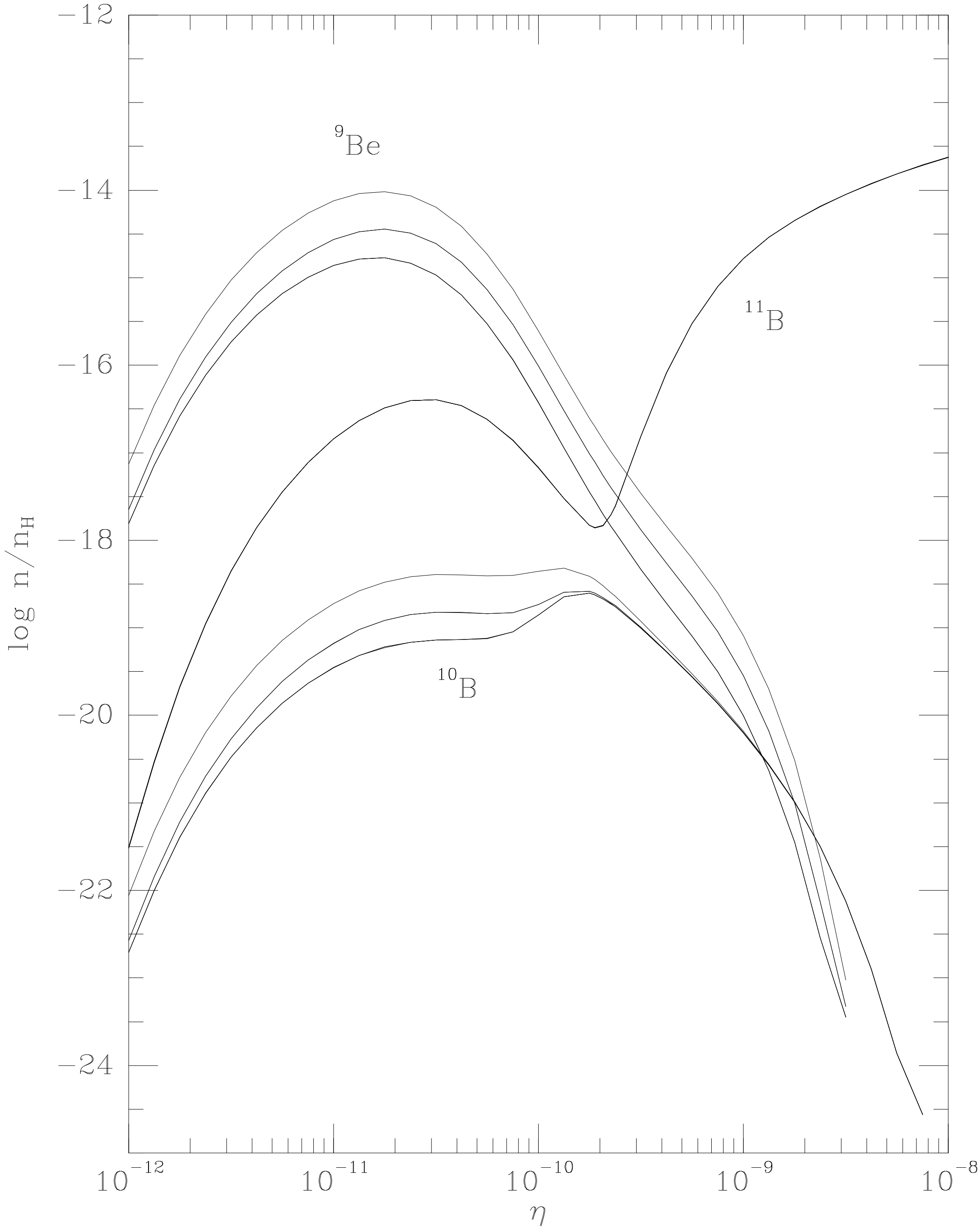
^{4}He mainly dependent on

Detailed Computations, IV



Light-element abundances as function of η (Olive, 1999, Fig. 4)

Detailed Computations, V



Intermediate mass abundances as function of η (Olive, 1999, Fig. 5).

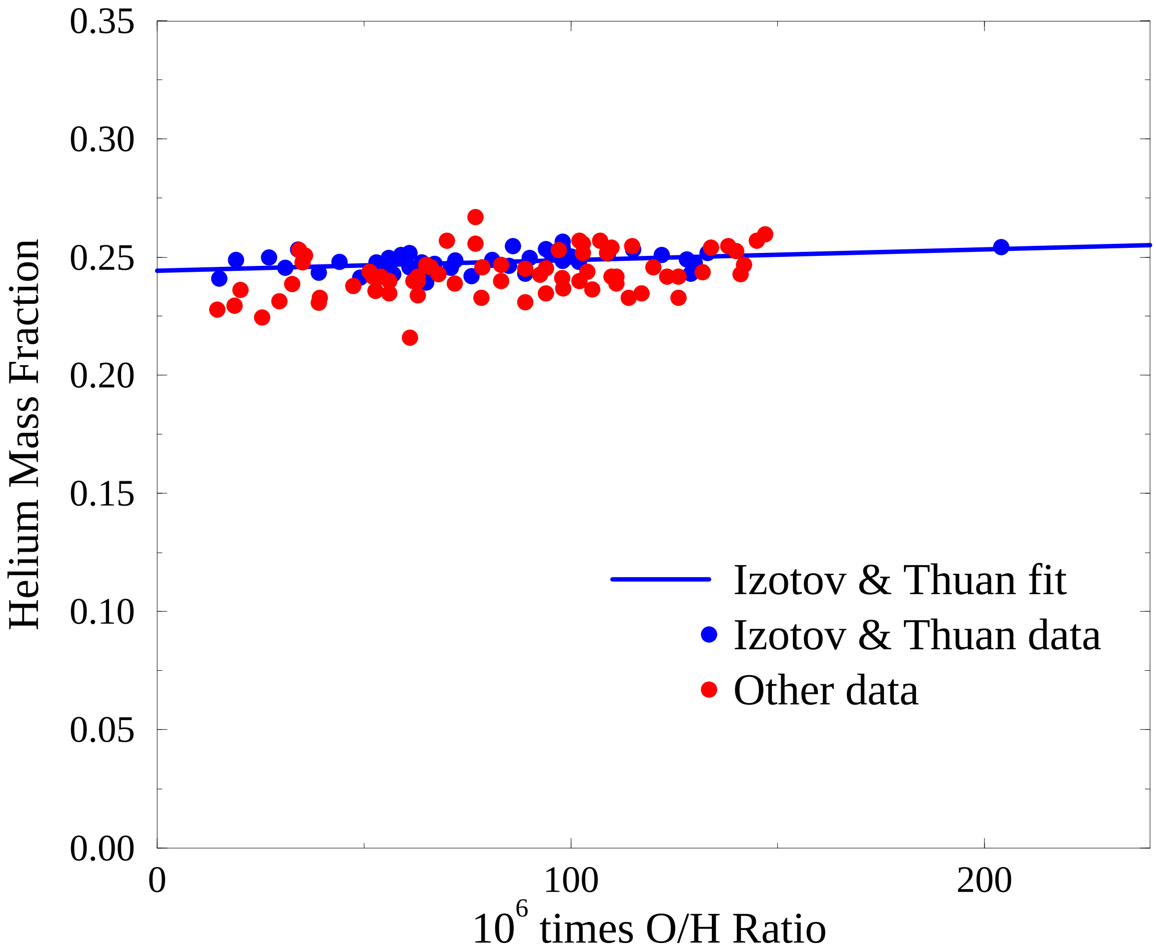
Remarkable Things

Note the following coincidences:

1. Freeze out of nucleons simultaneous to freeze out of neutrinos.
2. . . . and parallel to electron-positron annihilation.
3. Expansion slow enough that neutrons can be bound to nuclei.

$\Longrightarrow$ Long chain of coincidences makes our current universe possible!

^{4}He



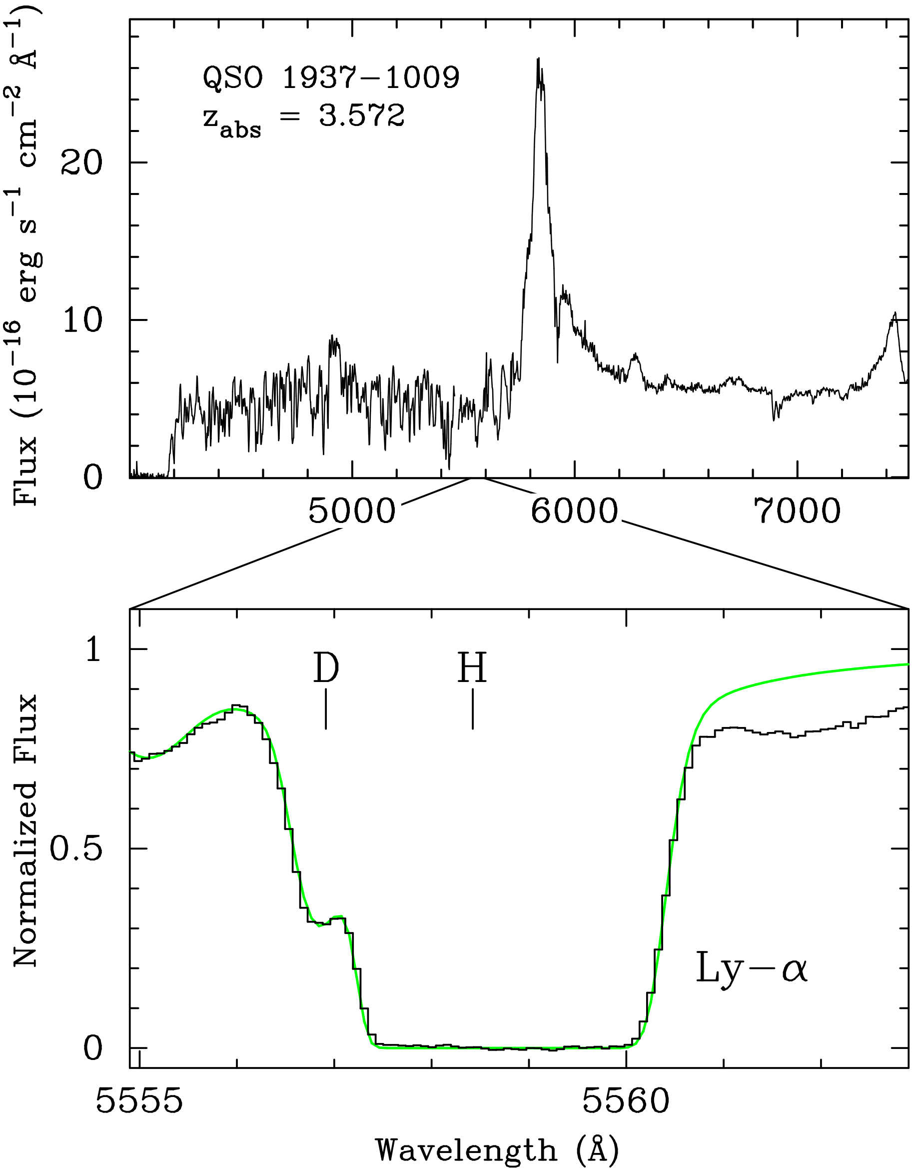
(Burles, Nollett & Turner, 1999, Fig. 4)

^{4}He produced in stars $\Longrightarrow$ extrapolate to zero metallicity in systems of low metallicity (i.e., minimize stellar processing).

Best determination from He II$\longrightarrow$He I recombination lines in H II regions (metallicity $\sim$ 20$\%$ solar).

Result: Linear correlation He vs. O
$\Longrightarrow$ extrapolate to zero oxygen to obtain primordial abundances. Result: $Y = 0.234 \pm 0.005$ (Olive, 1999).

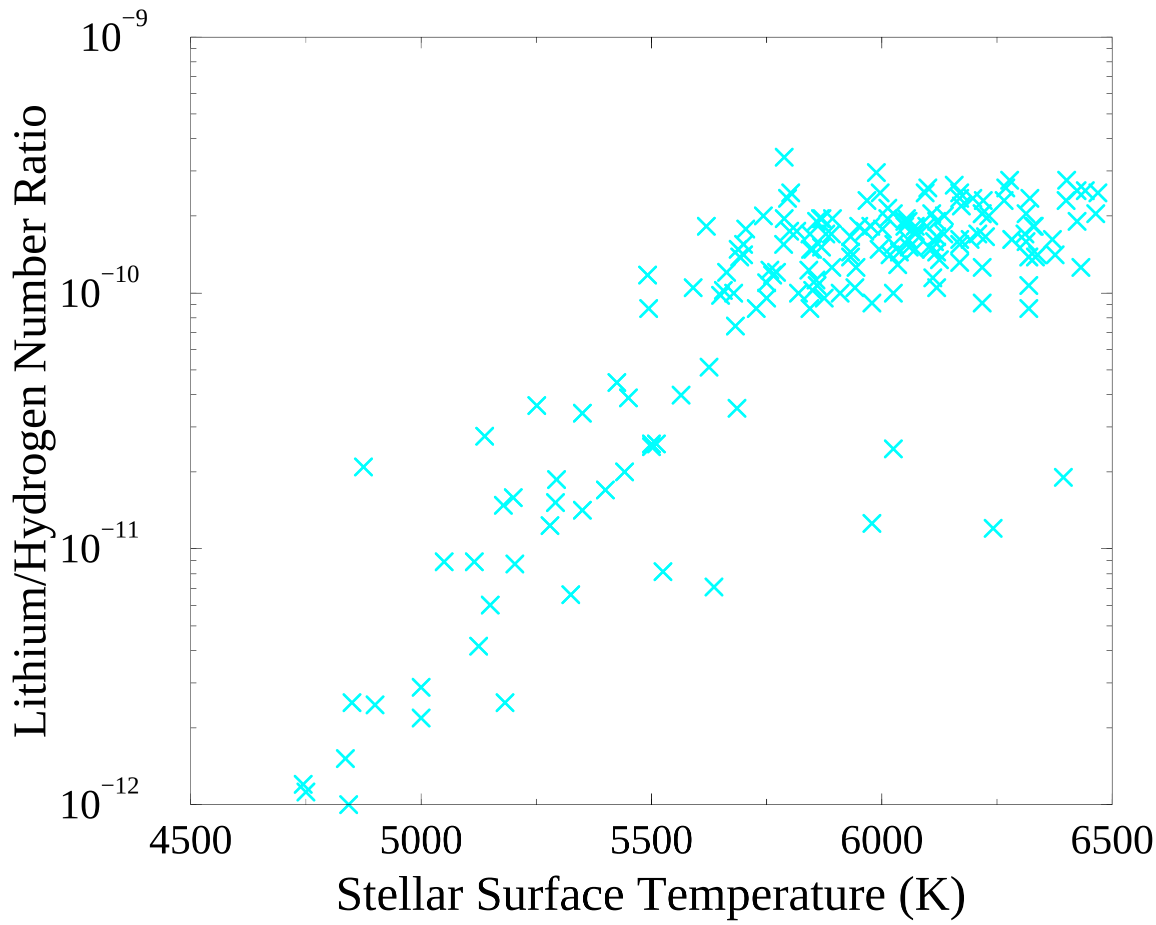
Deuterium



(Quasar 1937−1009; top: 3 m Lick, bottom: Keck; Burles, Nollett & Turner, 1999, Fig. 2)

Stars destroy D $\Longrightarrow$ use as non-processed material as possible!
Lyα forest (absorption of quasar light by intervening material)
$\Longrightarrow$ Structure caused by primordial deuterium, analysis of spectrum gives $\mathrm{D}/\mathrm{H} = (3.3 \pm 0.3) \times 10^{-5}$ (by number). Currently best measurement of primordial D-abundance.

Lithium



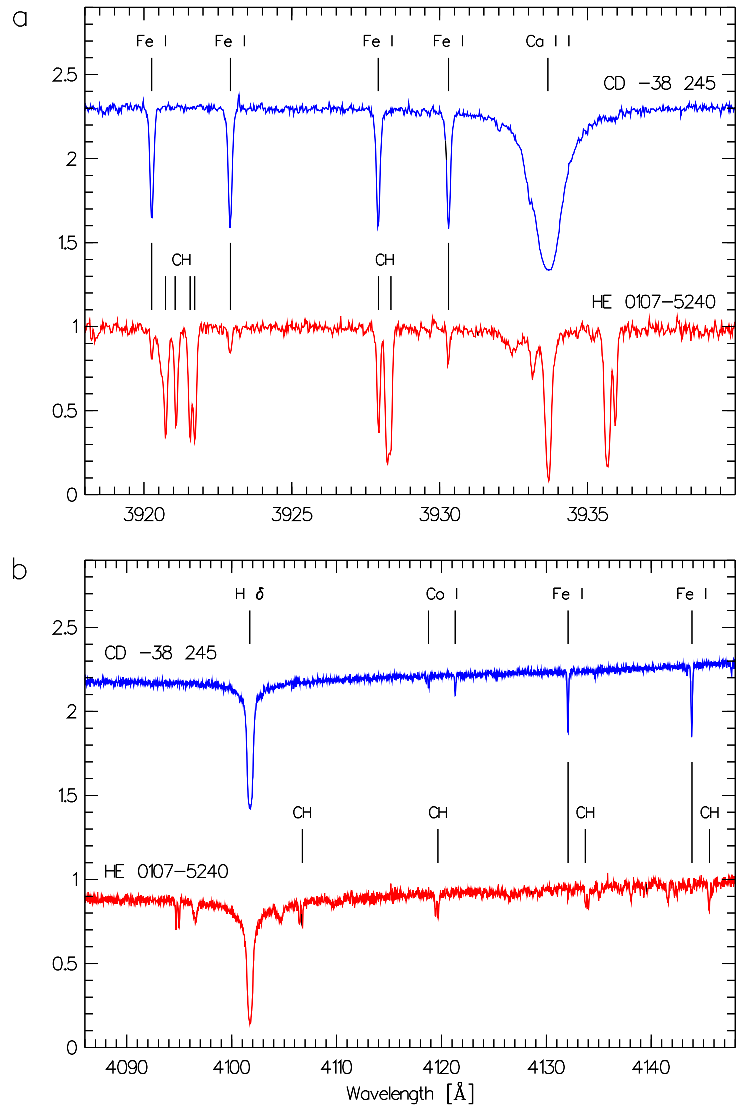
(Burles, Nollett & Turner, 1999, Fig. 5)

Stars with very low metallicity (old halo stars) show same Lithium abundance, ${}^{7}\mathrm{Li}/\mathrm{H} = 1.6 \times 10^{-10} \Longrightarrow$ close to primordial.

Cannot use galactic objects since Li also produced by spallation of heavier nuclei by cosmic rays ($10\times$ primordial produced this way).

Lower temperature stars: outer convection zone
$\Longrightarrow$ Li burning destroys Li.

Population III



(after Christlieb et al., 2002, Fig. 1)

Earliest stars should only have H, He, i.e., $Z = 0$

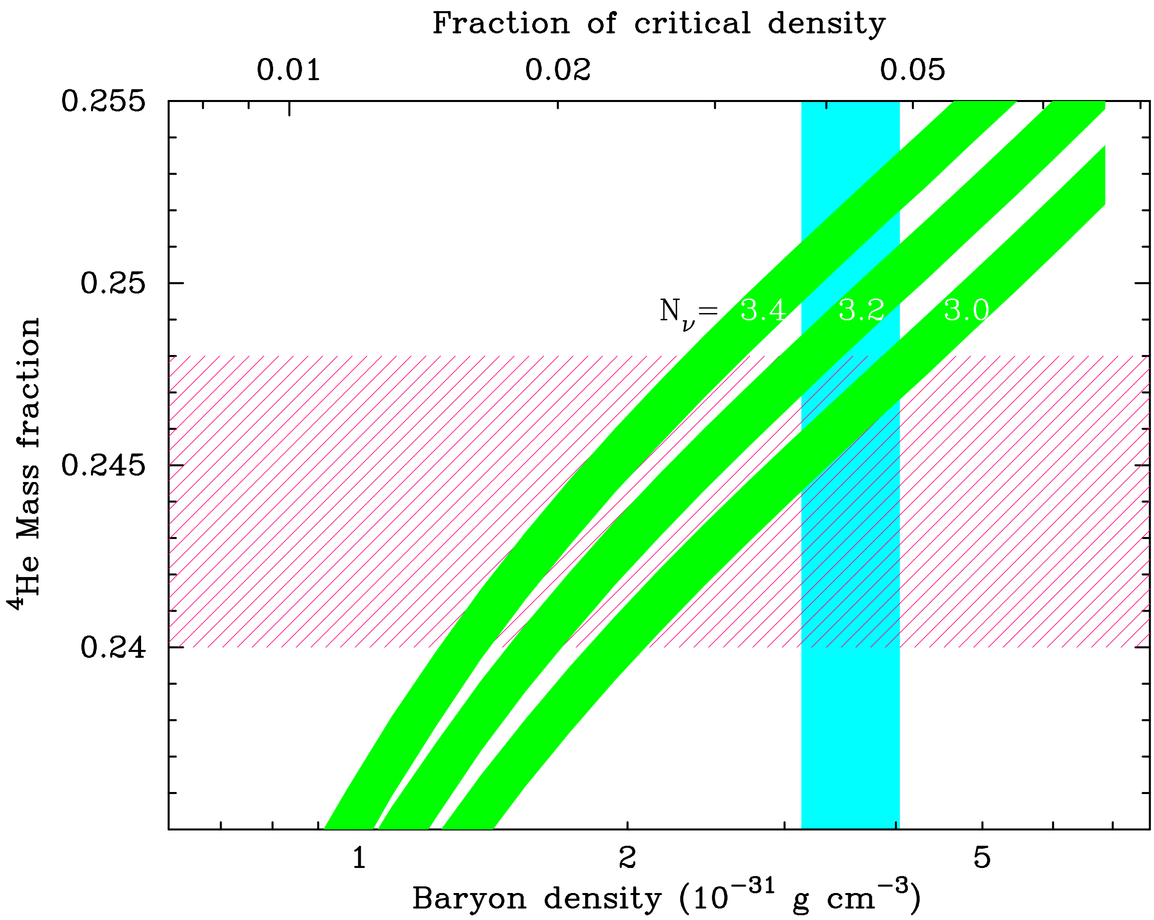
$\Longrightarrow$ would enable *direct* measure of primordial abundances.

Lowest metallicity known: HE0107$-$5240, with

Fe-abundance of 1/200000 solar

$\Longrightarrow$ "population III star", formed either from primordial gas cloud (and got some elements later through accretion from ISM), or from debris from type II SN explosion.

Neutrino Species

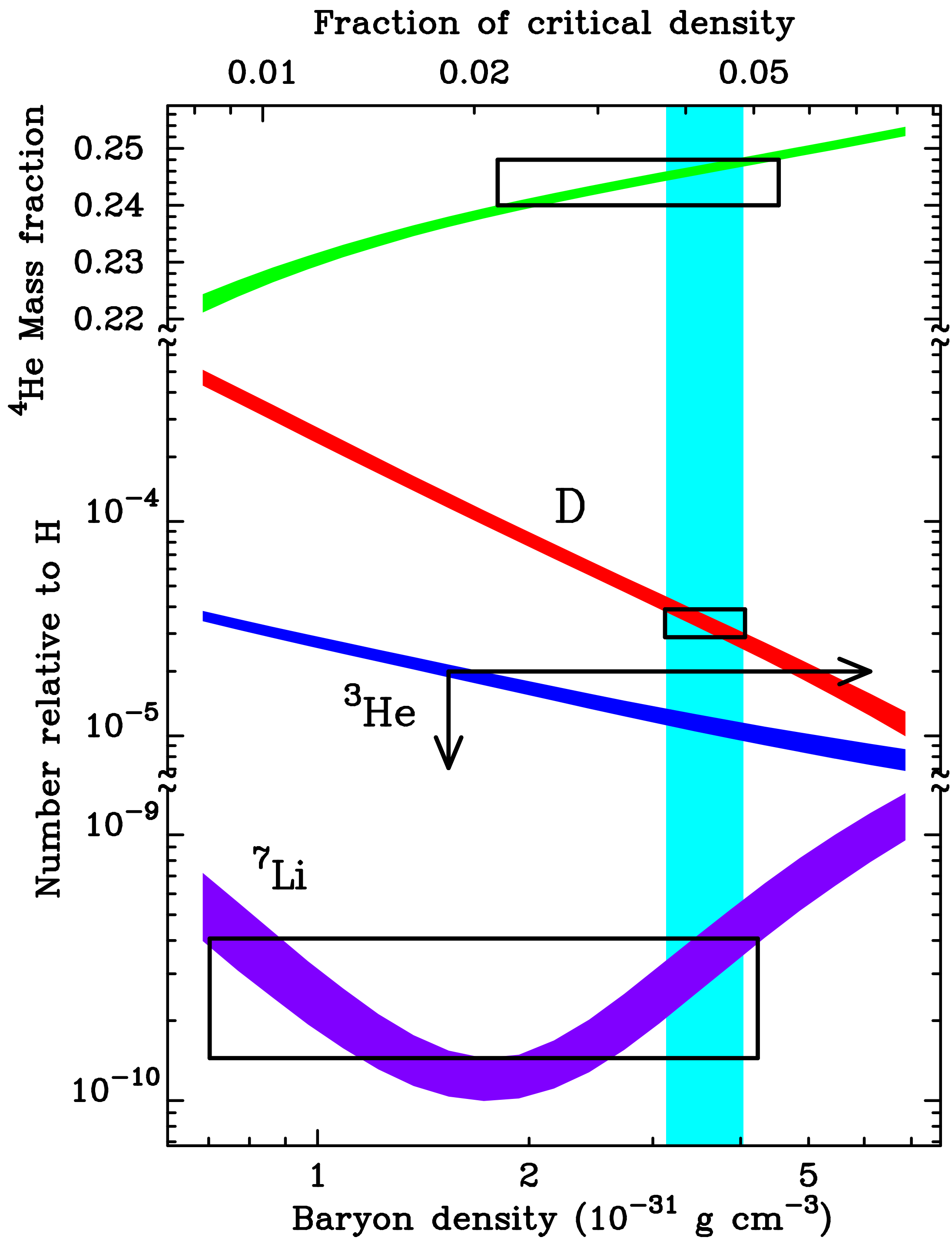


(Burles, Nollett & Turner, 1999, Fig. 7)

Number of neutrino species enters $\Omega \Longrightarrow$ Models for BBN constrain number of neutrino species to $N_\nu = 3$.

For a long time, BBN provided harder constraints on N_ν than laboratory experiments.

Summary



(Burles, Nollett & Turner, 1999, Fig. 1)

BBN strongly constrains $\Omega_{\mathrm{Baryons}}$.

Summary

Summary: History of the universe after its first 0.01 s (after Islam, 1992, Ch. 7, see also Weinberg, The first three minutes).

$t = 0.01\,\mathbf{s}$ $\quad T = 10^{11}\,\mathbf{K}$ $\quad \rho \sim 4 \times 10^{11}\,\mathbf{g\,cm^{-3}}$

Main constitutents: γ, ν, $\bar{\nu}$, e^--e^+ pairs.

No nuclei (instable). n and p in thermal balance.

$t = 0.1\,\mathbf{s}$ $\quad T = 3 \times 10^{10}\,\mathbf{K}$ $\quad \rho \sim 3 \times 10^{7}\,\mathbf{g\,cm^{-3}}$

Main constitutents: γ, ν, $\bar{\nu}$, e^--e^+ pairs. No nuclei.

$n + \nu \leftrightarrow p + e^-$: mass difference becomes important, 40% n, 60% p (by mass).

$t = 1.1\,\mathbf{s}$ $\quad T = 10^{10}\,\mathbf{K}$ $\quad \rho \sim 10^{5}\,\mathbf{g\,cm^{-3}}$

Neutrinos decouple, e^--e^+ pairs start to annihilate. No nuclei.

25% n, 75% p

Summary

$t = 13\,\mathbf{s}$ $T = 3 \times 10^9\,\mathbf{K}$ $\rho \sim 10^5\,\mathbf{g\,cm}^{-3}$

Reheating of photons, pairs annihilate, ν fully decoupled, deuterium still cannot form.

17% n, 83% p

$t = 3\,\mathbf{min}$ $T = 10^9\,\mathbf{K}$ $\rho \sim 10^5\,\mathbf{g\,cm}^{-3}$

Pairs are gone, neutron decay becomes important, start of nucleosynthesis

14% n, 86% p

$t = 35\,\mathbf{min}$ $T = 3 \times 10^8\,\mathbf{K}$ $\rho \sim 0.1\,\mathbf{g\,cm}^{-3}$

game over

Next important event: $t \sim 300000\,\text{years}$: Interaction CMB/matter stops ("last scattering", recombination).

Before we look at this, we look at

the first 0.01 s: the very early universe

Inflation

So far, have seen that BB works remarkably well in explaining the observed universe.

There are, however, quite big problems with the classical BB theories:

Horizon problem: CMB looks too isotropic $\Longrightarrow$ Why?

Flatness problem: Density close to BB was very close to $\Omega = 1$ (deviation $\sim 10^{-16}$ during nucleosynthesis) $\Longrightarrow$ Why?

Hidden relics problem: There are no observed magnetic monopoles, although predicted by GUT, neither gravitinos and other exotic particles $\Longrightarrow$ Why?

Vacuum energy problem: Energy density of vacuum is 10^{120} times smaller than predicted $\Longrightarrow$ Why?

Expansion problem: The universe expands $\Longrightarrow$ Why?

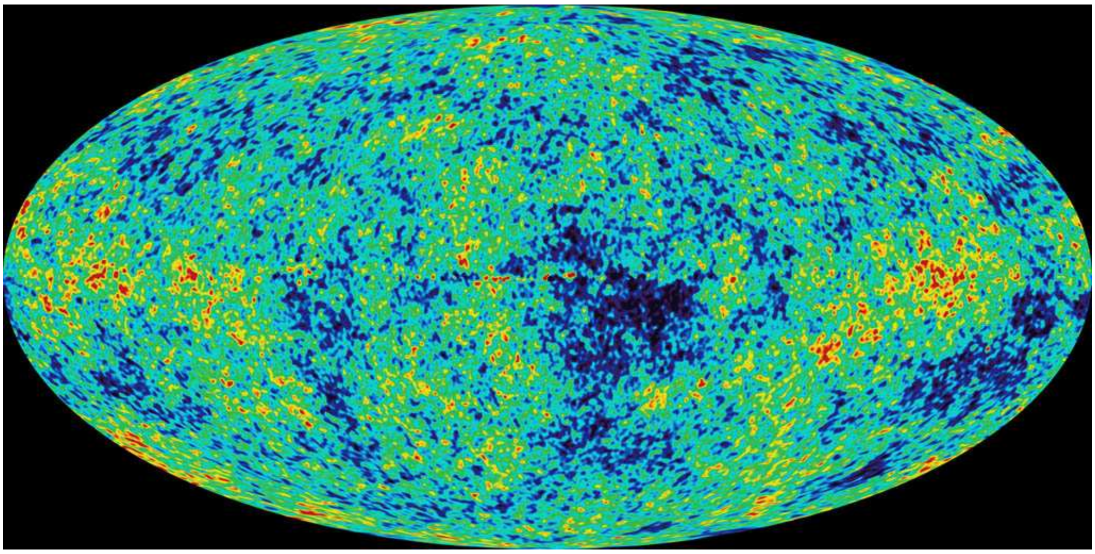
Baryogenesis: There is virtually no antimatter in the universe $\Longrightarrow$ Why?

Structure formation: Standard BB theory produces no explanation for lumpiness of universe.

Inflation attempts to answer all of these questions.

Recent Book: Liddle & Lyth (2000)

Horizon problem, I



(Bennett et al., 2003, temperature difference $\pm 200\,\mu$K)

COBE and WMAP: Temperature fluctuations in CMB on 10° scales:

$$\frac{\Delta T_{\text{CMB}}}{T_{\text{CMB}}} \sim 2 \times 10^{-5} \tag{6.107}$$

This is too small: Size of observable universe at given epoch ("particle horizon") is given by coordinate distance photons traveled since big bang (Eq. 4.46):

$$d_{\text{h}} = R_0 \cdot r_{\text{H}}(t) = \int_0^t \frac{c\,\mathrm{d}t}{a(t)} \tag{6.108}$$

For a matter dominated universe with $\Omega = 1$,

$$a(t) = \left(\frac{3H_0}{2}\,t\right)^{2/3} \tag{4.77}$$

such that for $t = t_0 = 2/(3H_0)$ (Eq. 4.78):

$$d_{\text{h}}(t_0) = \frac{3c}{(3H_0/2)^{2/3}}\,t_0^{1/3} = \frac{2c}{H_0} \tag{6.109}$$

Horizon problem, II

For matter dominated universes at redshift z, Eq. (6.109) works out to be (Peacock, 1999, eq. 11.2):

$$d_\mathrm{h} \approx \frac{6000}{\sqrt{\Omega z}}\, h^{-1}\,\mathrm{Mpc} \tag{6.110}$$

CMB decoupled from matter at $z \sim 1000$ (see later), such that then $d_\mathrm{h} \sim 200\,\mathrm{Mpc}$, while today $d_\mathrm{h} \sim 6000\,\mathrm{Mpc} \Longrightarrow$ current observable volume $\sim 30000\times$ larger!

Note: we use $a \Longrightarrow$ all scales refer to what they are *now*, not what they *were* when the photons started!

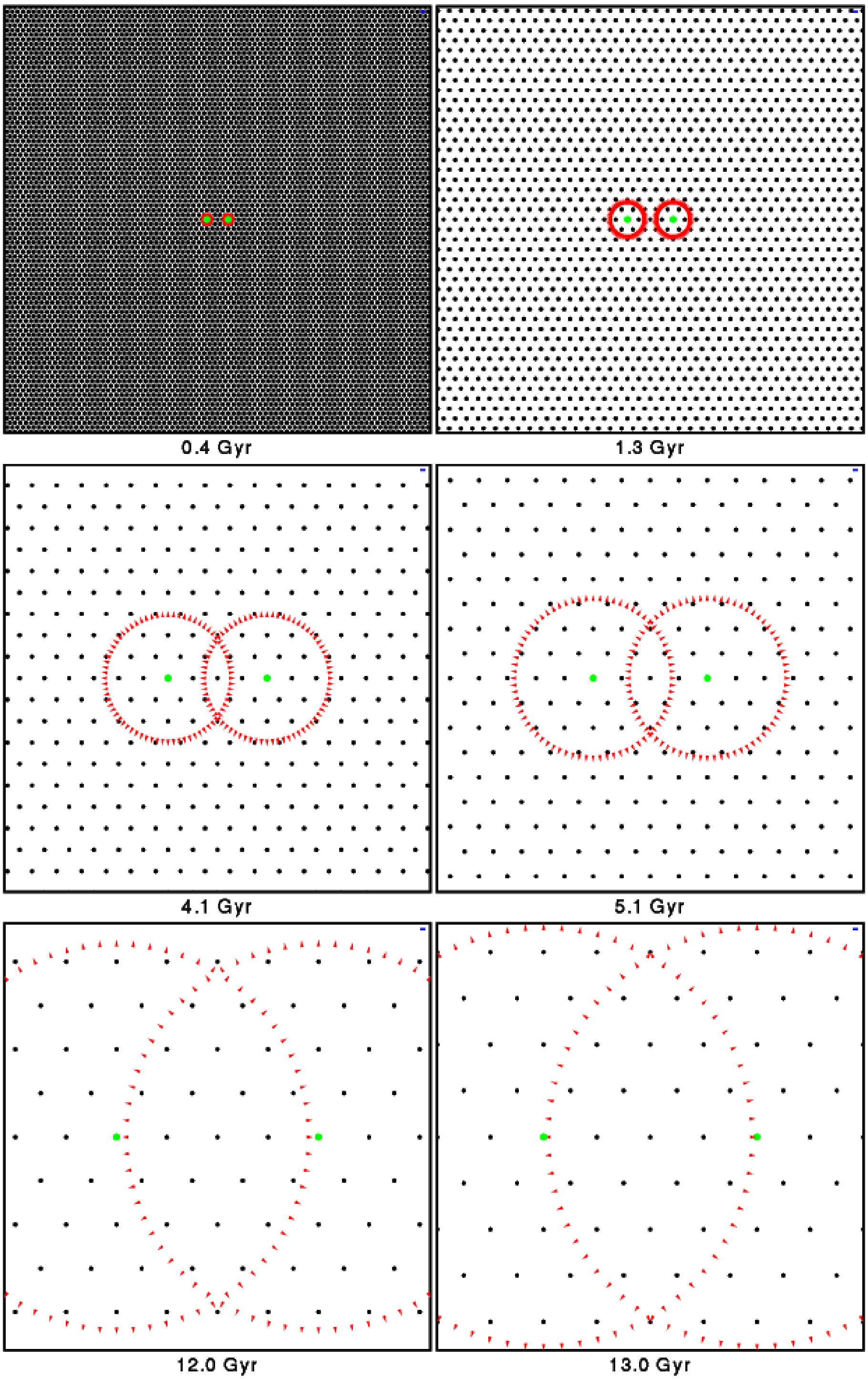
Horizon problem: Why were causally disconnected areas on the sky so similar when CMB last interacted with matter?

Note that the horizon distance is larger than Hubble length:

$$d_\mathrm{h} = \frac{2c}{H_0} > \frac{2c}{3H_0} = c \cdot t_0 = d_\mathrm{H} \tag{6.111}$$

Reason for this is that universe expanded while photons traveled towards us $\Longrightarrow$ Current observable volume larger than volume expected in a non-expanding universe.

Horizon problem, III



courtesy E. Wright.

Expansion of horizon in an expanding universe.

Flatness problem, I

Current observations of density of universe roughly imply

$$0.01 \lesssim \Omega \lesssim 2 \quad \text{i.e., } \Omega \sim 1 \tag{6.112}$$

(will be better constrained later).

$\Omega \sim 1$ imposes very strict conditions on initial conditions of universe:

The Friedmann equation (e.g., Eq. 4.61) can be written in terms of Ω:

$$\Omega - 1 = \frac{k}{a^2 H^2} = \frac{ck}{\dot{a}^2} \tag{6.113}$$

For a nearly flat, matter dominated universe, $a(t) \propto t^{2/3}$, such that

$$\frac{\Omega(t) - 1}{\Omega(t_0) - 1} = \left(\frac{t}{t_0}\right)^{2/3} \tag{6.114}$$

while for the radiation dominated universe with $a(t) \propto t$,

$$\frac{\Omega(t) - 1}{\Omega(t_0) - 1} = \frac{t}{t_0} \tag{6.115}$$

Flatness problem, II

Today: $t_0 = 3.1 \times 10^{17}\, h^{-1}$ s, i.e., observed flatness predicts for era of nucleosynthesis ($t = 1$ s):

$$\frac{\Omega(1\,\mathrm{s}) - 1}{\Omega(t_0) - 1} \sim 10^{-12} \dots 10^{-16} \tag{6.116}$$

i.e., very close to unity.

Flatness problem: It is very unlikely that Ω was so close to unity at the beginning without a physical reason.

Had Ω been different from 1, the universe would immediately have been collapsed or expanded too fast $\Longrightarrow$ Anthropocentric point of view *requires* $\Omega = 1$.

Hidden relics problem

Modern theories of particle physics predict the following particles to exist:

Gravitinos: From supergravity, spin $3/2$ particle with $mc^2 \sim 100\,\text{GeV}$, if it exists, then nucleosynthesis would not work if BB started at $kT > 10^9\,\text{GeV}$.

Moduli: Spin-0 particles from superstring theory, contents of vacuum at high energies.

Magnetic Monopoles: Predicted in grand unifying theories, but not observed.

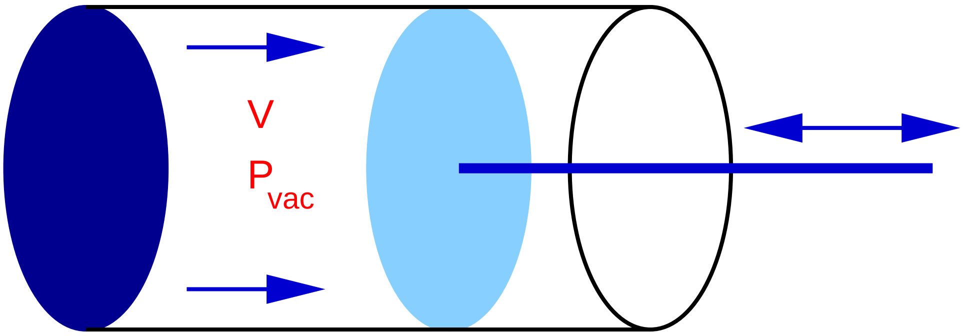
Hidden relics problem: If there was a normal big bang, then strange particles should exist, which are not observed today.

Vacuum, Λ, I

What is vacuum? *Not empty space* but rather ground state of some physical theory.

Reviews: Carroll, Press & Turner (1992), Carroll (2001).

Since ground state should be same in all coordinate systems $\Longrightarrow$ Vacuum is Lorentz invariant.



(after Peacock, 1999, Fig. 1.3)

Equation of state (Zeldovich, 1968):

$$P_{\text{vac}} = -\rho_{\text{vac}} c^2 \tag{6.117}$$

This follows directly from 1st law of thermodynamics: ρ_{vac} should be constant if compressed or expanded, which is true only for this type of equation of state:

$$\mathrm{d}E = \mathrm{d}U + P\,\mathrm{d}V = \rho_{\text{vac}} c^2\,\mathrm{d}V - \rho_{\text{vac}} c^2\,\mathrm{d}V = 0 \tag{6.118}$$

An alternative derivation goes via the stress-energy momentum tensor of a perfect fluid, see Carroll, Press & Turner (1992).

Vacuum, Λ, II

ρ_{vac} defines Einstein's cosmological constant

$$\Lambda = -\frac{8\pi G \rho_{\text{vac}}}{c^4} \tag{6.119}$$

Adding ρ_{vac} to the Friedmann equations allows to define

$$\Omega_\Lambda = \frac{\rho_{\text{vac}}}{\rho_{\text{crit}}} = \frac{\rho_{\text{vac}}}{3H^2/8\pi G} = \frac{c^4\Lambda}{3H^2} \tag{6.120}$$

Classical physics: Particles have energy

$$E = T + V \tag{6.121}$$

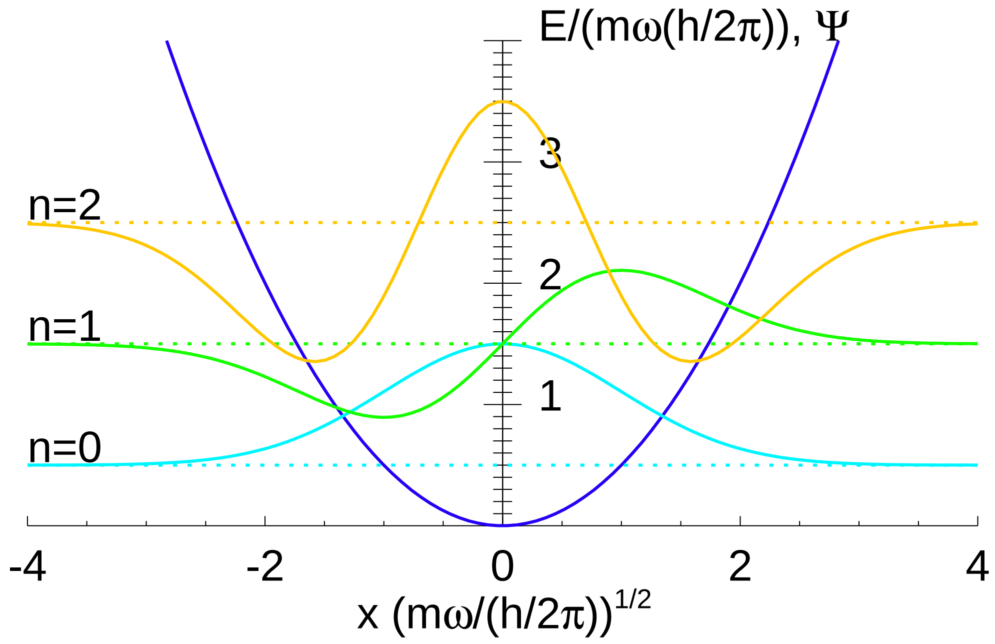
and force is $F = -\nabla V$, i.e., can add constant without changing equation of motion

$\Longrightarrow$ In classical physics, we are able to define $\rho_{\text{vac}} = 0$!

Quantum mechanics is (as usual) more difficult.

Vacuum, Λ, III

Vacuum in quantum mechanics:



Simplest case: harmonic oscillator:

$$V(x) = \frac{1}{2}m\omega^2x^2 \quad \text{i.e.,} \quad V(0) = 0 \tag{6.122}$$

However, particles can only have energies

$$E_n = \frac{1}{2}\hbar\omega + n\hbar\omega \quad \text{where } n \in \mathbb{N} \tag{6.123}$$

$\Longrightarrow$ Vacuum state has zero point energy

$$E_0 = \frac{1}{2}\hbar\omega \tag{6.124}$$

Simple consequence of uncertainty principle!

In QM, could normalize $V(x)$ such that $E_0 = 0$, important here is that vacuum state energy *differs* from classical expectation!

Vacuum, Λ, IV

Quantum field theory: Field as collection of harmonic oscillators of all frequencies. Simplest case: spinless boson ("scalar field", ϕ).

$\Longrightarrow$ Vacuum energy sum of all contributing modes:

$$E_0 = \sum_j \frac{1}{2}\hbar\omega_j \tag{6.125}$$

Compute sum by putting system in box with volume L^3, and then $L \longrightarrow \infty$.

Box $\Longrightarrow$ periodic boundary conditions:

$$\lambda_i = L/n_i \quad \Longleftrightarrow \quad k_i = 2\pi/\lambda_i = 2\pi n_i/L \tag{6.126}$$

for $n_i \in \mathbb{N} \Longrightarrow \mathrm{d}k_i L/2\pi$ discrete wavenumbers in $[k_i, k_i + \mathrm{d}k_i]$, such that

$$E_0 = \frac{1}{2}\hbar L^3 \int \frac{\omega_{\mathbf{k}}}{(2\pi)^3}\mathrm{d}^3\mathbf{k} \quad \text{where} \quad \omega_k^2 = k^2 + m^2/\hbar^2 \tag{6.127}$$

Imposing cutoff k_{max}:

$$\rho_{\mathrm{vac}}c^2 = \lim_{L\to\infty}\frac{E_0}{L^3} = \hbar\frac{k_{\mathrm{max}}^4}{16\pi^3} \tag{6.128}$$

Divergent for $k_{\mathrm{max}} \longrightarrow \infty$ ("ultraviolet divergence").

Not worrisome: Expect QM to break down at large energies anyway (ignored collective effects, etc.).

Vacuum, Λ, V

When does classical quantum mechanics break down?

Estimate: Formation of "Quantum black holes":

$$\lambda_{\text{de Broglie}} = \frac{2\pi\hbar}{mc} < \frac{2Gm}{c^2} = r_{\text{Schwarzschild}} \tag{6.129}$$

$\Longrightarrow$ Defines Planck mass:

$$m_{\text{P}} = \sqrt{\frac{\hbar c}{G}} \mathrel{\widehat{=}} 1.22 \times 10^{19}\,\text{GeV} \tag{6.130}$$

Corresponding length scale: Planck length:

$$l_{\text{P}} = \frac{\hbar}{m_{\text{P}}} = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-37}\,\text{cm} \tag{6.131}$$

...and time scale (Planck time):

$$t_{\text{P}} = \frac{l_{\text{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \sim 10^{-47}\,\text{s} \tag{6.132}$$

$\Longrightarrow$ Limits of current physics until successful theory of quantum gravity.

The system of units based on l_{P}, m_{P}, t_{P} is called the system of Planck units.

Vacuum, Λ, VI

To compute QFT vacuum energy density, choose

$$k_{\text{max}} = m_{\text{P}}c^2/\hbar \tag{6.133}$$

Inserting into Eq. (6.128) gives

$$\rho_{\text{vac}}c^2 = 10^{74}\,\text{GeV}\,\hbar^{-3} \quad \text{or} \quad \rho_{\text{vac}} \sim 10^{92}\,\text{g}\,\text{cm}^{-3} \tag{6.134}$$

a tad bit on the high side ($\sim 10^{120}$ higher than observed).

Inserting ρ_{vac} in Friedmann equation:
$T < 3\,\text{K}$ at $t = 10^{-41}\,\text{s}$ after Big Bang.
To obtain current universe, require $k_{\text{max}} = 10^{-2}\,\text{eV} \Longrightarrow$ Less than binding energy of Hydrogen, where QM definitively works!

> Vacuum energy problem: Contributions from virtual fluctuations of all particles must cancel to very high precision to produce observable universe.

Casimir effect: force between conducting plates of area A and distance a in vacuum is $F_{\text{Casimir}} = \hbar c A \pi^2/(240 a^4) \Longrightarrow$ caused by incomplete cancellation of quantum fluctuations. Confirmed by Lamoreaux in 1996 at 5% level.

Expansion problem

Cosmological Expansion:

GR predicts expansion of the universe, but initial conditions for expansion are not set!

Classical cosmology: “The unverse expands since it has expanded in the past”

$\Longrightarrow$ Hardly satisfying. . .

> Cosmological Expansion Problem: What is the physical mechanism responsible for the expansion of the universe?

To put it more bluntly:

“The Big Bang model explains nothing about the origin of the universe as we now perceive it, because all the most important features are ‘predestined’ by virtue of being built into the assumed initial conditions near to $t = 0$.” (Peacock, 1999, p. 324)

Baryogenesis

Quantitatively: Today:

$$\frac{N_{\mathrm{p}}}{N_\gamma} \sim 10^{-9} \quad \text{but} \quad \frac{N_{\bar{\mathrm{p}}}}{N_\gamma} \sim 0 \tag{6.135}$$

Assuming isotropy and homogeneity, this is puzzling: Violation of Copernican principle!

> Antimatter problem: There are more particles than antiparticles in the observable universe.

Sakharov (1968): Asymmetry implies three fundamental properties for theories of particle physics:

1. CP violation (particles and antiparticles must behave differently in reactions, observed, e.g., in the K^0 meson),
2. Baryon number violating processes (more baryons than antibaryons $\Longrightarrow$ Prediction by GUT),
3. Deviation from thermal equilibrium in early universe (CPT theorem: $m_{\mathrm{X}} = m_{\bar{\mathrm{X}}} \Longrightarrow$ *same* number of particles and antiparticles in thermal equilibrium).

Structure formation

Final problem: structure formation

In the classical BB picture, the initial conditions for structure formation observed are not explained. Furthermore, assuming the observed Ω_{baryons}, the observed structures (=us) cannot be explained.

The theory of inflation attempts to explain all of the problems mentioned by invoking phase of exponential expansion in the very early universe ($t \lesssim 10^{-16}$ s).

Basic Idea, I

Use the Friedmann equation with a cosmological constant:

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \qquad (6.136)$$

Basic assumption of inflationary cosmology:

During the big bang there was a phase where Λ dominated the Friedmann equation.

$$H(t) = \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3}} = \text{const.} \qquad (6.137)$$

since $\Lambda = \text{const.}$ (probably...).

Solution of Eq. (6.137):

$$a \propto \mathrm{e}^{Ht} \qquad (6.138)$$

and inserting into Eq. (6.113) shows that

$$\Omega - 1 = \frac{k}{a^2H^2} \propto \mathrm{e}^{-2Ht} \qquad (6.139)$$

Basic Idea, II

When did inflation happen?

Typical assumption: Inflation $=$ phase transition of a scalar field ("inflaton") associated with Grand Unifying Theories.

Therefore the assumptions:

- temperature $kT_{\text{GUT}} = 10^{15}$ GeV, when $1/H \sim 10^{-34}\,\mathrm{sec}$ ($t_{\text{start}} \sim 10^{-34}\,\mathrm{sec}$).
- inflation lasted for 100 Hubble times, i.e., for $\Delta T = 10^{-32}\,\mathrm{sec}$.

With Eq. (6.138):

$$\boxed{\text{Inflation: Expansion by factor } \mathrm{e}^{100} \sim 10^{43}.}$$

... corresponding to a volume expansion by factor $\sim 10^{130} \Longrightarrow$ solves hidden relics problem!

Furthermore, Eq. (6.139) shows

$$\Omega - 1 = 10^{-86} \tag{6.140}$$

$\Longrightarrow$ solves flatness problem!

Basic Idea, III

Temperature behavior: *During* inflation universe supercools:

Remember: entropy density

$$s = \frac{\rho c^2 + P}{T} \tag{6.79}$$

But for Λ:

$$p = -\rho c^2 \tag{6.117}$$

so that the entropy density of vacuum

$$s_{\text{vac}} = 0 \tag{6.141}$$

Trivial result since vacuum is just one quantum state $\Longrightarrow$ *very* low entropy.

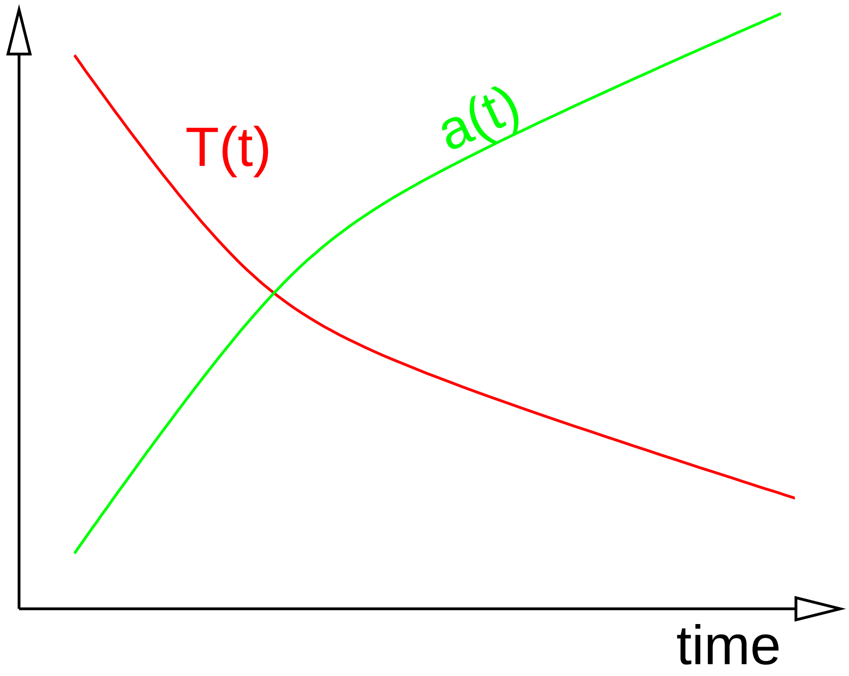
Inflation produces no entropy $\Longrightarrow$ S existing *before* inflation gets diluted, since entropy *density* $s \propto a^{-3}$.
But for relativistic particles $s \propto T^3$ (Eq. 6.82), such that

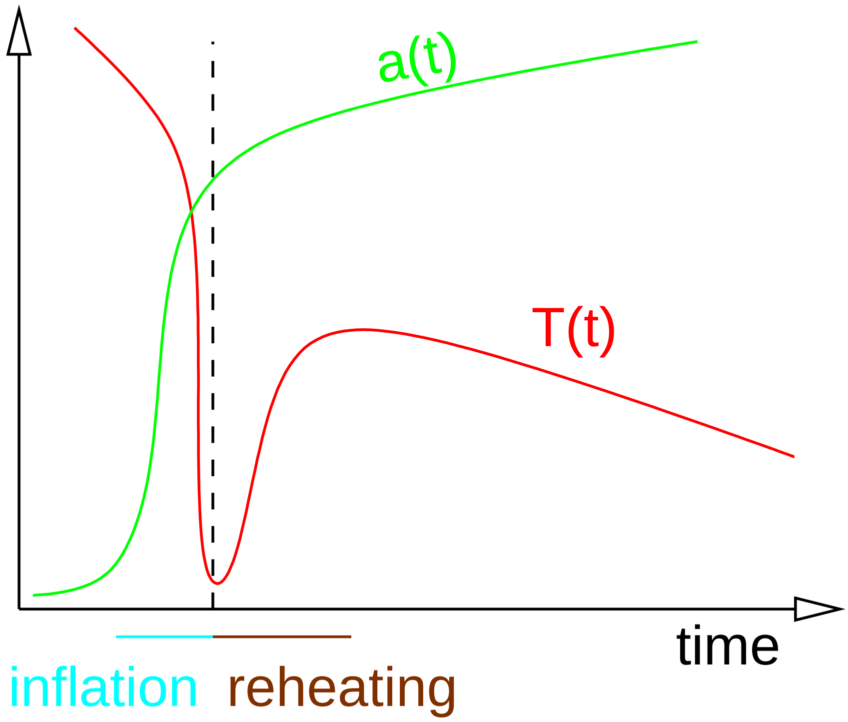
$$aT = \text{const.} \quad \Longrightarrow \quad T_{\text{after}} = 10^{-43} T_{\text{before}} \tag{6.142}$$

When inflation stops: vacuum energy of inflaton field transferred to normal matter
$\Longrightarrow$ "Reheating" to temperature

$$T_{\text{reheating}} \sim 10^{15}\,\text{GeV} \tag{6.143}$$

Summary





(after Bergström & Goobar, 1999, Fig. 9.1, and Kolb & Turner, Fig. 8.2)

Scalar Fields, I

For inflation to work: need short-term cosmological constant, i.e., need particles with negative pressure.

Basic idea (Guth, 1981): cosmological phase transition where suddenly a large Λ happens.

How? $\Longrightarrow$ Quantum Field Theory!

Describe hypothetical particle with a time-dependent quantum field, $\phi(t)$, and potential, $V(\phi)$.

Simplest example from QFT ($\hbar = c = 1$):

$$V(\phi) = \frac{1}{2}m^2\phi^2 \qquad (6.144)$$

where m: "mass of field".

Particle described by ϕ: "inflaton".

For all scalar fields, particle physics shows:

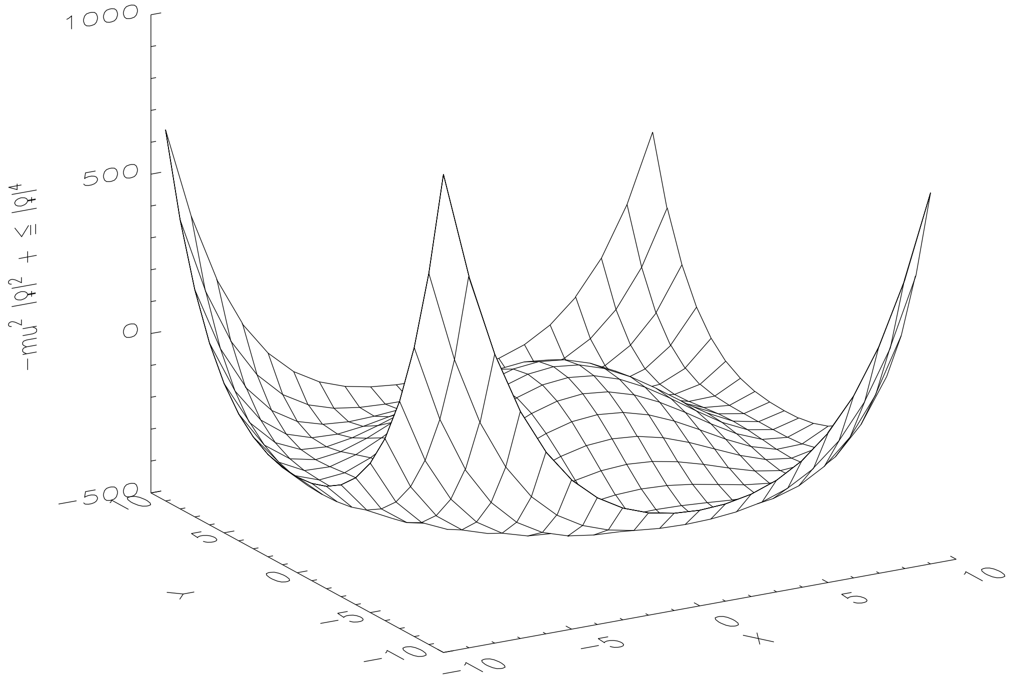
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad (6.145)$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad (6.146)$$

i.e., obeys vacuum EOS!

"Vacuum": particle "sits" at minimum of V.

Scalar Fields, II



Typically: potential looks more complicated.
Due to symmetry, after harmonic oscillator, 2nd simplest potential: Mexican hat potential ("Higgs potential"),

$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4 \tag{6.147}$$

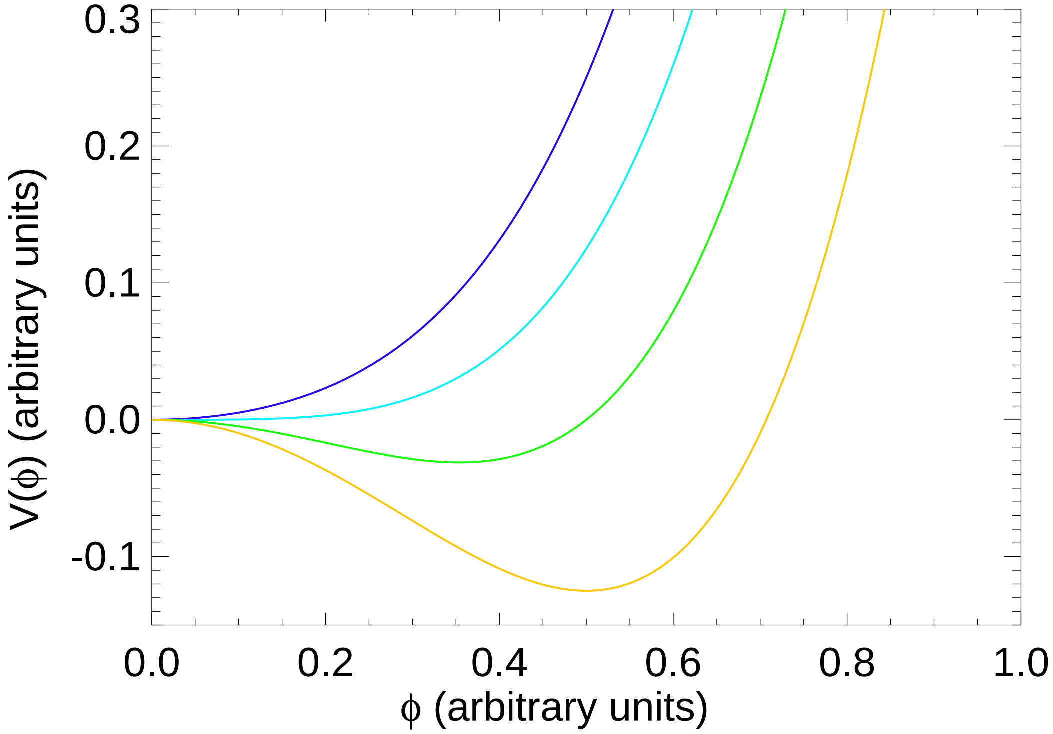
$\Longrightarrow$ Minimum of V still determines vacuum value.
For $T \neq 0$, need to take interaction with thermal bath into account $\Longrightarrow$ Temperature dependent potential!

$$V_{\text{eff}}(\phi) = -(\mu^2 - aT^2)\phi^2 + \lambda\phi^4 \tag{6.148}$$

where a some constant.

(minimization of Helmholtz free energy, see Peacock, 1999, , p. 329ff., for details)

Scalar Fields, III



The minimum of V is at

$$\phi = \begin{cases} 0 & \text{for } T > T_\mathrm{c} \\ \sqrt{(\mu^2 - aT^2)/(2\lambda)} & \text{for } T < T_\mathrm{c} \end{cases} \tag{6.149}$$

where the critical temperature

$$T_\mathrm{c} = \mu/\sqrt{a} \tag{6.150}$$

and

$$V_\text{min} = \begin{cases} 0 & \text{for } T > T_\mathrm{c} \\ -\frac{(\mu^2 - aT^2)^2}{4\lambda} & \text{for } T < T_\mathrm{c} \end{cases} \tag{6.151}$$

Since switch happens suddenly: phase transition

Scalar Fields, IV

Minimum V_{min} for $T > T_{\text{c}}$ smaller than "vacuum minimum" $\Longrightarrow$ Behaves like a cosmological constant!

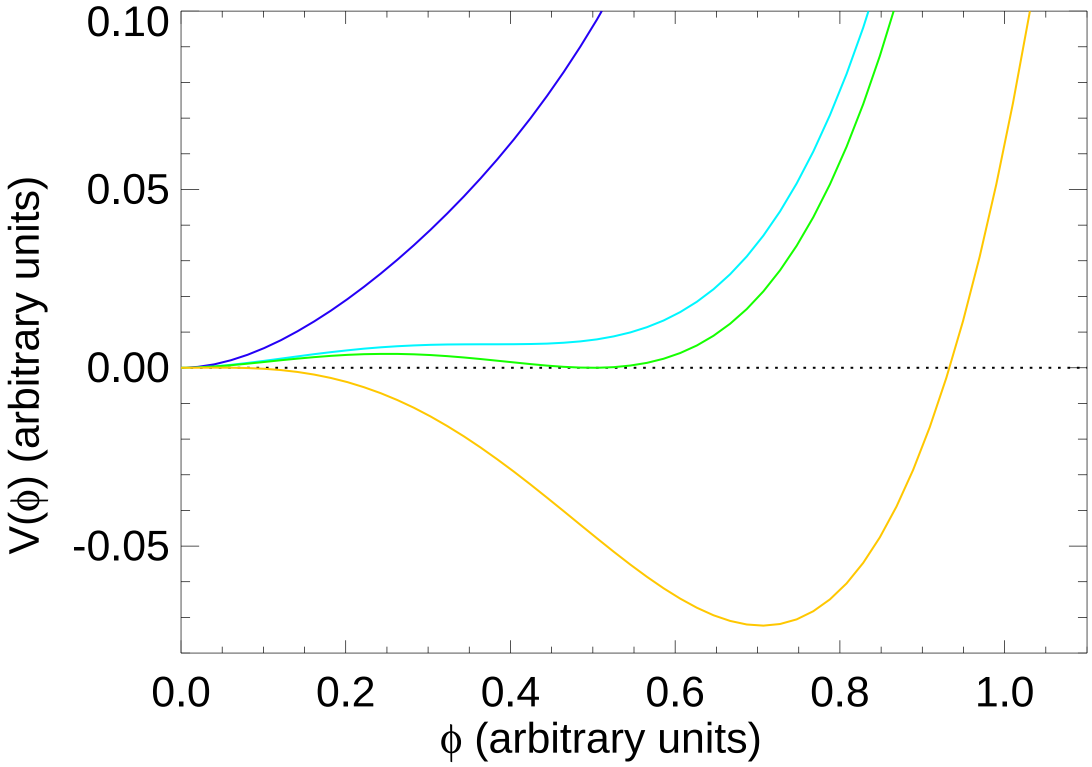
Since $T_{\text{c}} \propto \mu$,

> Inflation sets in at mass scale of whatever scalar field produces inflation.

Grand Unifying Theories: $m \sim 10^{15}$ GeV.

The problem is, what $V(\phi)$ to use...

First-Order Inflation



(after Peacock, 1999, Fig. 11.2)

Original idea (Guth, 1981):

$$V(\phi, T) = \lambda|\phi|^4 - b|\phi|^3 + aT^2|\phi|^2 \qquad (6.152)$$

has two minima for T greater than a critical temperature:

$V_{\text{min}}(\phi = 0)$: false vacuum

$V_{\text{min}}(\phi > 0)$: true vacuum iff < 0.

Particle can tunnel between both vacua: first order phase transition $\Longrightarrow$ first order inflation.

Problem: vacuum tunnels between false and true vacua $\Longrightarrow$ formation of bubbles.

Outside of bubbles: inflation goes infinitely ("graceful exit problem").

First order inflation is not feasible.

Summary

First order inflation does not work $\Longrightarrow$ Potentials derived from GUTs do not work.

$\Longrightarrow$ However, many empirical potentials do not suffer from these problems $\Longrightarrow$ inflation is *still* theory of choice.

Catchphrases (Liddle & Lyth, 2000, Ch. 8):

- supersymmetry/-gravitation $\Longrightarrow$ tree-level potentials,
- renormalizable global susy,
- chaotic inflation,
- power-law inflation,
- hybrid inflation (combination of *two* scalar fields) $\Longrightarrow$ spontaneous or dynamical susy breaking,
- scalar-tensor gravity

. . . and many more

All are somewhat *ad hoc*, and have more or less foundations in modern theories of QM and gravitation.

Information on what model correct from

1. predicted seed to structure formation, and
2. values of Ω and Λ.

$\Longrightarrow$ Determine Ω and Λ!

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Determination of Omega and Lambda

Inflation

Previous lectures: Inflation requires

$$\Omega = \Omega_\mathrm{m} + \Omega_\Lambda = 1 \tag{7.1}$$

Here,

Ω_m: Ω due to gravitating stuff,

Ω_Λ: Ω due to vacuum energy or other exotic stuff.

To decide whether that is true:

- need inventory of gravitating material in the universe,
- need to search for evidence of non-zero Λ

Also search for evidence in structure formation $\Longrightarrow$ Later...

Often, express Ω in terms of a mass to luminosity ratio.

Using canonical luminosity density of universe, one can show (Peacock, 1999, p. 368, for the B-band):

$$\left.\frac{M}{L}\right|_\mathrm{crit} = 1390\,h\frac{M_\odot}{L_\odot} \tag{7.2}$$

Introduction

Constituents of Ω_m:

- Radiation (CMBR)
- Neutrinos
- Baryons ("normal matter", Ω_b)
- Other, non-radiating, gravitating material ("dark matter")

Radiation: From temperature of CMBR, using $u = a_\mathrm{rad} T^4$:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \tag{7.3}$$

for $h = 0.72$, $\Omega_\gamma = 4.8 \times 10^{-5}$

Massless Neutrinos have

$$\Omega_\nu = 3 \cdot \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} \Omega_\gamma = 0.68\, \Omega_\gamma \tag{7.4}$$

Photons and massless neutrinos are unimportant for todays Ω.

Massive Neutrinos

Sudbury Neutrino Observator (SNO) and Super-Kamiokande: Neutrinos are not massless.

From neutrino decoupling and expansion:

Current neutrino density: 113 neutrinos/cm^3 per neutrino family.

In terms of Ω:

$$\Omega_\nu h^2 = \frac{\sum m_i}{93.5\,\mathrm{eV}} \tag{7.5}$$

$\Longrightarrow$ For $h = 0.75$, $m \sim 17\,\mathrm{eV}$ sufficient to close universe

Current mass limits:

ν_e: $m < 2.2\,\mathrm{eV}$

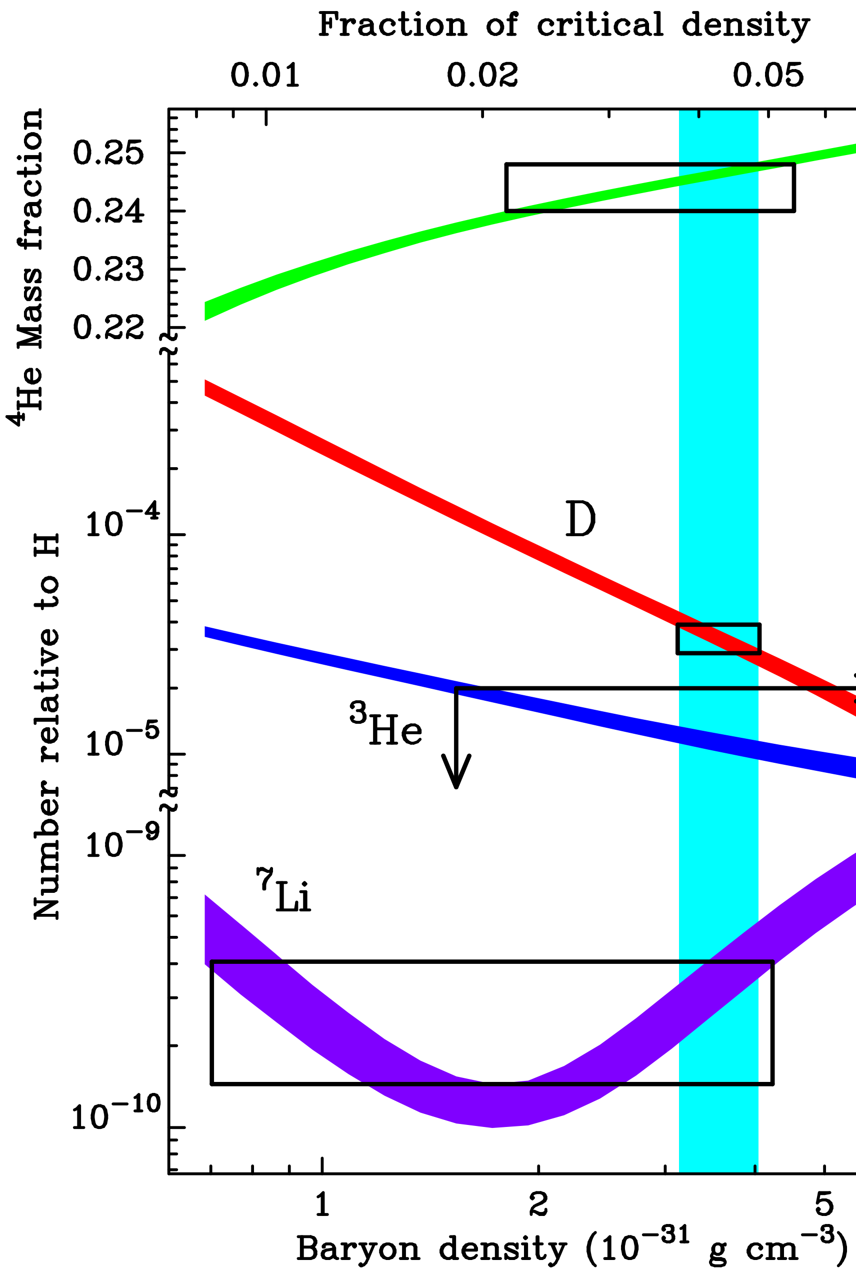
ν_μ: $m < 0.19\,\mathrm{MeV}$

ν_τ: $m < 18.2\,\mathrm{MeV}$

Source: `http://cupp.oulu.fi/neutrino/nd-mass.html` and Particle Physics Booklet 2000

Note that solar neutrino oscillations imply Δm between ν_e and ν_μ is $\sim 10^{-4}\,\mathrm{eV}$, i.e., most probable mass for ν_μ much smaller than direct experimental limit.

Baryons



(Burles, Nollett & Turner, 1999, Fig. 1)

Best evidence for mass in baryons, Ω_b: primordial nucleosynthesis.

$$\Omega_b h^2 = 0.02 \pm 0.002 \tag{7.6}$$

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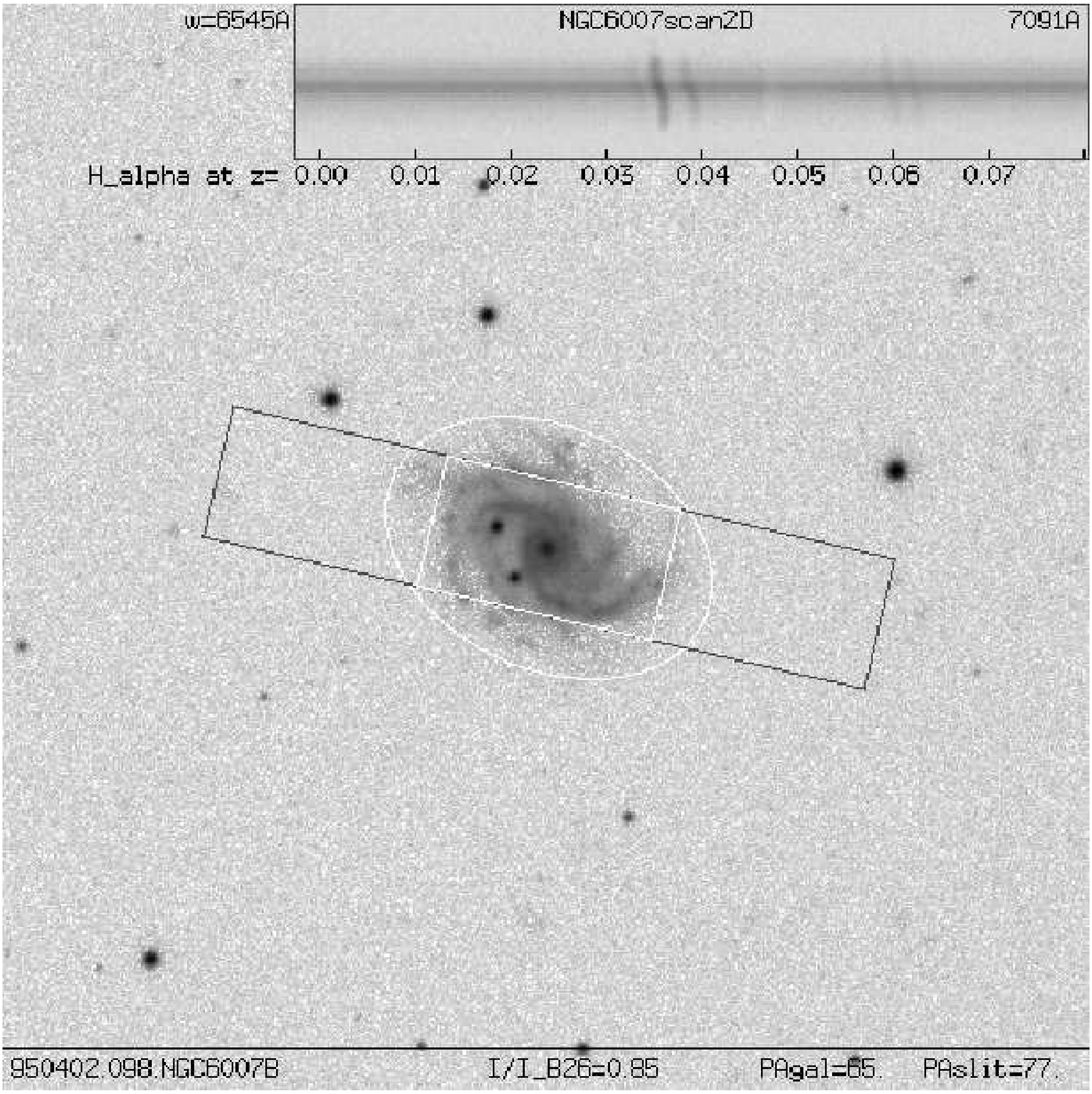
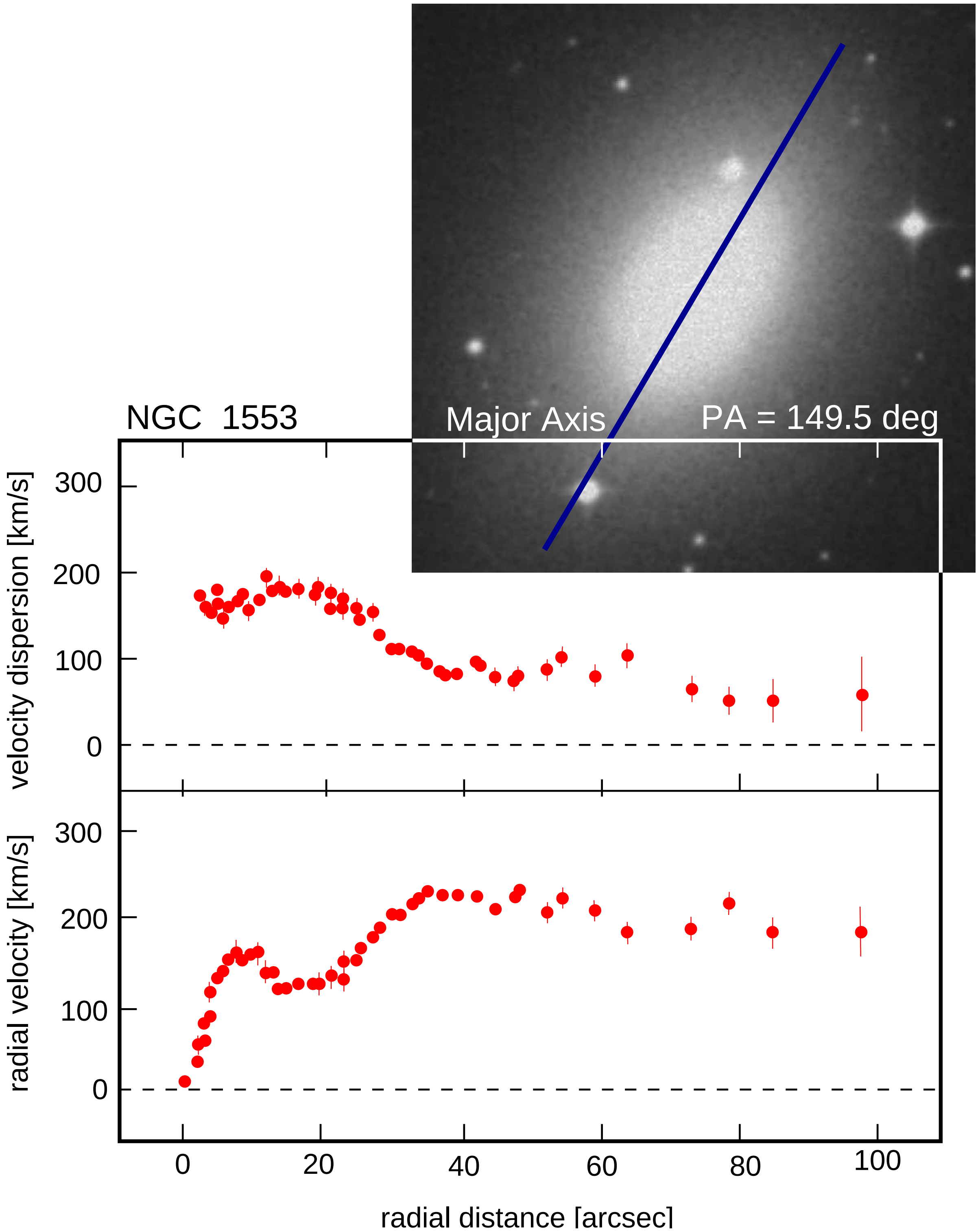
Galaxy Rotation Curves, I



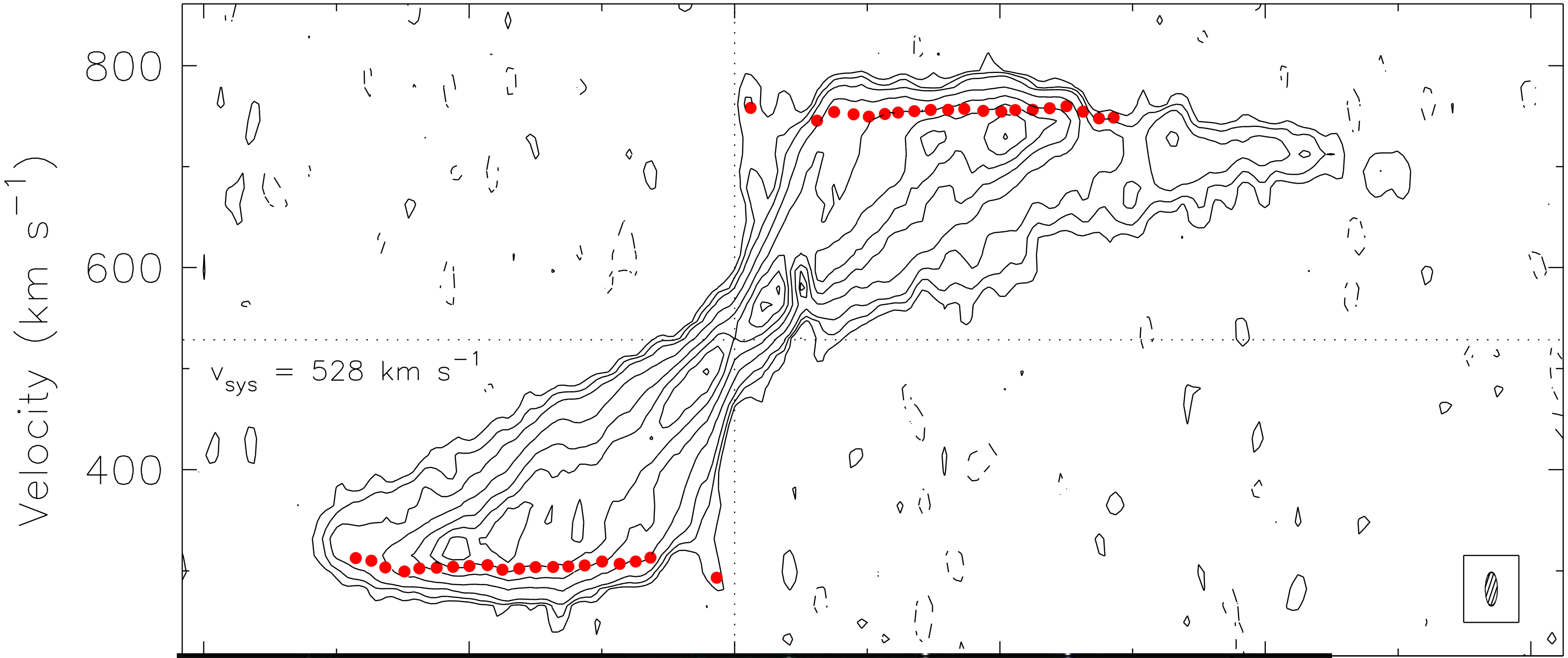
NGC 6007 (Jansen; `http://www.astro.rug.nl/~nfgs/`)

Galaxy Rotation Curves, II



NGC 1553 (S0) after Kormendy (1984, ApJ 286, 116)

Galaxy Rotation Curves, III



NGC 891 (Swaters et al., 1997, ApJ 491, 140 / Paul LeFevre, S&T Nov. 2002)

Galaxy Rotation Curves, IV

NGC 891, KPNO 1.3 m
Barentine & Esquerdo

Stellar motion due to mass within r:

$$\frac{GM(\le r)}{r^2} = \frac{v_{\text{rot}}^2(r)}{r}$$

$$\Longrightarrow M(\le r) = \frac{v_{\text{rot}}^2 r}{G}$$

therefore:

$v \sim$ const. $\Longrightarrow M(\le r) \propto r$.

For disk in spiral galaxies, $I(r) = I_0 \exp(-r/h)$ such that

$$L(r < r_0) = I_0 \int_0^{r_0} 2\pi r \exp(-r/h)\,\mathrm{d}r$$
$$\propto h^2 - h(r+h)\exp(-r/h) \quad (7.7)$$

such that for $r \longrightarrow \infty$: $L(r < r_0) \to$ const..

If $M/L \sim$ const. $\Longrightarrow$ contradiction with observations!
(would expect $v \propto r^{-1/2}$)

Result for galaxies compared to stars

$$\left.\frac{M}{L}\right|_{\text{galaxies}} = 10\ldots 20\,\frac{M_\odot}{L_\odot} \quad \text{vs.} \quad \left.\frac{M}{L}\right|_{\text{stars}} = 1\ldots 3\,\frac{M_\odot}{L_\odot}$$

Only about 10% of the gravitating matter in universe radiates.

Galaxy Clusters, I

For mass of galaxy clusters, make use of the virial theorem:

$$E_{\mathrm{kin}} = -E_{\mathrm{pot}}/2 \tag{7.8}$$

in statistical equilibrium.

Measurement: assume isotropy, such that

$$\left\langle v^2 \right\rangle = \left\langle v_x^2 \right\rangle + \left\langle v_y^2 \right\rangle + \left\langle v_z^2 \right\rangle = 3 \left\langle v_\parallel^2 \right\rangle \tag{7.9}$$

assuming that velocity dispersion independent of m_i gives:

$$E_{\mathrm{kin}} = \frac{1}{2} \sum_i m_i \mathbf{v}_i^2 = \frac{3}{2} M \left\langle v_\parallel^2 \right\rangle \tag{7.10}$$

where M total mass.

If cluster is spherically symmetric $\Longrightarrow$ Define weighted mean separation R_{cl}, such that

$$E_{\mathrm{pot}} = \frac{GM^2}{R_{\mathrm{cl}}} \tag{7.11}$$

From Eqs. (7.10) and (7.11):

$$M = \frac{3}{G} \left\langle v_\parallel^2 \right\rangle R_{\mathrm{cl}} \tag{7.12}$$

Typical values: $v_\parallel \sim 1000\,\mathrm{km\,s^{-1}}$, $R \sim 1\,\mathrm{Mpc}$.

Derivation of the Virial Theorem

Assume system of particles, each with mass m_i. Acceleration on particle i:

$$\ddot{\mathbf{r}} = \sum_{j\neq i} \frac{Gm_j(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{7.13}$$

...scalar product with $m_i\mathbf{r}_i$

$$m_i\mathbf{r}_i \cdot \ddot{\mathbf{r}}_i = \sum_{j\neq i} \frac{Gm_im_j\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{7.14}$$

...since

$$\frac{1}{2}\frac{\mathrm{d}^2\mathbf{r}_i^2}{\mathrm{d}t^2} = \frac{\mathrm{d}}{\mathrm{d}t}(\dot{\mathbf{r}}_i \cdot \mathbf{r}_i) = \ddot{\mathbf{r}}_i \cdot \mathbf{r}_i + \dot{\mathbf{r}}_i \cdot \mathbf{r}_i \tag{7.15}$$

...therefore Eq. (7.14)

$$\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}(m_i\mathbf{r}_i^2) - m_i\dot{\mathbf{r}}_i{}^2 = \sum_{j\neq i} \frac{Gm_im_j\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{7.16}$$

Summing over all particles in the system gives

$$\frac{1}{2}\sum_i \frac{\mathrm{d}^2}{\mathrm{d}t^2}(m_i\mathbf{r}_i^2) - \sum_i m_i\dot{\mathbf{r}}_i{}^2 = \sum_i\sum_{j\neq i} \frac{Gm_im_j\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \tag{7.17}$$

$$= \frac{1}{2}\left(\sum_i\sum_{j\neq i} Gm_im_j \frac{\mathbf{r}_i \cdot (\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j\sum_{i\neq j} Gm_jm_i \frac{\mathbf{r}_j \cdot (\mathbf{r}_i - \mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{r}_i|^3}\right) \tag{7.18}$$

$$= \frac{1}{2}\left(\sum_i\sum_{j\neq i} Gm_im_j \frac{\mathbf{r}_i \cdot \mathbf{r}_j - \mathbf{r}_i^2}{|\mathbf{r}_i - \mathbf{r}_j|^3} + \sum_j\sum_{i\neq j} Gm_jm_i \frac{\mathbf{r}_j \cdot \mathbf{r}_i - \mathbf{r}_j^2}{|\mathbf{r}_j - \mathbf{r}_i|^3}\right) \tag{7.19}$$

$$= -\frac{1}{2}\sum_{\substack{i,j\\ i\neq j}} \frac{Gm_im_j}{|\mathbf{r}_i - \mathbf{r}_j|} \tag{7.20}$$

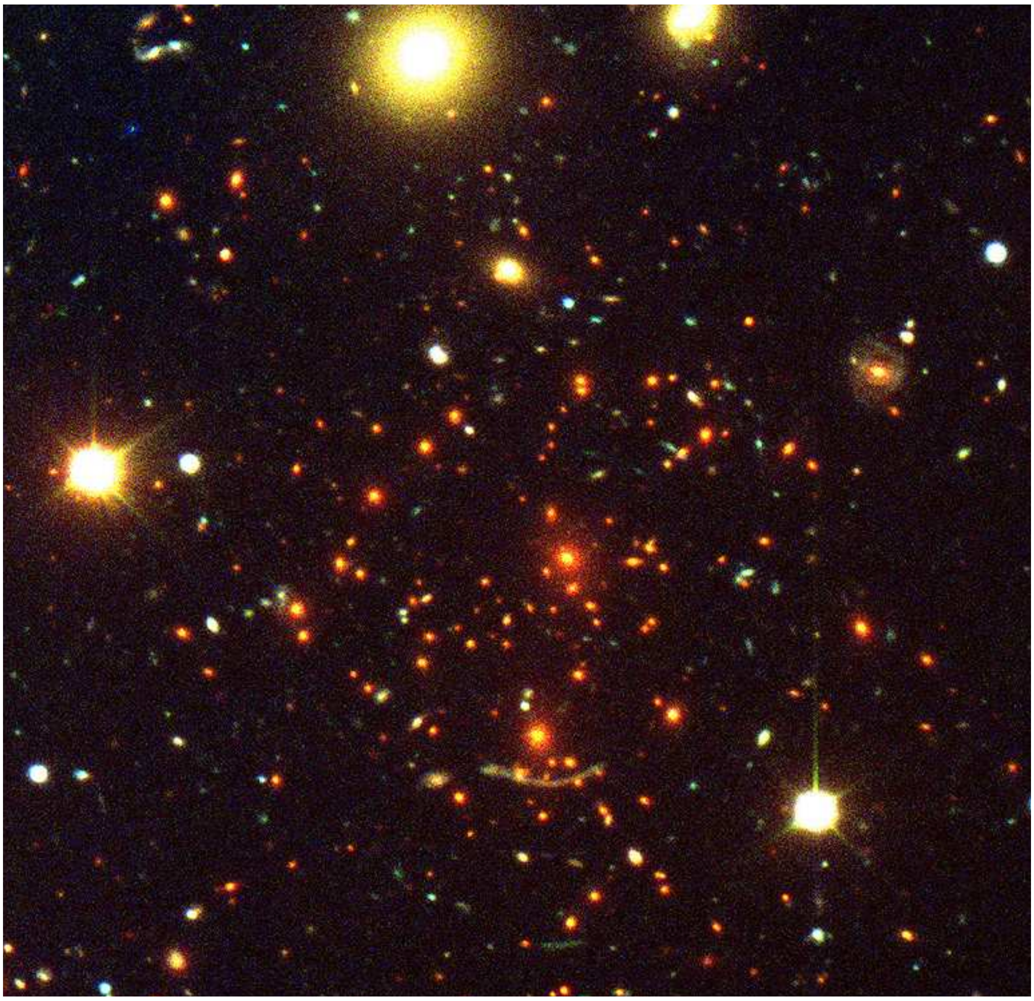
Thus, identifying the total kinetic energy, T, and the gravitational potential energy, U, gives

$$2T - U = \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\sum_i m_i\mathbf{r}_i^2 = 0 \tag{7.21}$$

in statistical equilibrium.

Thus we find the virial theorem: $T = \frac{1}{2}|U|$

Galaxy Clusters, II



Abell 370 (VLT UT1+FORS)

More detailed analysis using more complicated mass models gives (Merritt, 1987):

$$\frac{M}{L} \sim 350h^{-1}\frac{M_\odot}{L_\odot} \quad (7.22)$$

would have expected $M/L = 10\ldots20$ as for galaxies

Dark matter is an important constituent in galaxy clusters

X-ray emission, I

X-ray emission from galaxy clusters gives mass to higher precision:

Assume gas in potential of galaxy cluster. Hydrostatic equilibrium:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2} \tag{7.23}$$

Pressure from equation of state:

$$P = nkT = \frac{\rho kT}{\mu m_{\mathrm{H}}} \tag{7.24}$$

where m_{H}: mass of H-atom, μ mean molecular weight of gas ($\mu = 0.6$ for fully ionized).

Eq. (7.24) gives

$$\frac{\mathrm{d}P}{\mathrm{d}r} = \frac{k}{\mu m_{\mathrm{H}}}\left(T\frac{\mathrm{d}\rho}{\mathrm{d}r} + \rho\frac{\mathrm{d}T}{\mathrm{d}r}\right) = \frac{\rho kT}{\mu m_{\mathrm{H}}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{7.25}$$

Inserting into Eq. (7.23) and solving gives

$$M_r = -\frac{kTr^2}{G\mu m_{\mathrm{H}}}\left(\frac{\mathrm{d}\log\rho}{\mathrm{d}r} + \frac{\mathrm{d}\log T}{\mathrm{d}r}\right) \tag{7.26}$$

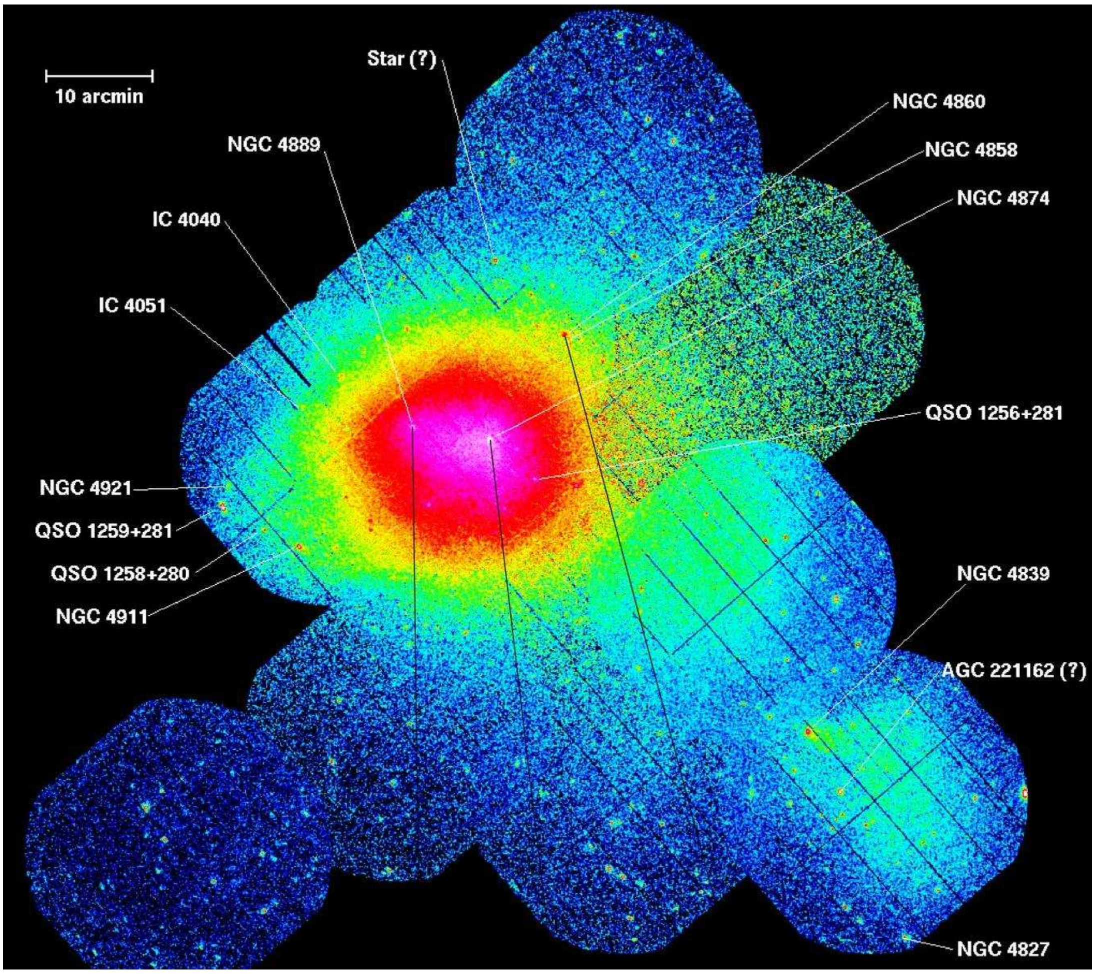
Cluster gas mainly radiates by bremsstrahlung emission, with a spectrum

$$\epsilon(E) \propto \left(\frac{m_{\mathrm{e}}}{kT}\right)^{1/2} g(E,T)\, N\, N_{\mathrm{e}}\, \exp\left(-\frac{E}{kT}\right) \tag{7.27}$$

where N: number density of nuclei, $g(E,T)$: Gaunt factor (roughly constant).

$\Longrightarrow$ T from X-ray spectrum, N from measured flux $\Longrightarrow$ M_r.

X-ray emission, II



XMM-Newton, EPIC-pn

Result for Coma:

$$\frac{M_{\mathrm{B}}}{M_{\mathrm{tot}}} = 0.01 + 0.05\,h^{-3/2} \tag{7.28}$$

Technical problems:

- see through cluster $\Longrightarrow$ integrate over line of sight, assuming spherical geometry.
- spherical geometry is assumed
- Gas cools by radiating was wrong ("cooling flow")

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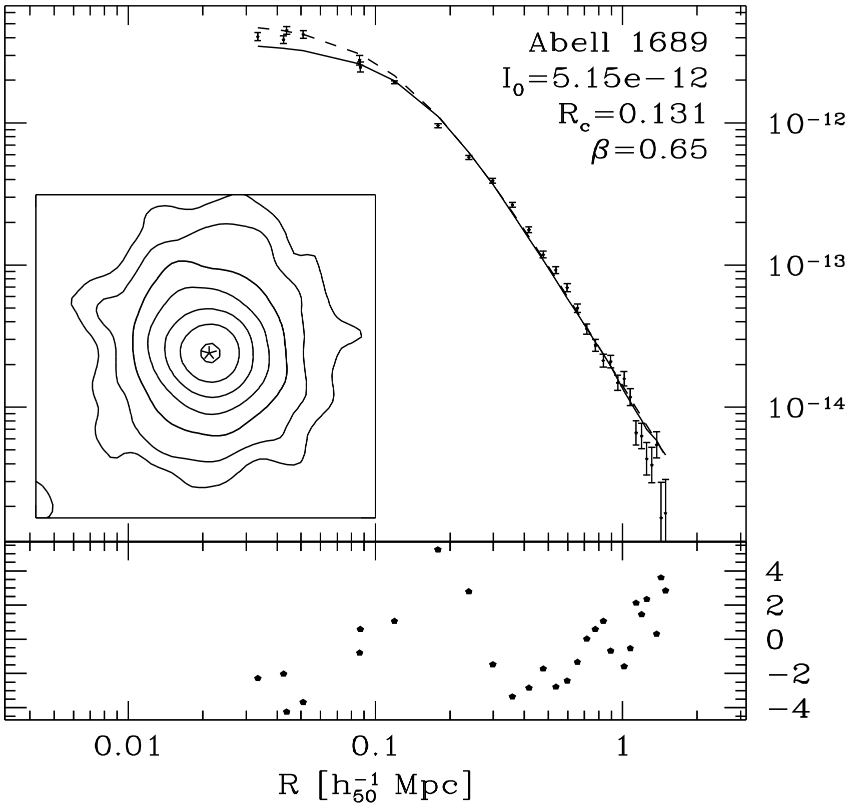
X-ray emission, III



(Mohr, Mathiesen & Evrard, 1999)

Generally: assume intensity profile from β-model,

$$\frac{I(r)}{I_0} = \left(1 + \left(\frac{r}{R_c}\right)^2\right)^{-3\beta+\frac{1}{2}} \tag{7.29}$$

and obtain T from fitting X-ray spectra to "shells" $\Longrightarrow$ technically complicated...

Summary for X-ray mass determination for 45 clusters (Mohr, Mathiesen & Evrard, 1999):

$$f_{\text{gas}} = (0.07 \pm 0.002)h^{-3/2} \tag{7.30}$$

resulting in

$$\Omega_m = \Omega_b / f_{\text{gas}} = (0.3 \pm 0.05)\, h^{-1/2} \tag{7.31}$$

Sunyaev-Zeldovich, I

Gas in cooling flow influences CMBR by Compton upscattering $\Longrightarrow$ Sunyaev-Zeldovich effect.

Derivation of following formulae follows from Fokker-Planck equation and Kompaneets equation, see, e.g., Peacock (1999, p. 375ff.).

Compton y-parameter (=optical depth for Compton scattering):

$$y = \int \left(\frac{kT_\mathrm{e}}{m_\mathrm{e}c^2} \right) \sigma_\mathrm{T} N_\mathrm{e} \, \mathrm{d}l \qquad (7.32)$$

Intensity change in Rayleigh-Jeans regime due to Compton upscattering:

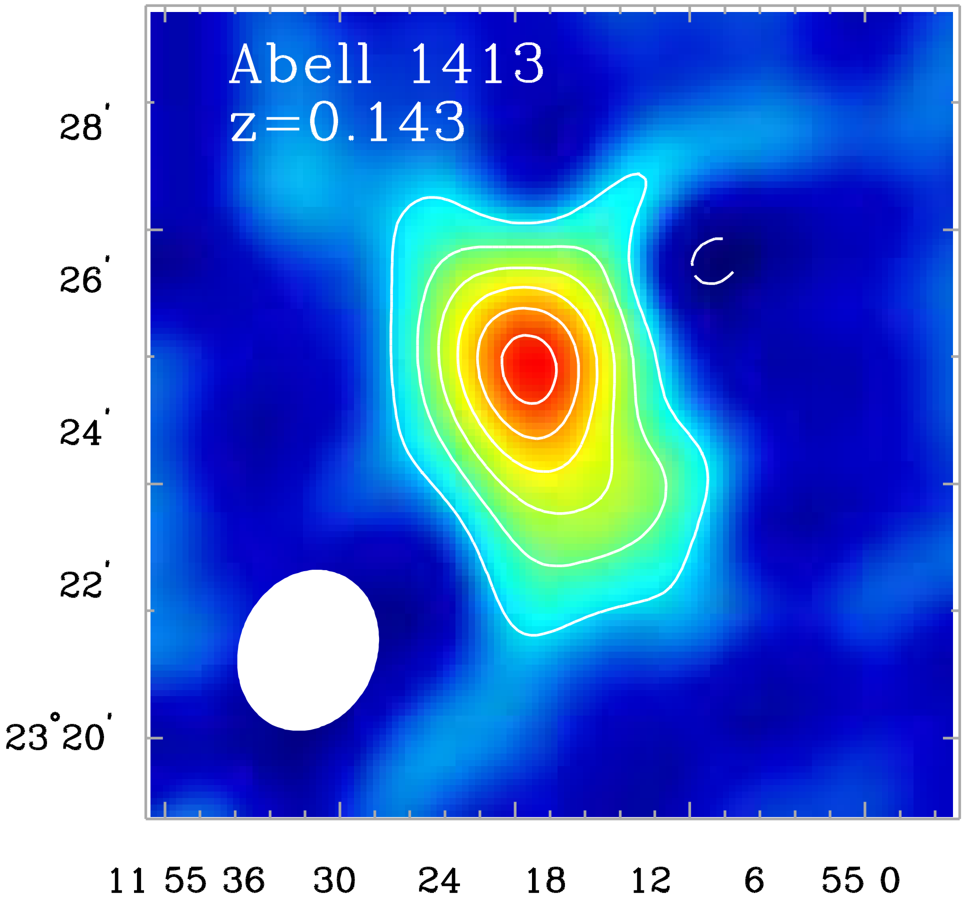
$$\frac{\Delta I}{I} = -2y \sim 10^{-4} \qquad (7.33)$$

(for typical parameters).

$\Longrightarrow$ Measure of $\int N_\mathrm{e} T_\mathrm{e} \, \mathrm{d}l \Longrightarrow$ Mass!

T is known from X-ray spectrum.

Sunyaev-Zeldovich, II



(decrement from 3 K background, Carlstrom et al., 2000, Fig. 3)

SZ analysis gives gas fraction for 27 clusters

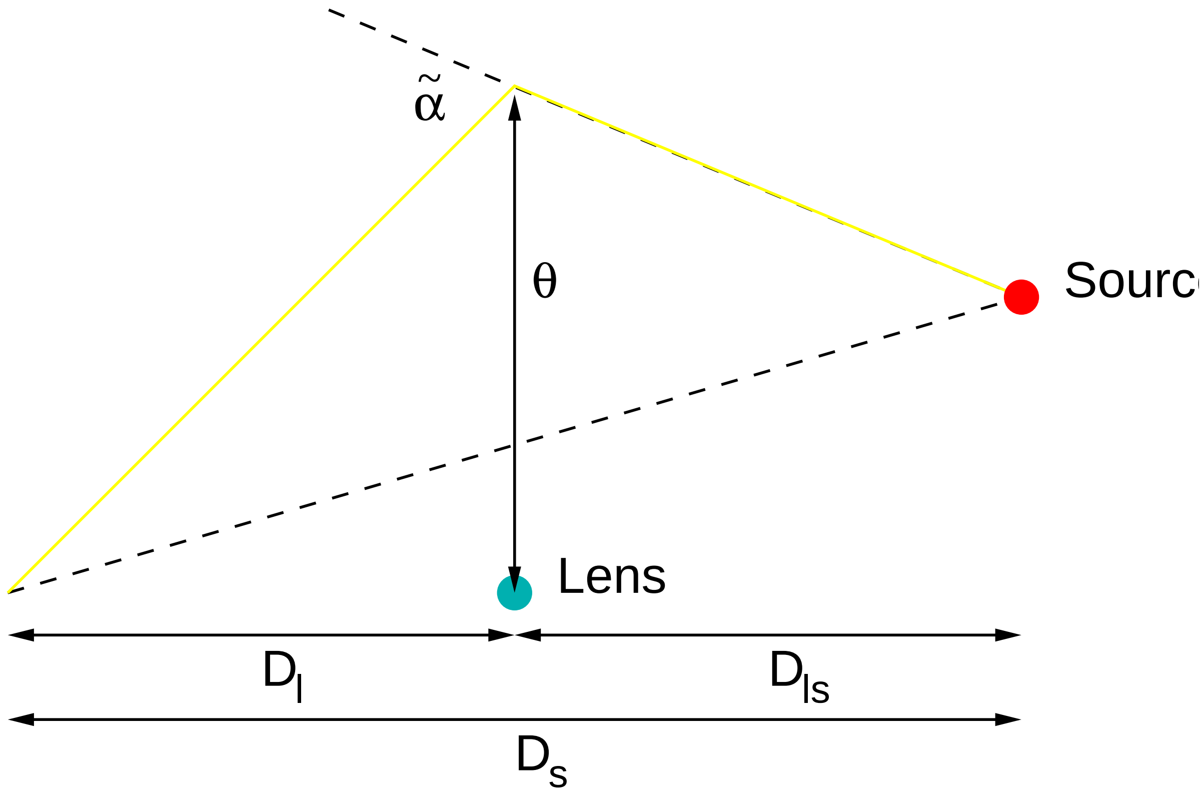
$$f_{\mathrm{gas}} = (0.06 \pm 0.006)h^{-3/2} \tag{7.34}$$

remarkably similar to X-ray result $\Longrightarrow$ clumping of gas does not influence results! (SZ only traces real gas. . .)

f_{gas} translates to

$$\Omega_{\mathrm{m}} = (0.25 \pm 0.04)\, h^{-1} \tag{7.35}$$

Gravitational Lenses, I



(after Longair, 1998, Fig. 4.8a)

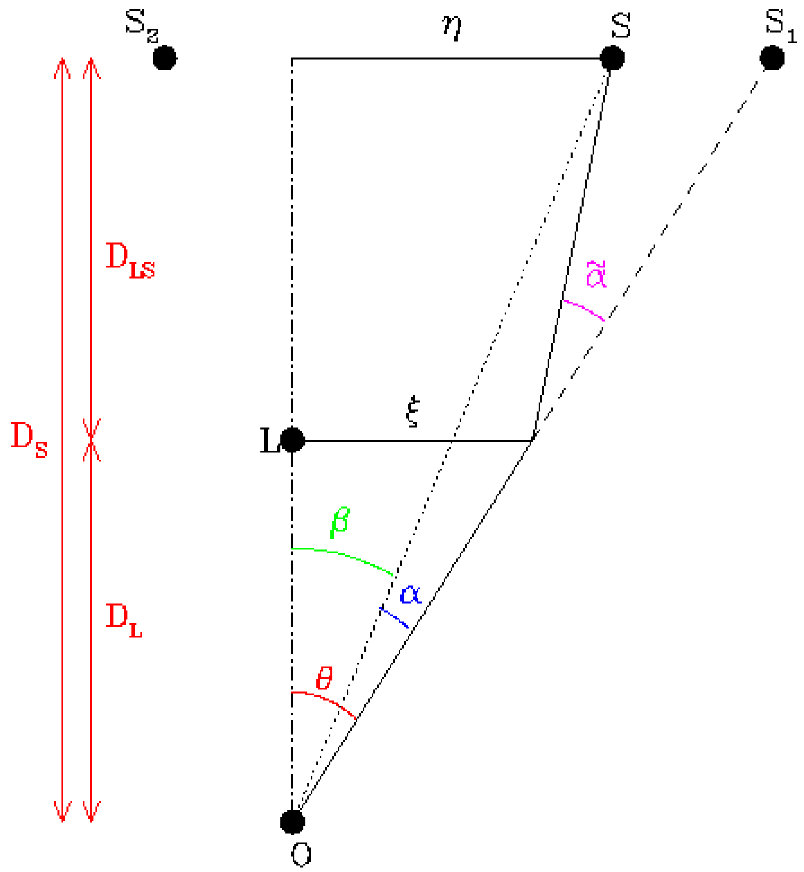
GR: Angular deflection due to mass M:

$$\tilde{\alpha} = \frac{4GM}{\theta c^2} = \frac{2}{c^2} \cdot \frac{2GM}{\theta} \qquad (7.36)$$

where θ distance of closest approach (*twice* classical result).

Measurement of deflection from solar eclipse 1919: most convincing observational evidence for reality of GR.

Gravitational Lenses, II



Wambsganss, 1998, Fig. 3

In the small angle approximation:

$$\theta D_\mathrm{s} = \beta D_\mathrm{s} + \tilde{\alpha} D_\mathrm{ls} \tag{7.37}$$

defining the reduced deflection angle,

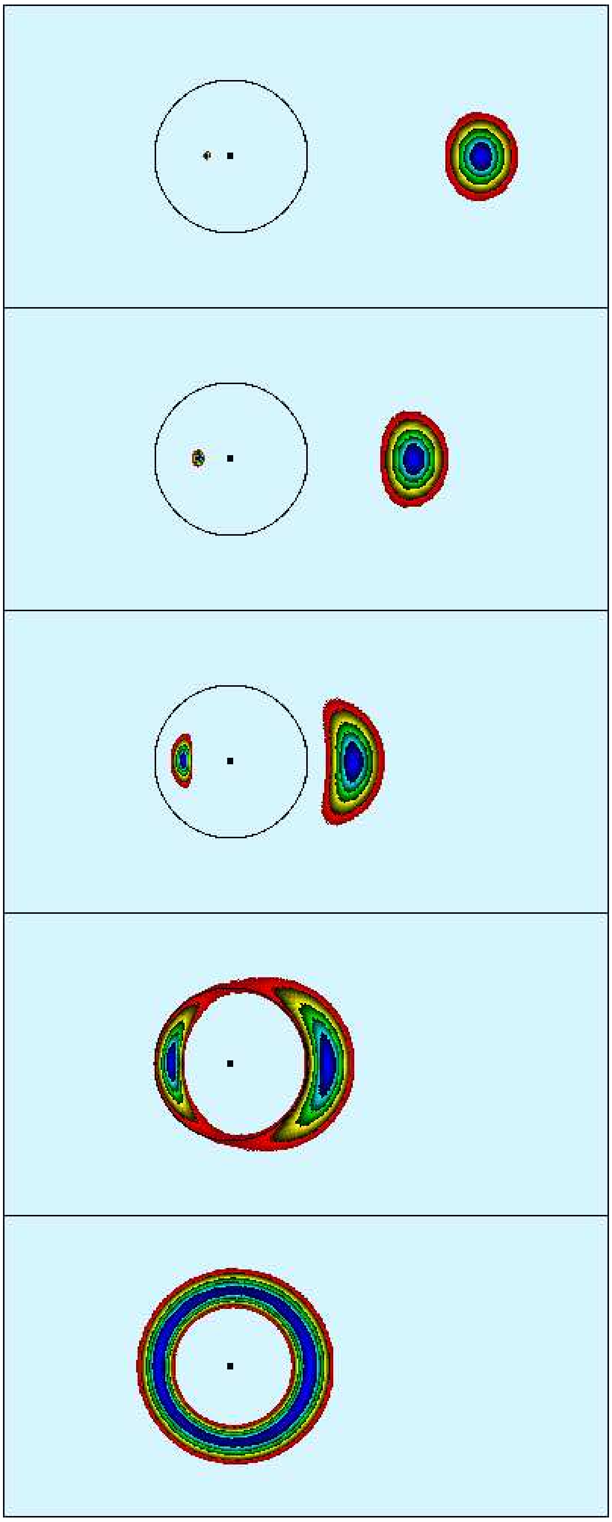
$$\alpha = \frac{D_\mathrm{ls}}{D_\mathrm{s}} \tilde{\alpha} \tag{7.38}$$

gives the lens equation

$$\beta = \theta - \alpha = \theta - \frac{D_\mathrm{ls}}{D_\mathrm{l} D_\mathrm{s}} \cdot \frac{4GM}{c^2 \theta} = \theta - \frac{1}{D} \cdot \frac{4GM}{c^2 \theta} \tag{7.39}$$

(last expression valid for a point-mass)

Gravitational Lenses, III



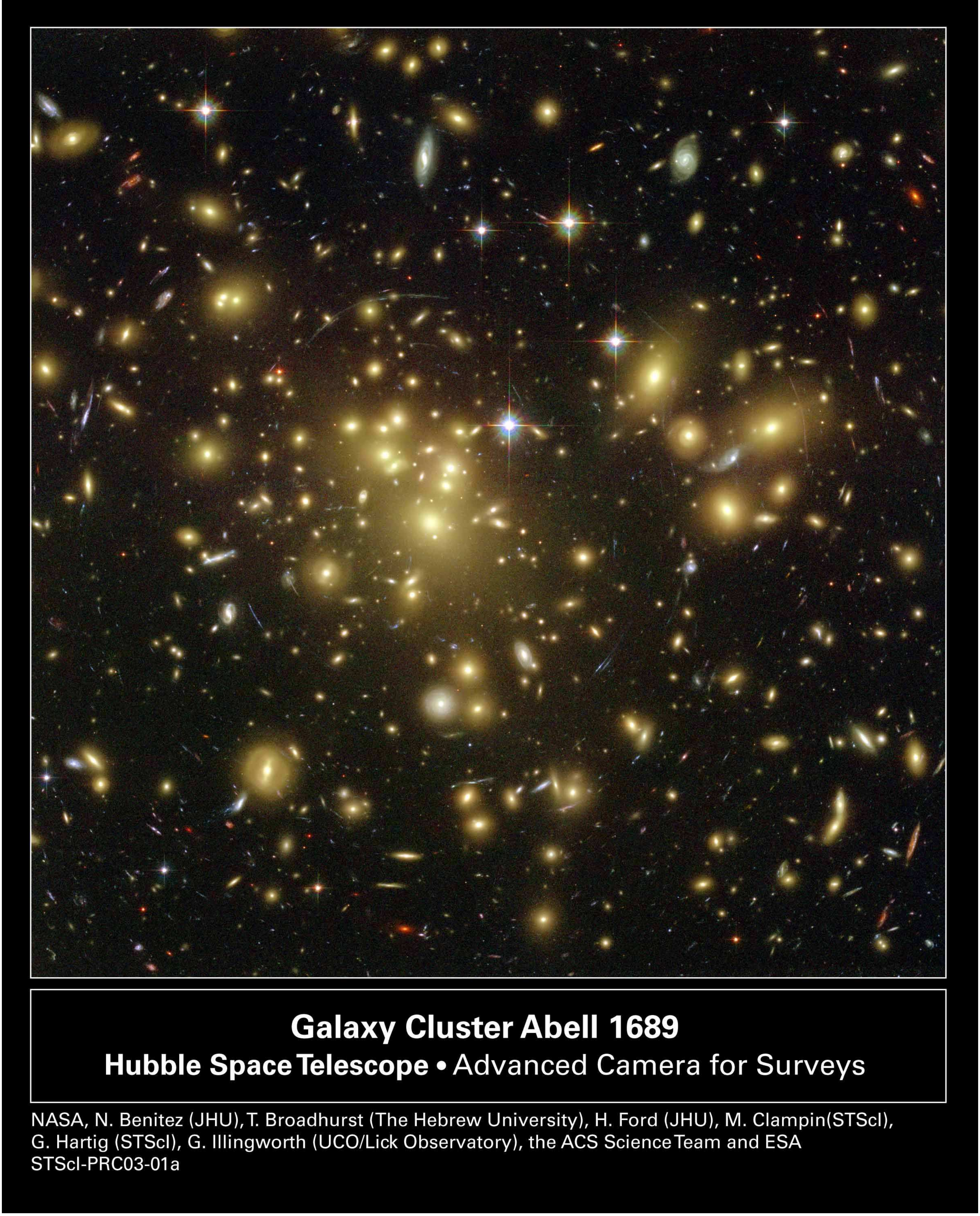
Einstein ring: source directly behind lens,
Obtain radius by setting $\beta = 0$ in lens-equation Eq. (7.39):

$$\theta_{\mathrm{E}}^2 = \frac{4GM}{c^2}\frac{1}{D} \qquad (7.40)$$

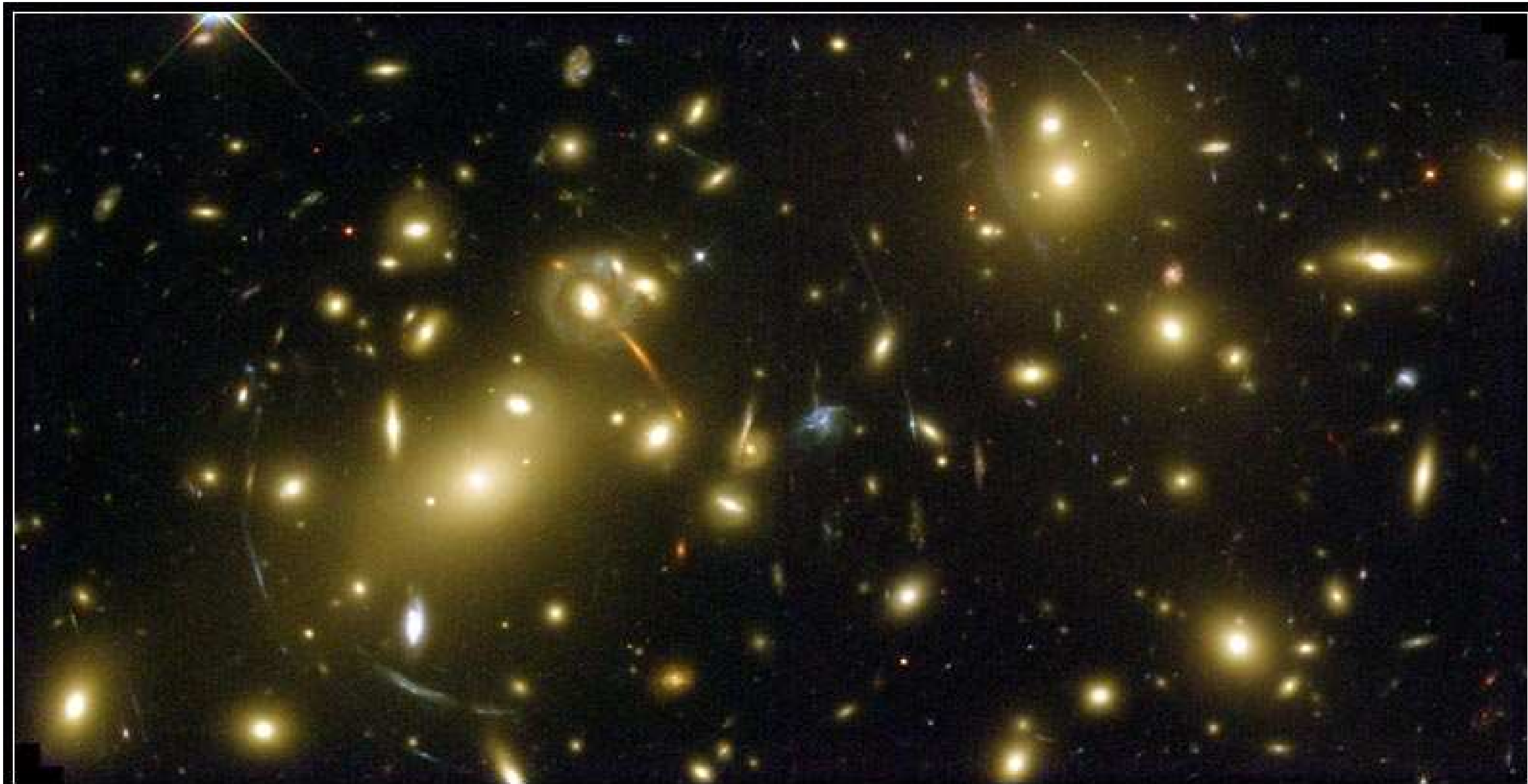
i.e.,

$$\theta_{\mathrm{E}} = 98.9'' \left(\frac{M}{10^{15}\,M_\odot}\right)^{1/2} \frac{1}{(D/1\,\mathrm{Gpc})^{1/2}} \qquad (7.41)$$

Mass measurements possible by observing "giant luminous arcs" and Einstein rings.



General results of mass determinations from lensing agree with other methods.



Galaxy Cluster Abell 2218 **HST • WFPC2**

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

Summary

So far, we have seen:

Photons:

$$\Omega_\gamma h^2 = 2.480 \times 10^{-5} \tag{7.42}$$

Neutrinos:

$$\Omega_\nu h^2 = 1.69 \times 10^{-5} \tag{7.43}$$

Baryons: (from nucleosynthesis)

$$\Omega_\mathrm{b} h^2 = 0.02 \tag{7.44}$$

where stars:

$$\Omega_\mathrm{stars} \sim 0.005 \ldots 0.01 \tag{7.45}$$

Baryons+dark matter: (from clusters)

$$\Omega_\mathrm{m} \sim 0.25 \tag{7.46}$$

(of which $\sim 10\%$ in baryons)

If we believe in $\Omega_\mathrm{total} \equiv 1 \Longrightarrow \Omega_\Lambda \sim 0.7$.

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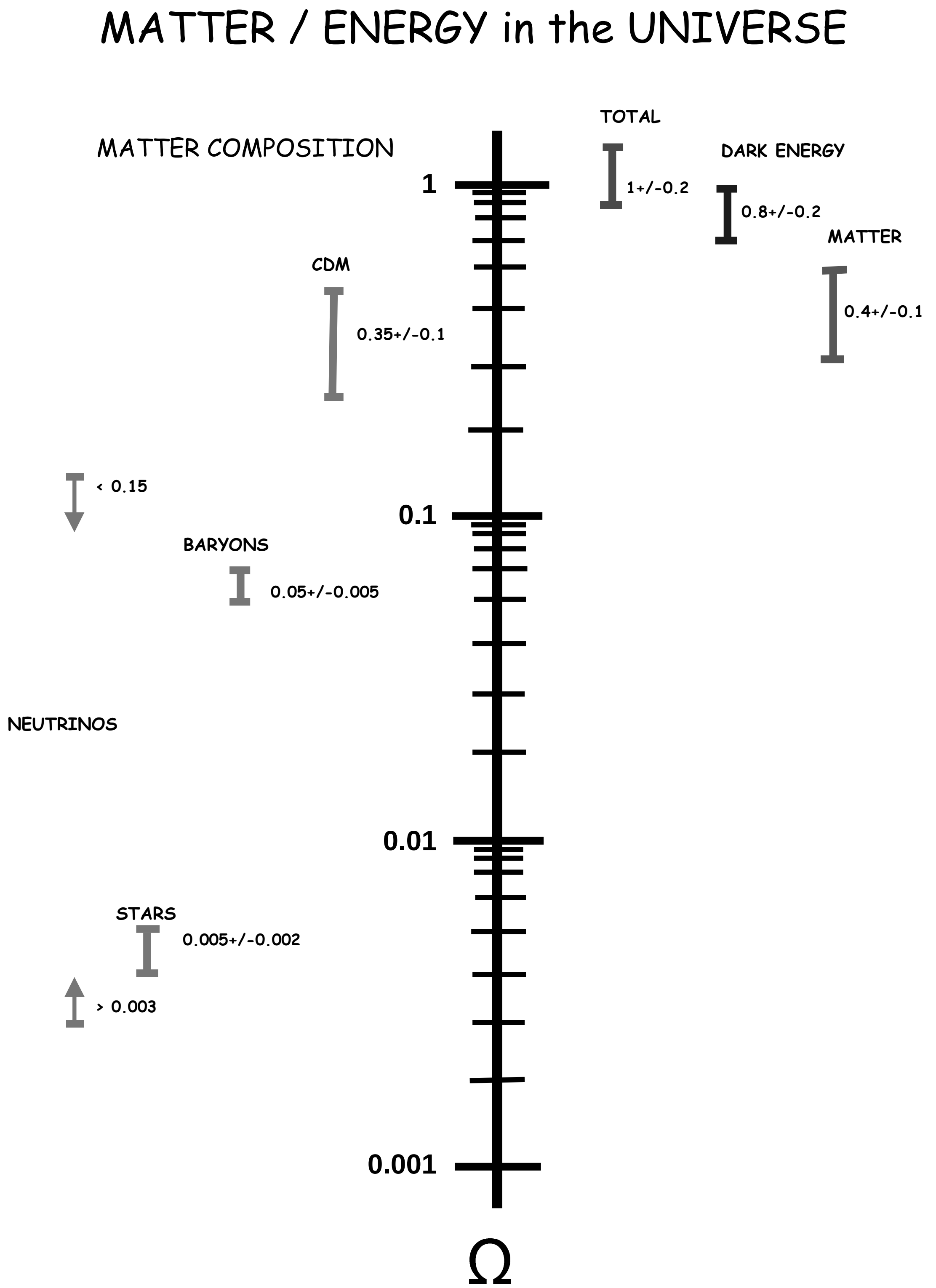
Summary



(Turner, 1999, Fig. 1, numbers slightly different to ours. . .)

Introduction

Clusters and galaxies: $\Omega_{\mathrm{m}} \sim 0.3$, but for baryons $\Omega_{\mathrm{b}} \sim 0.02 \Longrightarrow$ Rest of gravitating material is dark matter.

$\Longrightarrow$ Two dark matter problems:

$$\Omega_{\mathrm{m}} \xleftarrow{\text{nonbaryonic dark matter}} \Omega_{\mathrm{b}} \xleftarrow{\text{baryonic dark matter}} \Omega_{\mathrm{stars}}$$

baryonic dark matter= undetected baryons:

- diffuse hot gas?
- MACHOs (Massive compact halo objects; white dwarfs, neutron stars, black holes, brown dwarfs, jupiters,...)

nonbaryonic dark matter= exotic stuff:

- massive neutrinos
- axions
- neutralinos

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Baryonic Dark Matter, I

Intra Cluster Gas:

Pro:

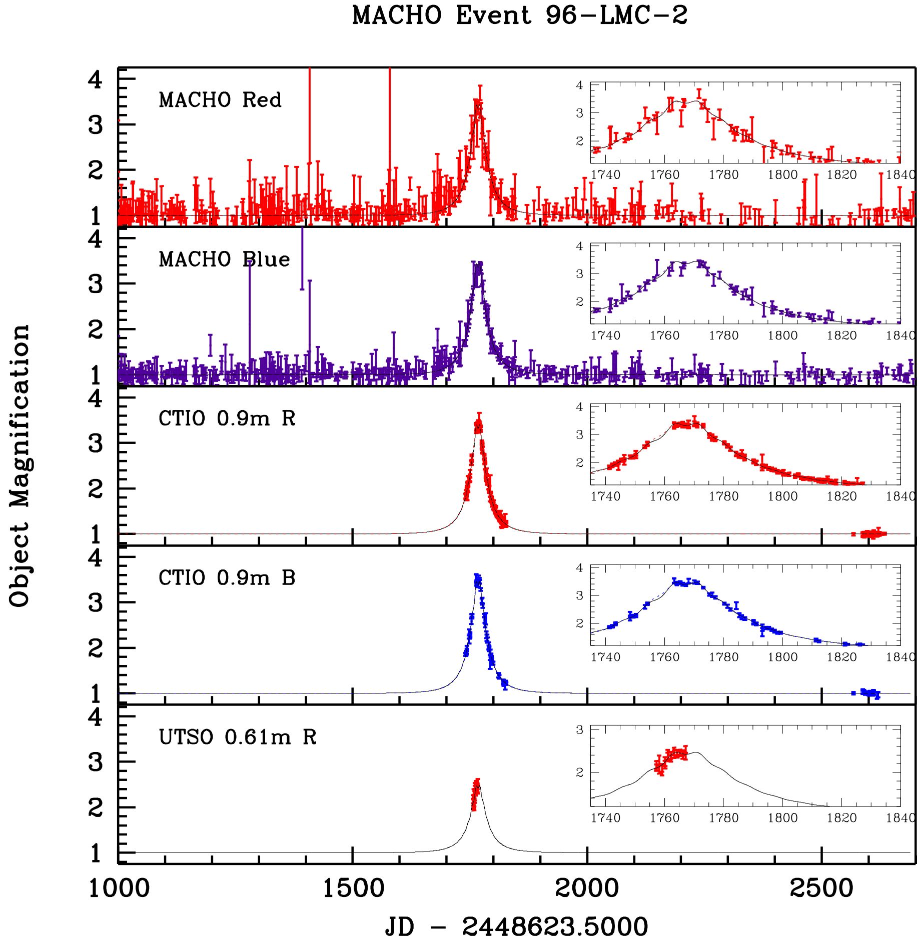
1. same location where the hot gas in clusters also found,
2. structure formation suggests most baryons are *not* in structures today

Contra:

1. 90% of the universe is *not* in clusters...
2. gas has not been detected at any wavelength

If gas cold enough, would not expect it to be detectable, so point 2 is not really valid.

Baryonic Dark Matter, II



(Alcock et al., 2001, Fig. 2)

MACHOS:

Pro:

1. detected by microlensing towards SMC and LMC (see figure) $\Longrightarrow$ MW halo consists of 50% WD

Contra:

1. possible "self-lensing" (by stars in MW or SMC/LMC; confirmed for a few cases)
2. where are white dwarfs?
3. WD formation rate too high (100 $\text{year}^{-1}\,\text{Mpc}^{-3}$)

Nonbaryonic Dark Matter

Nonbaryonic dark matter:

Requirements:

- gravitating
- non-interacting with baryons

$\Longrightarrow$ Grab-box of elementary particle physics:

1. Neutrinos with non-zero mass

   Pro: It exists, mass limits are a few eV, need only $\langle m_\nu \rangle \sim 10\,\mathrm{eV}$

   Contra: ν are relativistic $\Longrightarrow$ Hot dark matter $\Longrightarrow$ Forces top down structure formation, contrary to what is believed to have happened.

2. Axion

   (=Goldstone boson from QCD, invented to prevent strong CP violation in QCD; $m \sim 10^{-5\ldots-2}\,\mathrm{eV}$)

   Pro: It *could* exist, would be in Bose-Einstein condensate due to inflation ($\Longrightarrow$ Cold dark matter!), might be detectable in the next 10 years

   Contra: We do not know it exists...

3. Neutralino or other WIMPs (weakly interacting massive particles; masses $m \sim \mathrm{GeV}$)

   Pro: Also is CDM

   Contra: We do not know they exist...

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Friedmann with $\Lambda \neq 0$, I

$\Longrightarrow$ Need to study cosmology with $\Lambda \neq 0$.

Reviews: Carroll, Press & Turner (1992), Carroll (2000)

Friedmann equation with $\Lambda \neq 0$:

$$H^2(t) = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3} \quad (6.136)$$

And define the Ω's:

$$\Omega_\mathrm{m} = \frac{8\pi G\rho_\mathrm{m}}{3H_0^2} \quad (7.47)$$

$$\Omega_\Lambda = \frac{\Lambda c^4}{3H_0^2} \quad (6.120)$$

$$\Omega_k = -\frac{k}{R_0^2 H_0^2} \quad (7.48)$$

Because of Eq. (6.136),

$$\Omega_\mathrm{m} + \Omega_\Lambda + \Omega_k = \Omega + \Omega_k = 1 \quad (7.49)$$

Friedmann with $\Lambda \neq 0$, II

It is easier to work with the dimensionless scale factor,

$$a = \frac{R(t)}{R_0} \tag{4.30}$$

$\Longrightarrow$ Friedmann:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_{\mathrm{m},0}}{a^3} - \frac{k}{a^2 R_0^2} + \frac{\Lambda}{3} \tag{7.50}$$

since $\rho_\mathrm{m} = \rho_{\mathrm{m},0} a^{-3}$ (Eq. 4.67).

Inserting the Ω's

$$\left(\frac{\dot{a}/H_0}{a}\right)^2 = \frac{\Omega_\mathrm{m}}{a^3} + \frac{1 - \Omega_\mathrm{m} - \Omega_\Lambda}{a^2} + \Omega_\Lambda \tag{7.51}$$

Substituting the time in units of todays Hubble time,

$$\tau = H_0 \cdot t \tag{7.52}$$

results in

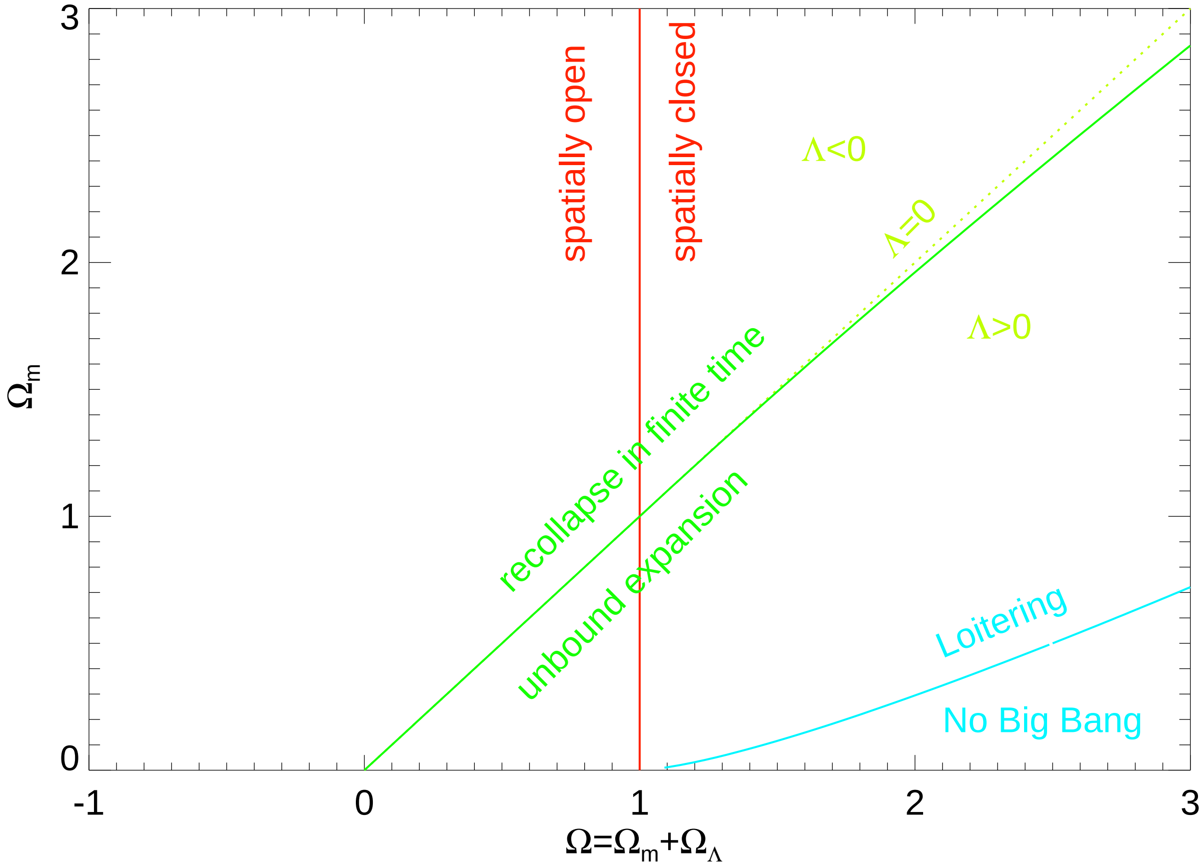
$$\left(\frac{\mathrm{d}a}{\mathrm{d}\tau}\right)^2 = 1 + \Omega_\mathrm{m}\left(\frac{1}{a} - 1\right) + \Omega_\Lambda(a^2 - 1) \tag{7.53}$$

with the boundary conditions

$$a(\tau = 0) = 1 \quad \text{and} \quad \left.\frac{\mathrm{d}a}{\mathrm{d}\tau}\right|_{\tau=0} = 1 \tag{7.54}$$

For most combinations of Ω_m and Ω_Λ, need to solve numerically.

Friedmann with $\Lambda \neq 0$, III

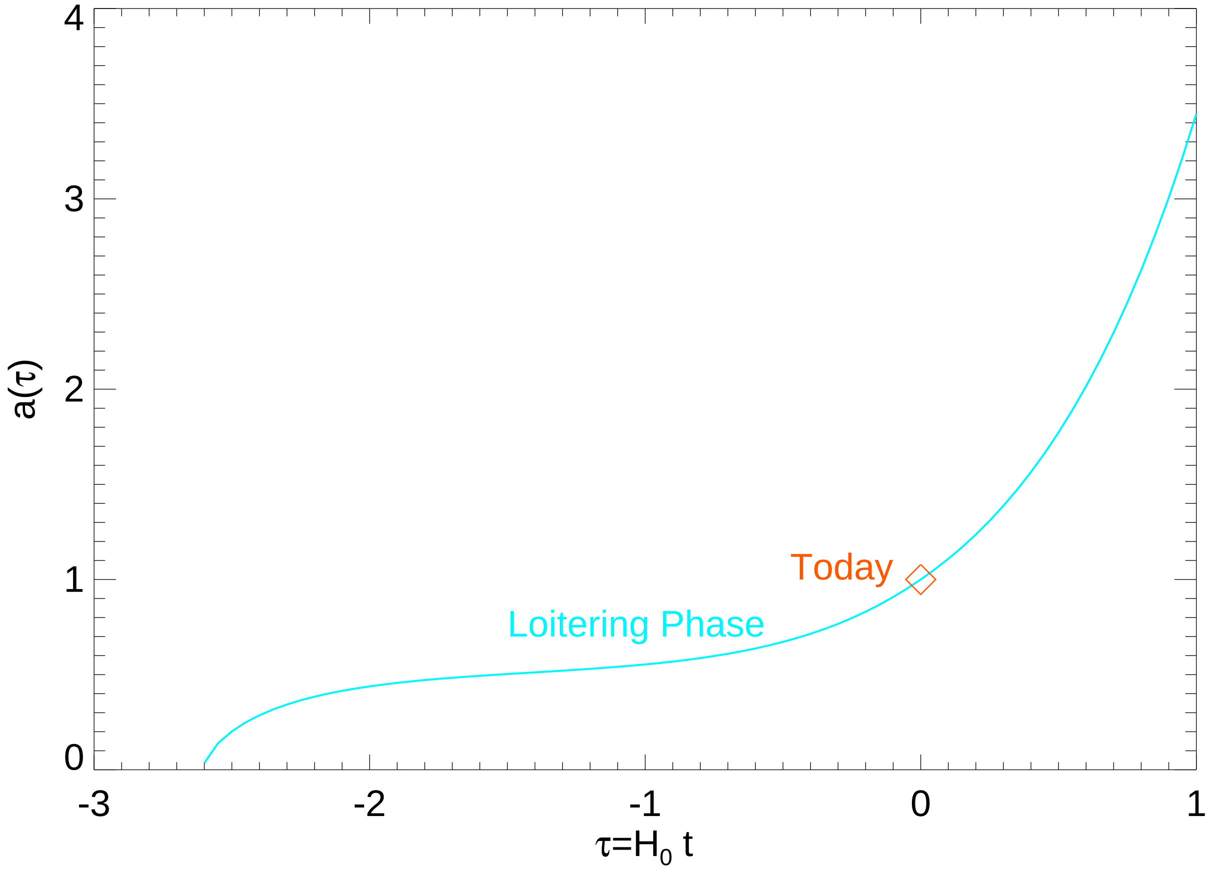


(after Carroll, Press & Turner, 1992, Fig. 1)

With Λ, evolution of universe is more complicated than without:

- unbound expansion possible for $\Omega < 1$,
- For Ω_Λ large: no big bang!
- For Ω_Λ large: possible "loitering phase"

$\Omega_\Lambda > 1$, I



"Loitering universe" with $\Omega_\mathrm{m} = 0.55$, $\Omega_\Lambda = 2.055$

For large Ω_Λ: contraction from $+\infty$ and reexpansion

$\Longrightarrow$ no big bang.

For slightly smaller Ω_Λ: phase where $\dot{a} \sim 0$ in the past

$\Longrightarrow$ loitering universe.

Threshold for presence of turning-point (Carroll, Press & Turner, 1992, Eq. 12):

$$\Omega_\Lambda \geq \Omega_{\Lambda,\mathrm{thresh}} = 4\Omega_\mathrm{m}\left\{C_\kappa\left[\frac{1}{3}C_\kappa^{-1}\left(\frac{1-\Omega_\mathrm{m}}{\Omega_\mathrm{m}}\right)\right]\right\}^3 \quad (7.55)$$

where $\kappa = \mathrm{sgn}(0.5 - \Omega_\mathrm{m})$ and $C_\kappa(\theta)$ was defined in Eq. (4.25).

$\Omega_\Lambda > 1$, II

QSO at $z = 5.82$, courtesy SDSS

For $\Omega_\Lambda = \Omega_{\Lambda,\text{thresh}}$: turning-point, i.e., there is a minimal a.

Since

$$1 + z = \frac{1}{a} \tag{4.43}$$

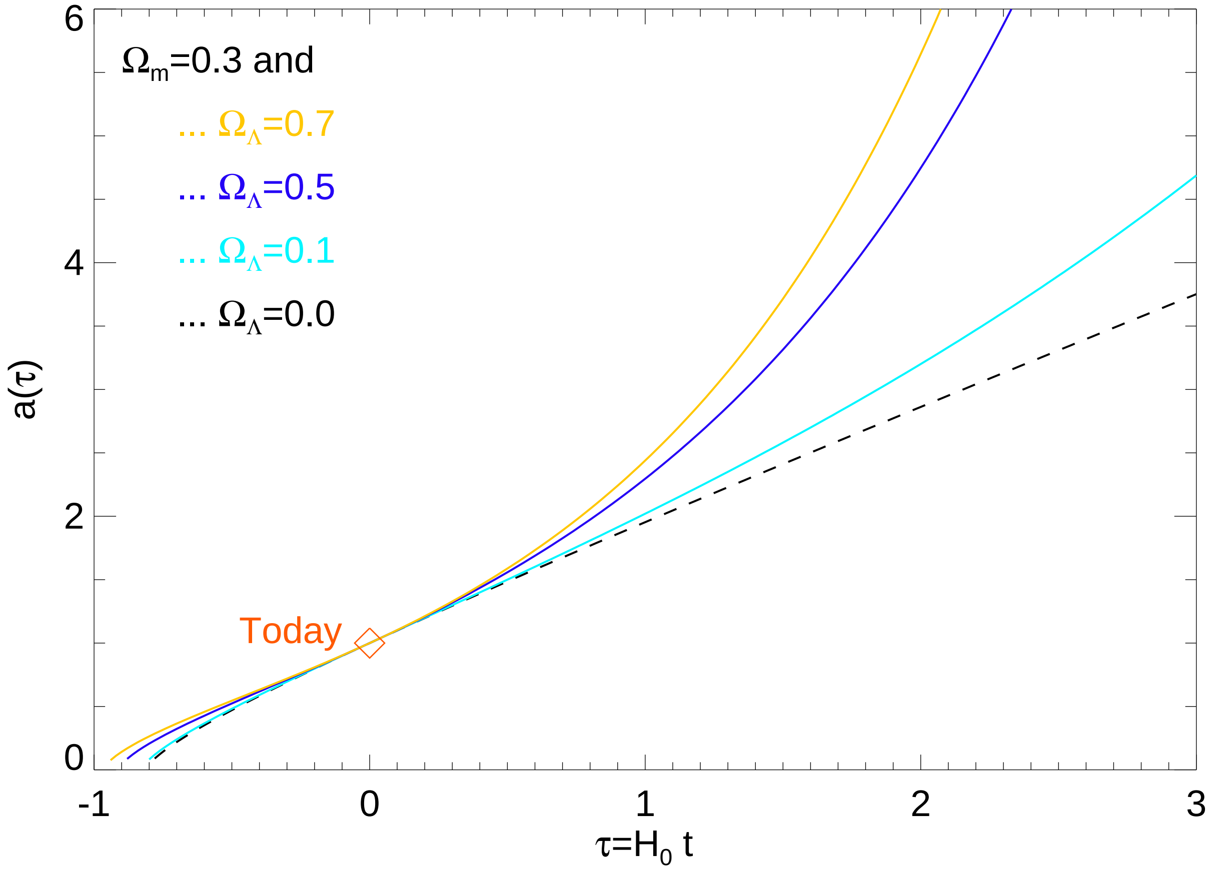
existence of turning-point $\Longrightarrow$ maximal possible z:

$$z \leq 2C_\kappa \left(\frac{1}{3} C_\kappa^{-1} \left\{ \frac{1 - \Omega_\text{m}}{\Omega_\text{m}} \right\} \right) - 1 \tag{7.56}$$

(Carroll, Press & Turner, 1992, Eq. 14).

Since quasars observed with $z = 5.82$, this means that $\Omega_\text{m} < 0.007$, clearly not what is observed $\Longrightarrow$ $\Omega_\Lambda < 1$.

$\Omega_\Lambda < 1$



For $\Omega_\Lambda < 1$ evolution has two parts:

- matter domination, similar to earlier results
- Λ domination, exponential rise.

Exponential rise called by some workers the "second inflationary phase"...

Note accelerating effect of Ω_Λ!

$\Omega_\Lambda < 1$

Computation of age similar to $\Omega_\Lambda = 0$ case (see, e.g., Eq. 4.86), but generally only possible numerically.

Result:

Universes with $\Omega_\Lambda > 0$ are *older* than those with $\Omega_\Lambda = 0$.

This solves the age problem, that some globular clusters have age comparable to age of universe if $\Omega_\Lambda = 0$.

Analytical formula for age (Carroll, Press & Turner, 1992, Eq. 17):

$$t = \frac{2}{3H_0}\frac{\sinh^{-1}\left(\sqrt{(1-\Omega_a)/\Omega_a}\right)}{\sqrt{1-\Omega_a}} \qquad (7.57)$$

for $\Omega_a < 1$, where

$$\Omega_a = 0.7\Omega_\mathrm{m} + 0.3(1-\Omega_\Lambda) \qquad (7.58)$$

For $\Omega_\mathrm{m} = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$: $t = 13.5\,\mathrm{Gyr}$.

Remember that for $\Omega_\mathrm{m} = 1$, $t = 3/2H_0$!

Luminosity Distance

Influence of Λ most prominent at large distances!

$\Longrightarrow$ Expect influence on Hubble Diagram.

$\Longrightarrow$ Need to find relation between measured flux, emitted luminosity, and redshift.

Assume source with luminosity L at comoving coordinate r, emitting isotropically into 4π sr.

At time of detection today, photons are

- on sphere with proper radius $R_0 r$,
- redshifted by factor $1+z$,
- spread in time by factor $1+z$.

$\Longrightarrow$ observed flux is

$$F = \frac{L}{4\pi R_0^2 r^2 (1+z)^2} \tag{7.59}$$

The luminosity distance is defined as

$$d_\mathrm{L} = R_0 \cdot r \cdot (1+z) \tag{7.60}$$

The computation of d_L is somewhat technical, one can show that (Carroll, Press & Turner, 1992):

$$d_\mathrm{L} = \frac{c}{H_0} |\Omega_k|^{-1/2} \cdot S_{-\mathrm{sgn}(\Omega_k)} \Big\{ |\Omega_k|^{1/2} \int_0^z \left[(1+z)^2 (1+\Omega_\mathrm{m} z) - z(2+z)\Omega_\Lambda \right]^{1/2} \mathrm{d}z \Big\} \tag{7.61}$$

Supernovae

Best way to determine Ω_Λ:

Type Ia supernovae

Remember: SN Ia = CO WD collapse. . . (Hoyle, Fowler, Colgate, Wheeler,. . .

The distance modulus is

$$m - M = 5 \log \left(\frac{d_\mathrm{L}}{1\,\mathrm{Mpc}} \right) + 25 \qquad (7.62)$$

Use SNe as standard candles $\Longrightarrow$ Deviations from $d_\mathrm{L} \propto z$ indicative of Λ.

Two projects:

- High-z Supernova Team (STSCI, Riess et al.)
- Supernova Cosmology Project (LBNL, Perlmutter et al.)

Both find SNe out to $z \sim 1$.

Present mainly Perlmutter et al. results here, Riess et al. (1998) are similar.

Supernovae

Basic observations: easy:

- Detect SN in rise $\Longrightarrow$ CTIO 4 m
- Follow SN for $\sim$ 2–3 months with 2–4 m class telescopes, HST, Keck...

More technical problems in data analysis:

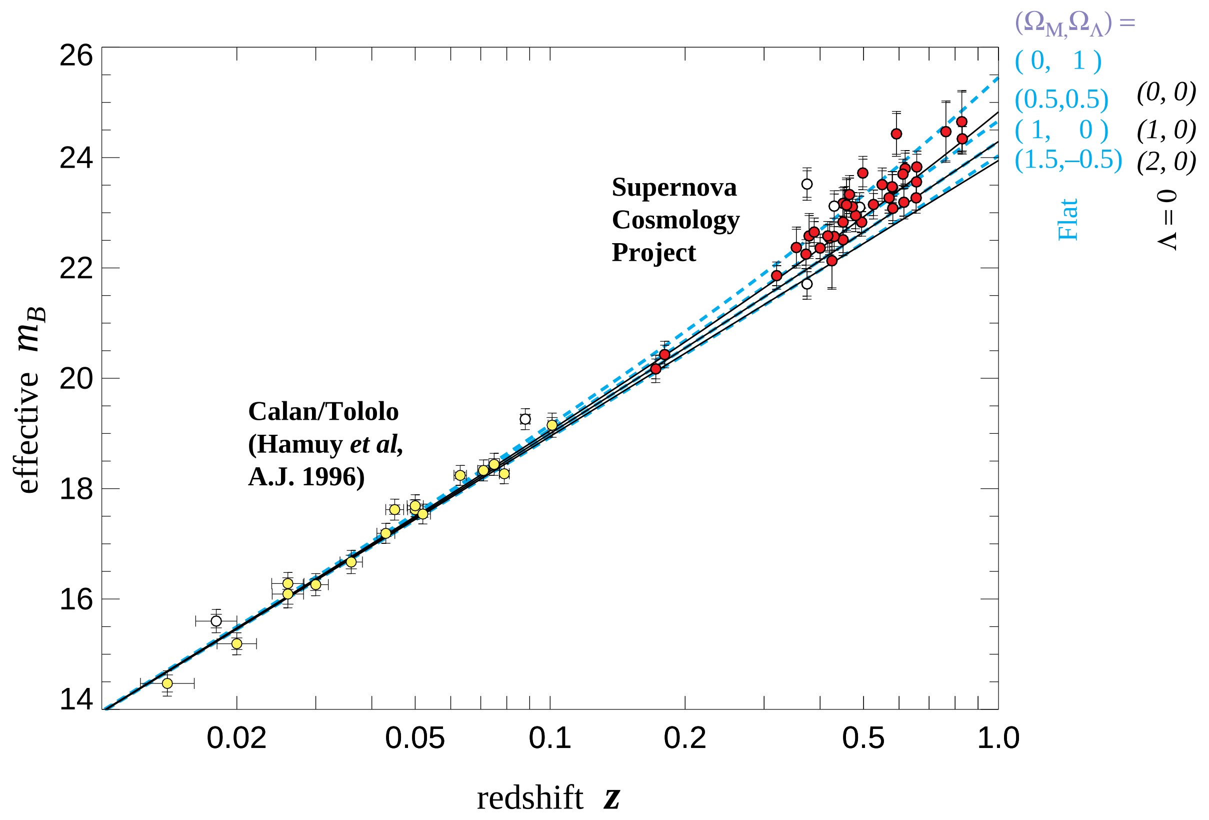
Conversion into source frame:

- Correction of photometric flux for redshift: "K-correction"
- Correct for time dilatation in SN light curve

Further things to check

- SN internal extinction
- Galactic extinction
- Galactic reddening
- Photometric cross calibration
- Peculiar motion of SN

Supernovae



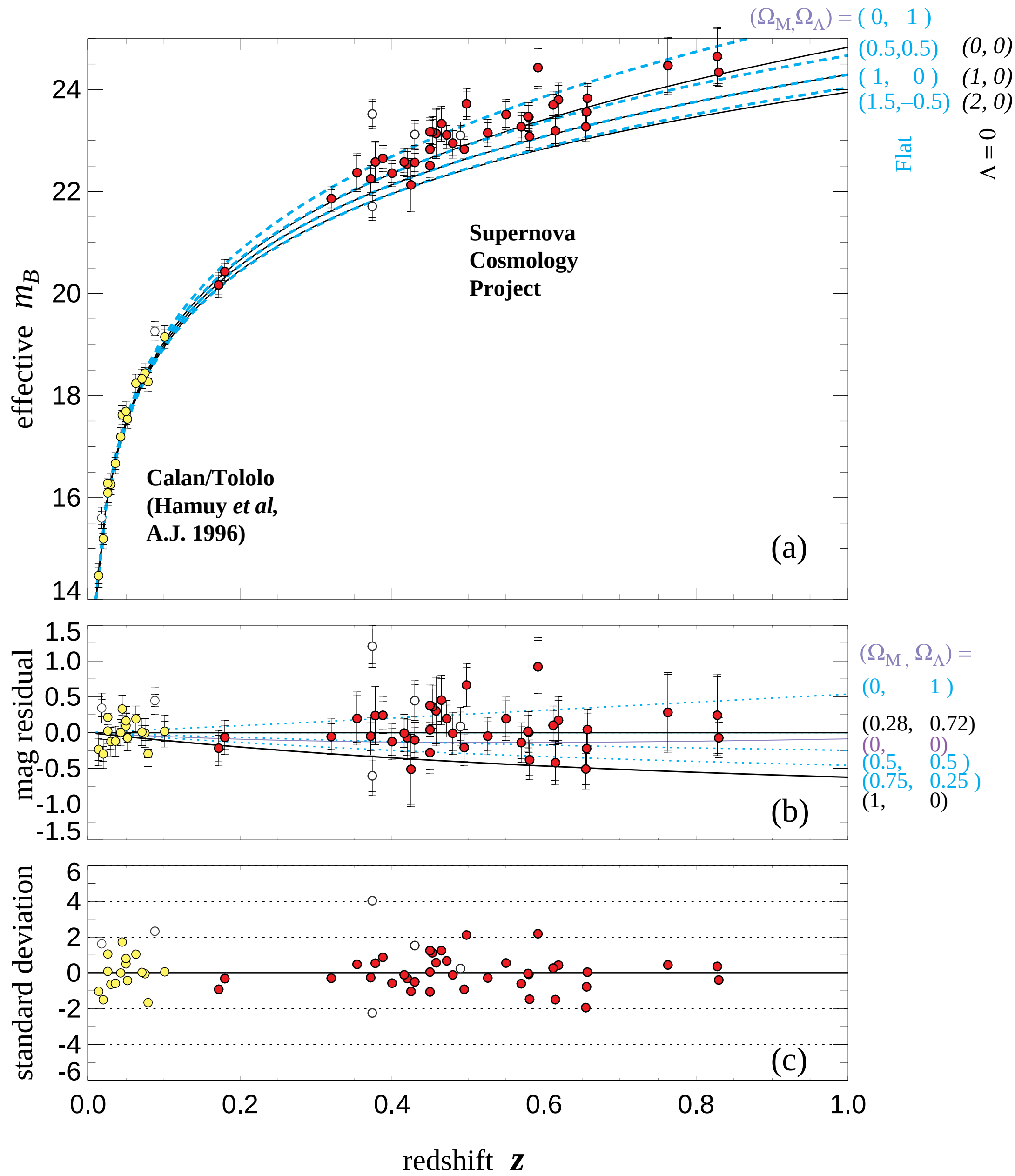
(Perlmutter et al., 1999, Fig. 1)

42 SNe from SCP, 18 low redshift from Calán/Tololo SN Survey

Vertical error bars: measurement uncertainty *plus* 0.17 mag intrinsic mag. dispersion

Horizontal error bars: $300\,\mathrm{km\,s^{-1}}$ peculiar velocity uncertainty

Supernovae

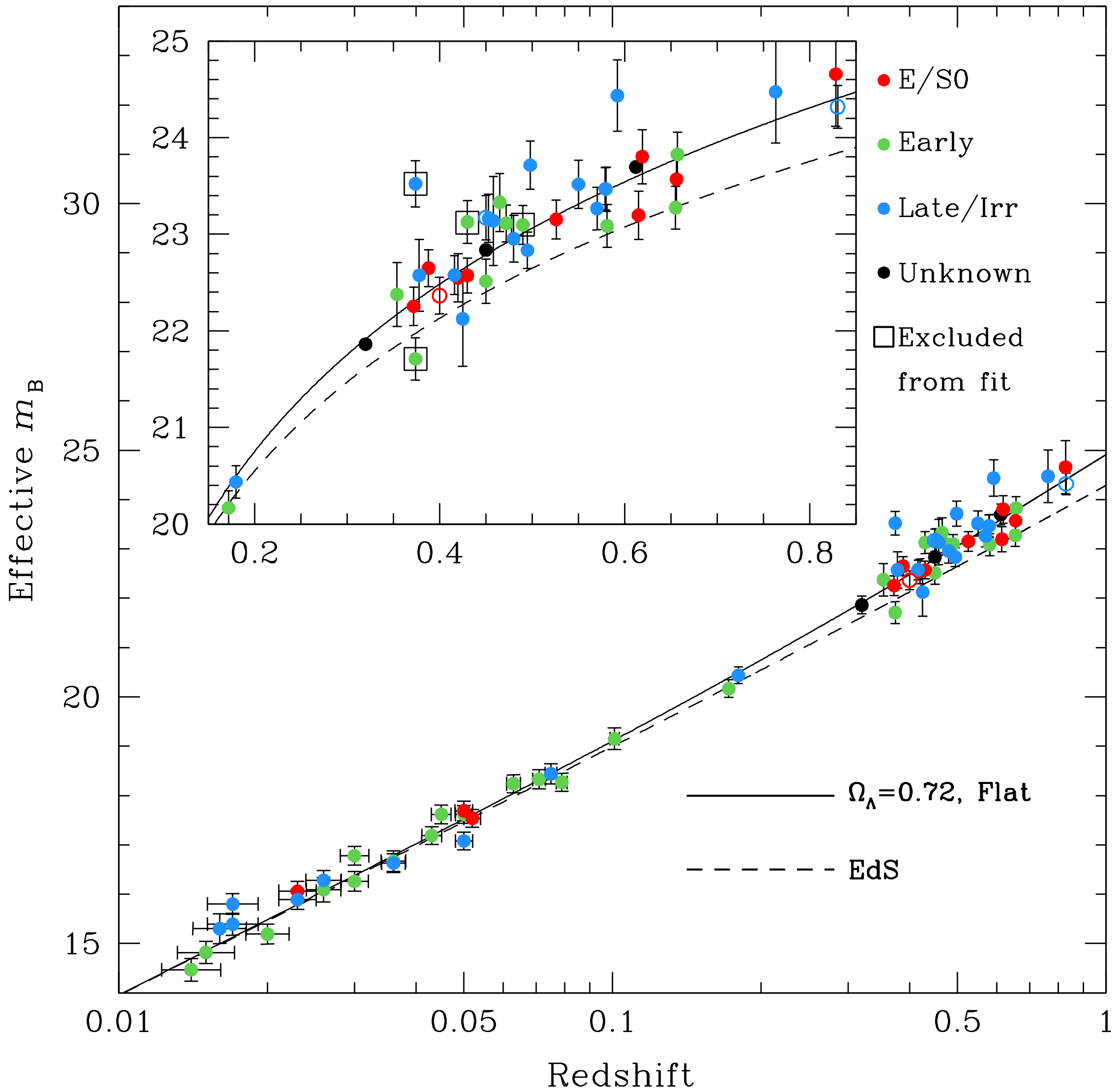


(Perlmutter et al., 1999, Fig. 2)

Best fit: $\Omega_{\text{m, flat}} = 0.28^{+0.09}_{-0.08}$, $\chi^2/\text{DOF} = 56/50$

corresponding best free fit: $(\Omega_\text{m}, \Omega_\Lambda) = (0.73, 1.32)$.

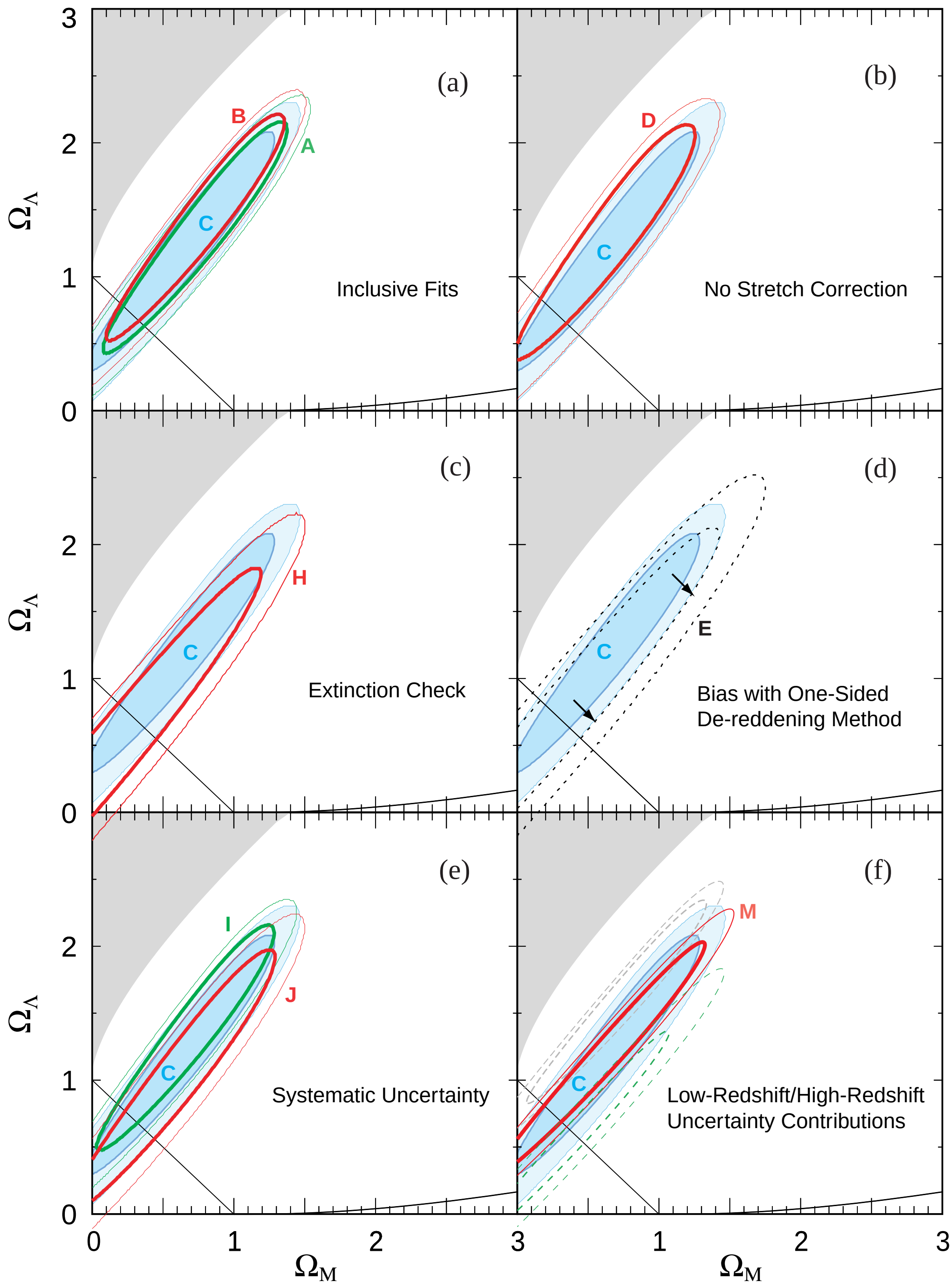
Supernovae



Sullivan et al., 2002

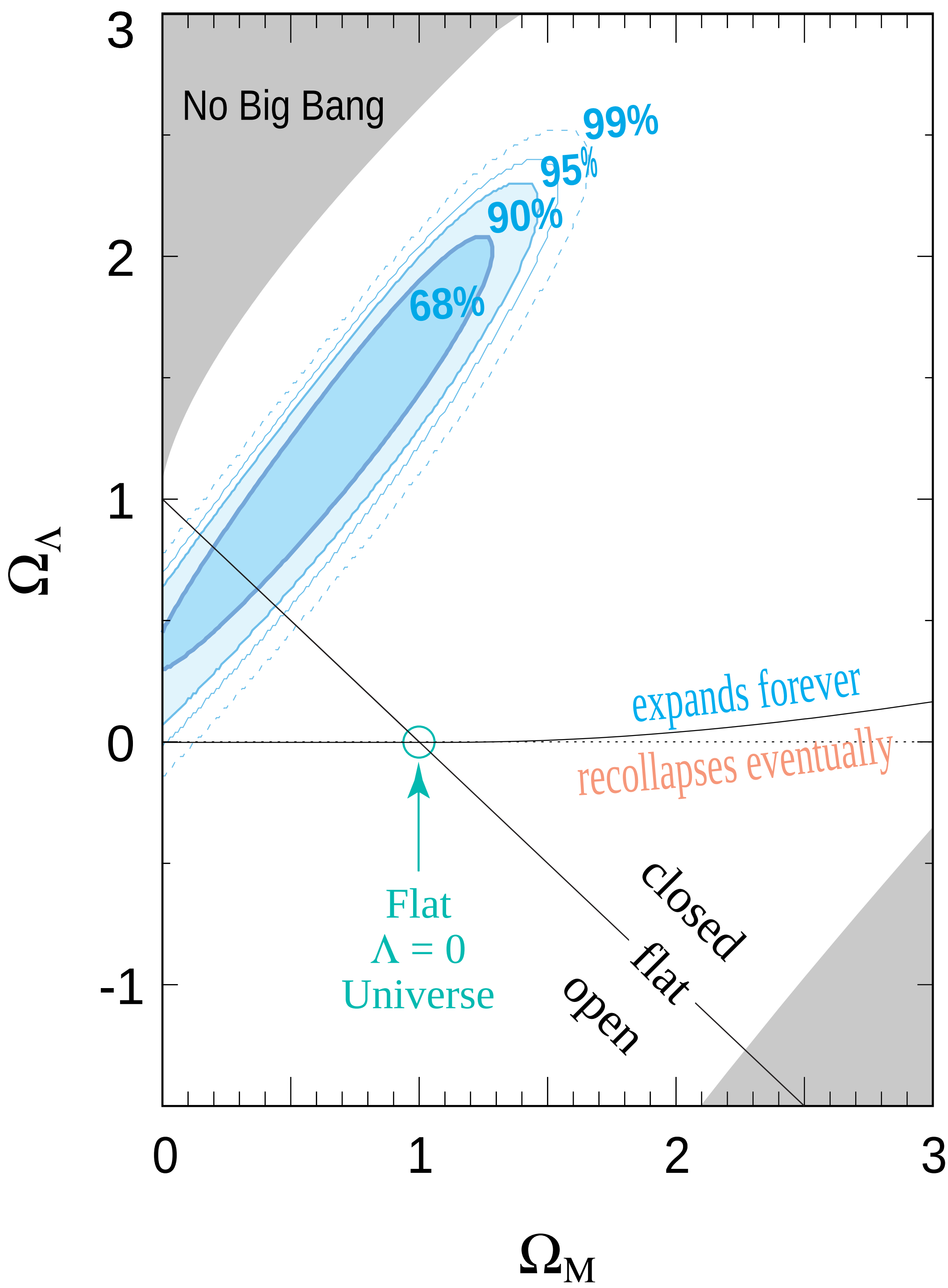
Updated 2002 Hubble diagram for SN Iae confirms Perlmutter 1999.

Supernovae



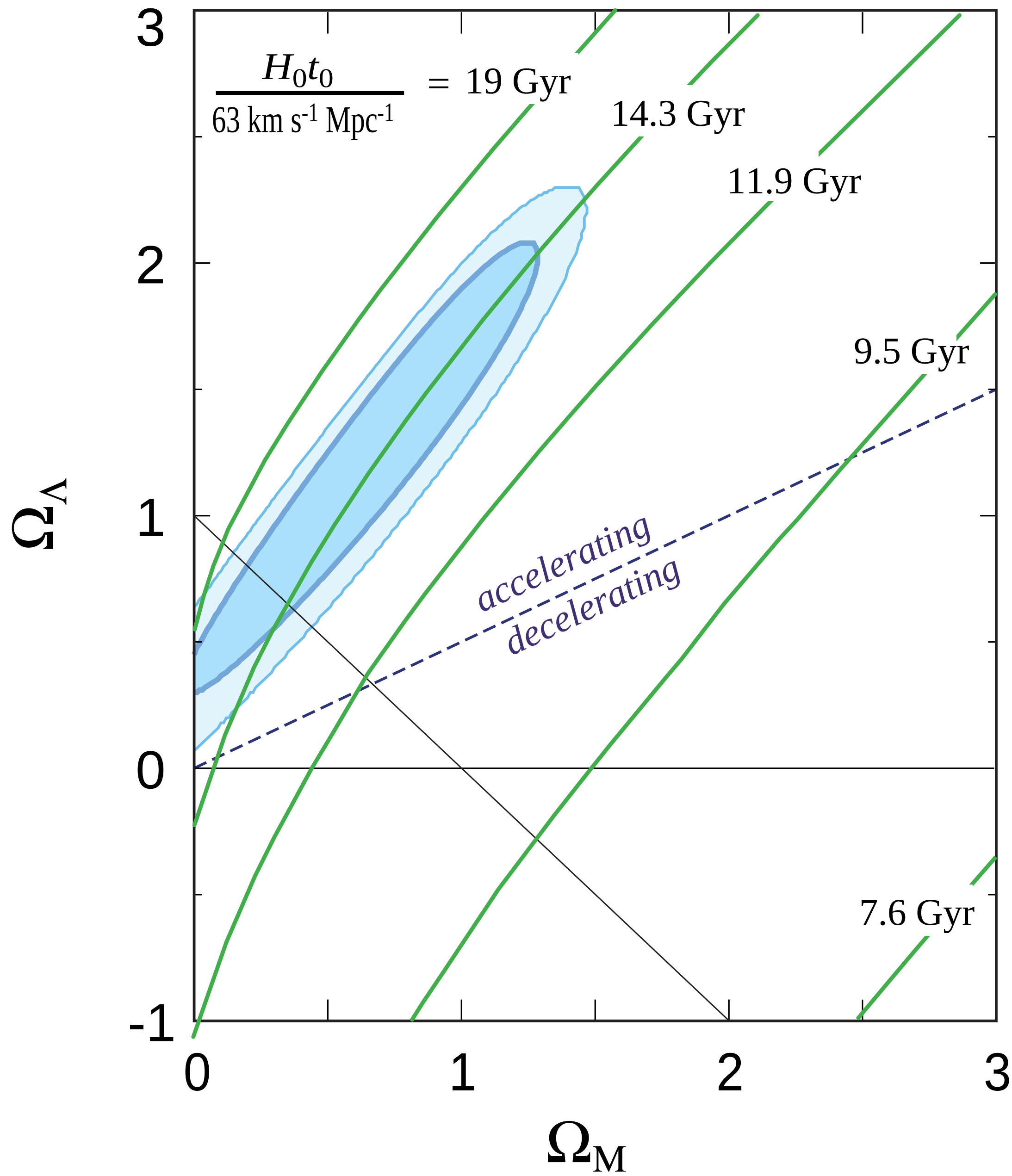
(68% and 90% confidence regions for sources of systematic error, Perlmutter et al., 1999, Fig. 5)

Supernovae



(Combined confidence region Perlmutter et al., 1999, Fig. 7 (lower right: universes that are younger than oldest heavy elements.))

Supernovae



(Perlmutter et al., 1999, Fig. 9)

Isochrones for age of universe for $H_0 = 63\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (for $h = 0.7$: age 10% smaller).

$\Longrightarrow$ Consistent with globular cluster ages!

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Summary

For all practical purposes, the currently best values are

$$\Omega_{\text{m}} \sim 0.3 \qquad \Omega_\Lambda = 0.7$$

Even if $\Omega \neq 1$:

$$\Omega_\Lambda \neq 0$$

And therefore

Baryons are an energetically unimportant constituent of the universe.

"The dark side of the force..." :-)

Small print: Influences of

- Metallicity evolution
- Dust
- Malmquist bias
- ???

...these are believed to be small, however, see Drell, Loredo & Wasserman (2000) for a critique

Outlook

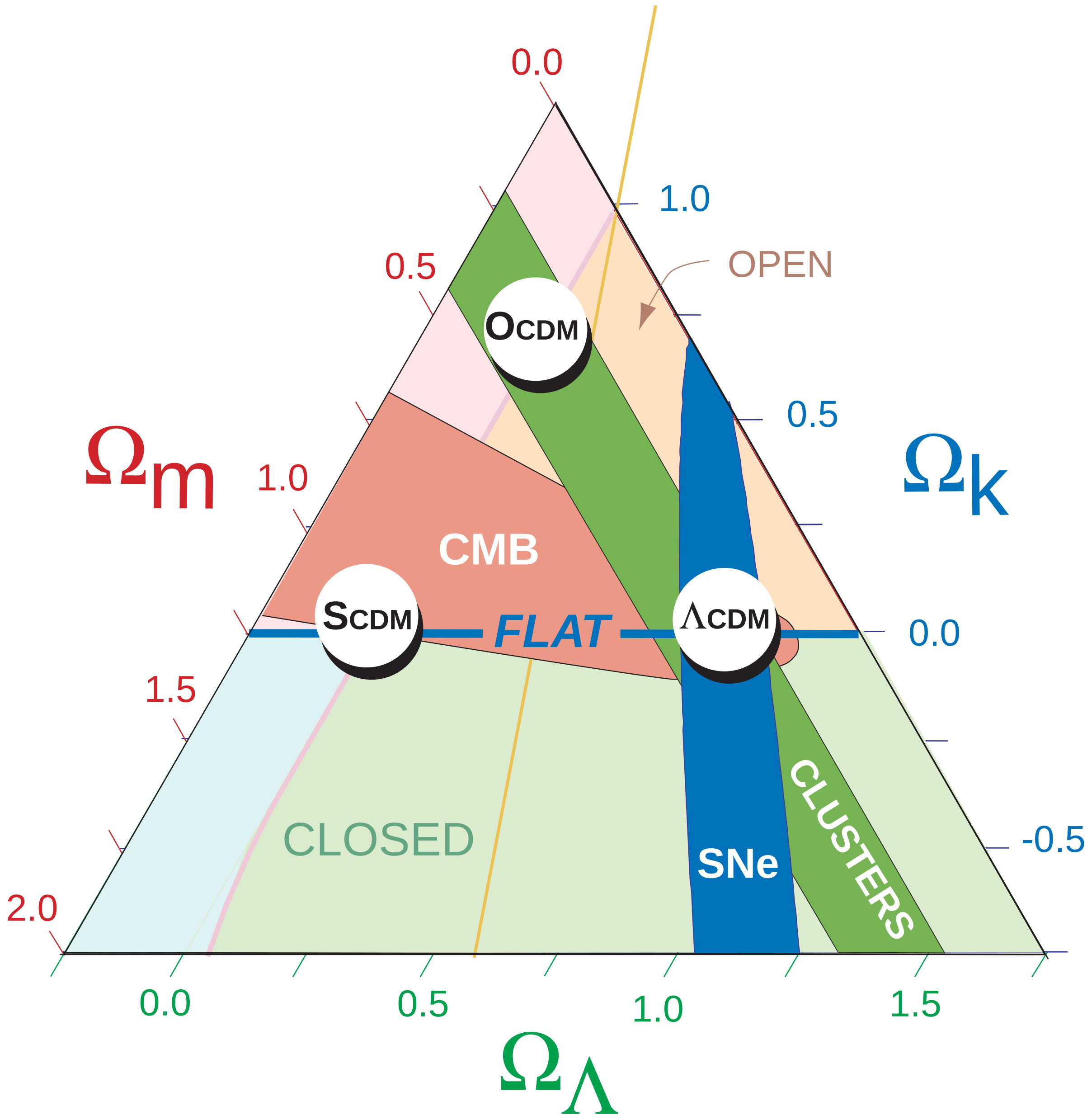
What is physical reason for $\Omega_\Lambda \neq 0$?
Currently discussed: quintessence: "rolling scalar field", corresponding to very lightweight particle ($\lambda_{\text{de Broglie}} \sim 1\,\text{Mpc}$), looks like time varying cosmological "constant".

Why? $\Longrightarrow$ More naturally explains why Ω_Λ so close to 0 (i.e., why matter and vacuum have so similar energy densities)

Motivated by string theory and M theory...

Still VERY SPECULATIVE, decision Λ vs. quintessence should be possible in next 5...10 years when new instruments become available.

Outlook



Bahcall et al.

Even better constraints come from combination of SNe data with structure formation.

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Large Scale Structures and Structure Formation

The Lumpy Universe

So far: treated universe as smooth universe.

In reality:

Universe contains structures!

Last part of this class:

1. What are structures?
2. How can we quantify them?
3. How do structures form?
4. How do structures evolve?

Will see that all these questions are deeply connected with parameters of the universe seen so far:

1. H_0
2. Ω_0, Ω_b, Ω_m, Ω_Λ,...
3. Existence and Nature of Dark Matter

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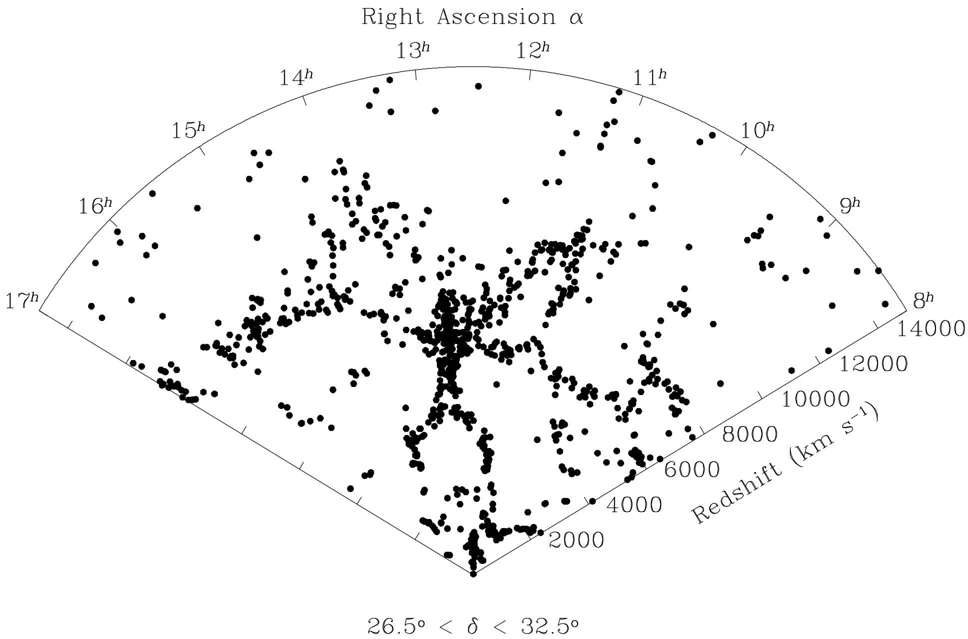
Introduction, I



(de Lapparent, Geller & Huchra, 1986, limiting mag $m_\mathrm{B} = 15.6$)

Lumpy universe: spatial distribution of galaxies and greater structures.

Observationally: need distance information for many (10^4) objects

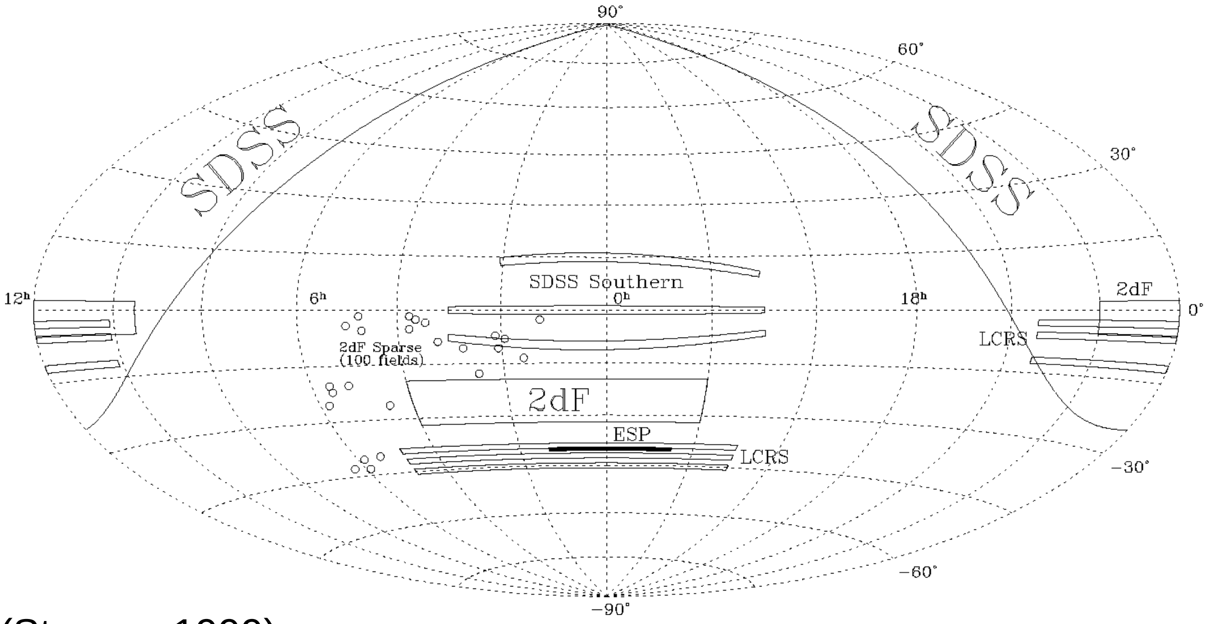
$\Longrightarrow$ Large redshift surveys

Review: Strauss & Willick (1995)

Redshift survey: Survey of (patch of) sky determining galaxy z and position to predefined magnitude or z.

First larger survey: de Lapparent, Geller & Huchra (1986)

Introduction, II



(Strauss, 1999)

Classification:

1D-surveys: very deep exposures of small patch of sky, e.g. HST Deep Field, Lockman Hole Survey, Marano Field.

2D-surveys: cover long strip of sky, e.g., CfA-Survey ($1.5 \times 100^\circ$), 2dF-Survey ("2 degree Field").

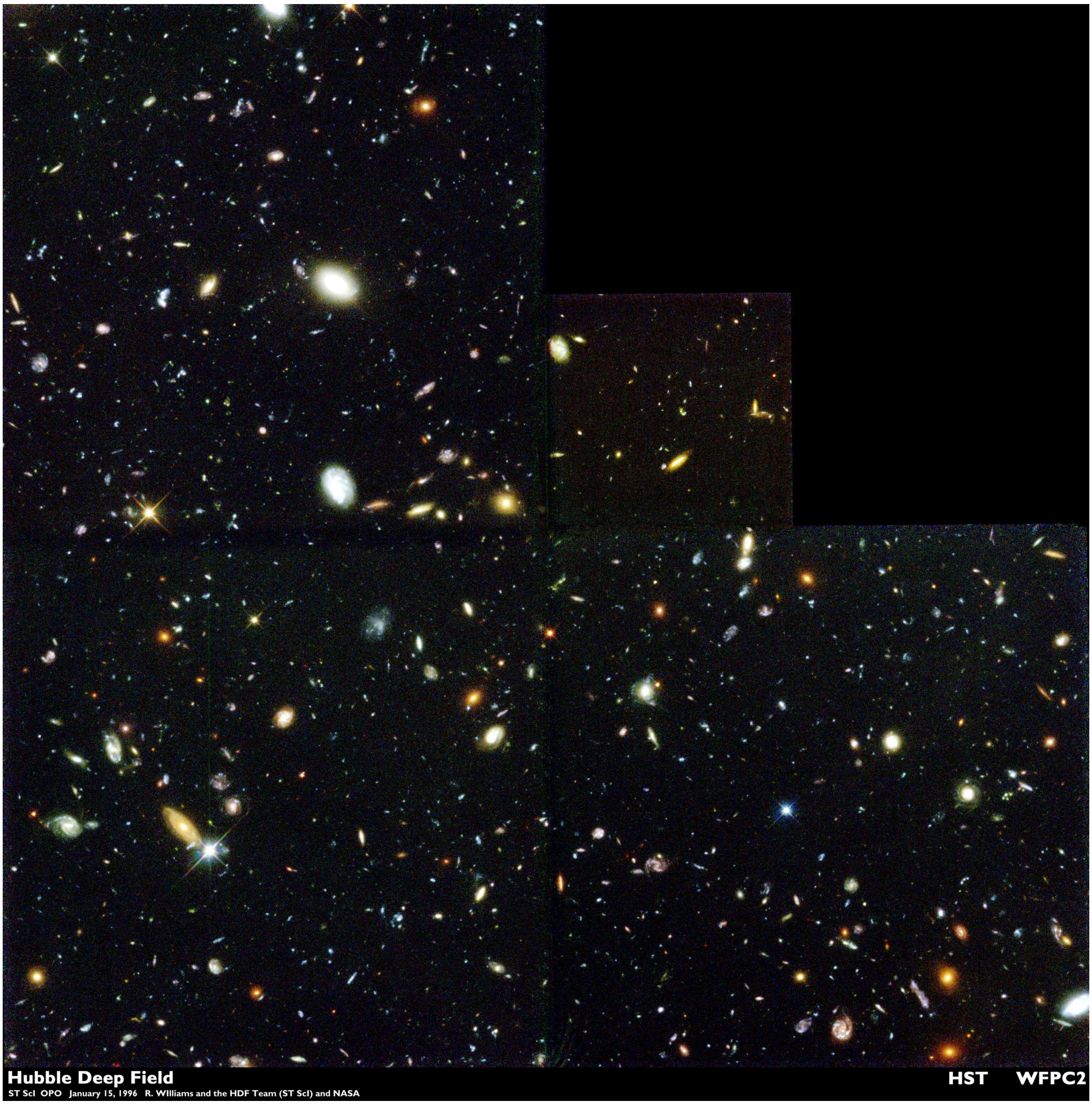
3D-surveys: cover part of the sky, e.g., Sloan Digital Sky Survey.

These surveys attempt to go to certain limit in z or m.

Other approaches: use pre-existing galaxy catalogues (e.g., QDOT Survey [IRAS galaxies], APM survey,...).

Will concentrate here on the larger surveys based on no other catalogue.

1D Surveys

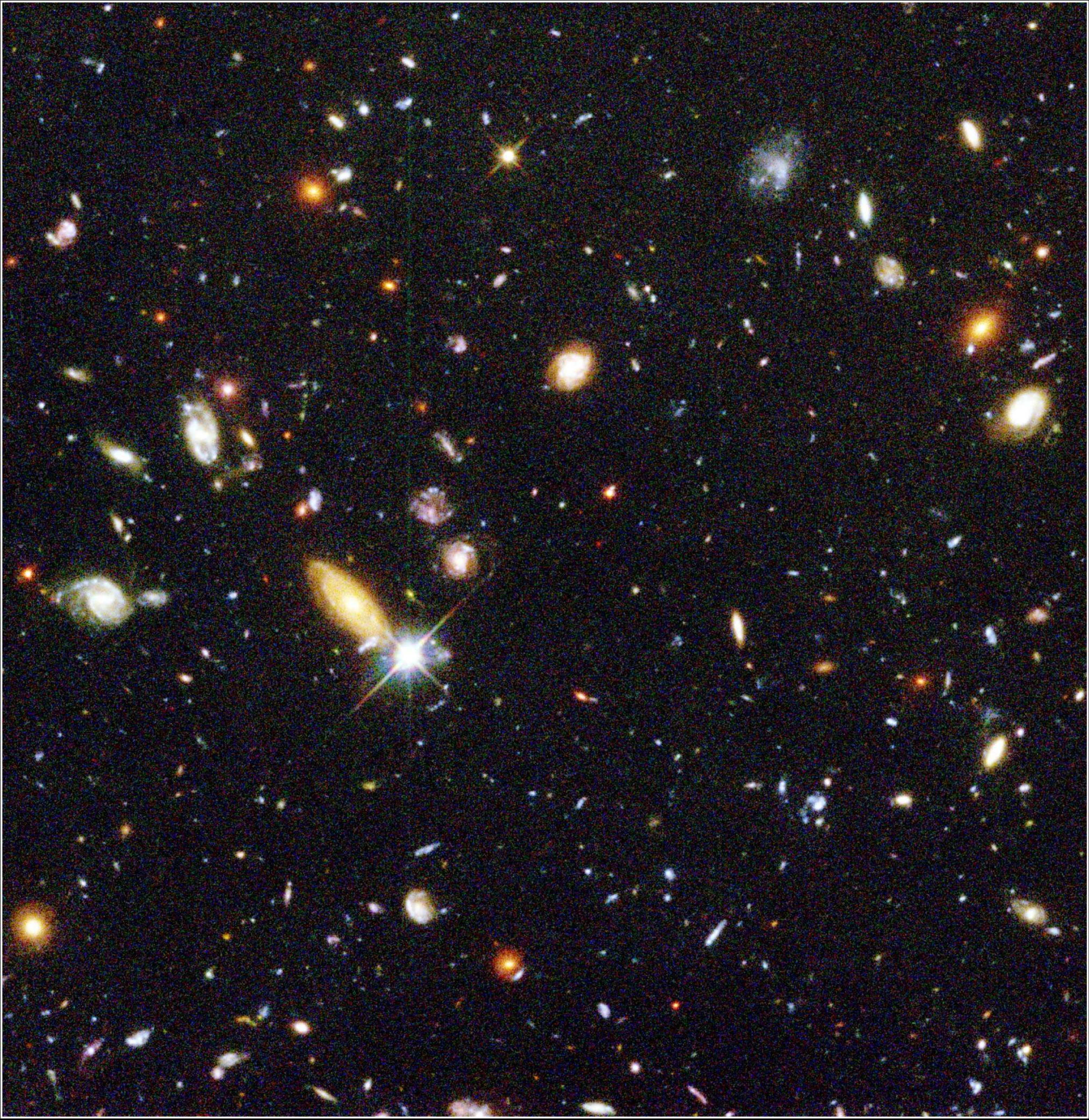


Hubble Deep Field, courtesy STScI

HDF: $\sim$ 150 ksec/Filter for 4 HST Filters made in 1995 December.

Many galaxies with weird shapes $\Longrightarrow$ protogalaxies!

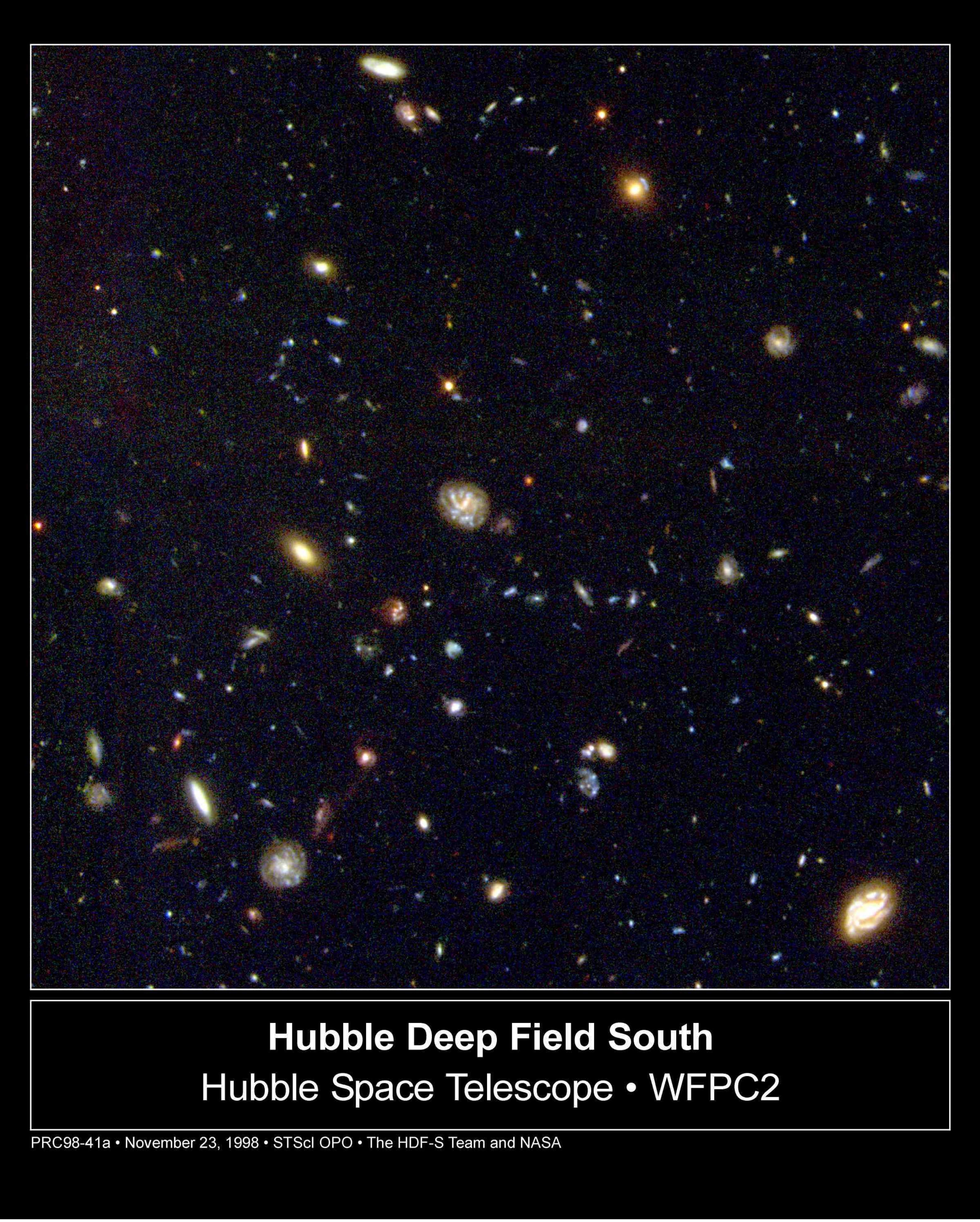
Redshifts: $z \in [0.5, 5.3]$ (Fernández-Soto et al., 1999)



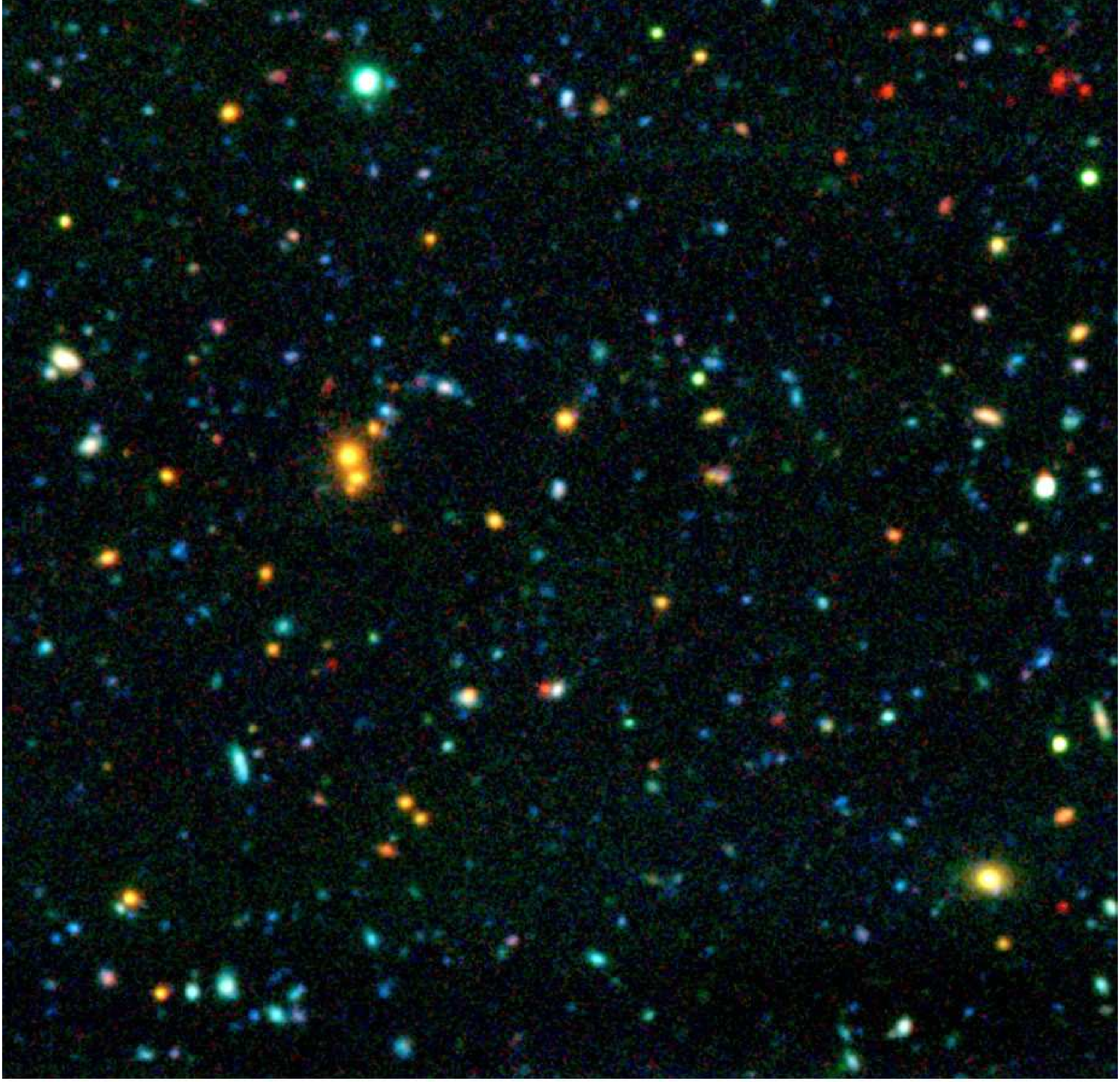
Hubble Deep Field

Hubble Space Telescope • WFPC2

PRC96-01a • ST ScI OPO • January 15, 1995 • R. Williams (ST ScI), NASA



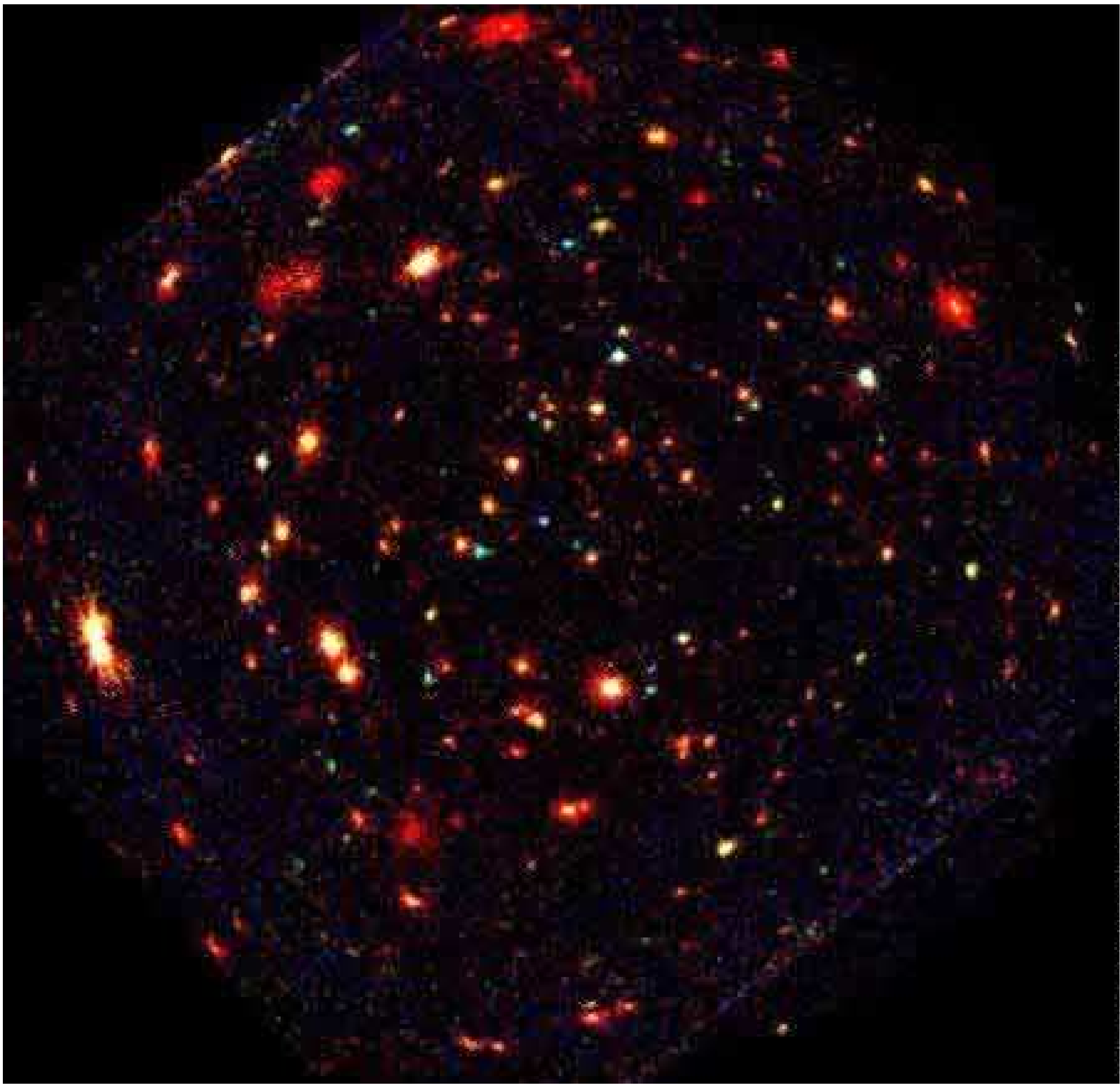
1998: Hubble Deep Field South, 10 d of total observing time!



Distant Galaxies in "AXAF Deep Field" (VLT ANTU / ISAAC + NTT / SUSI-2)

ESO PR Photo 06b/00 (17 February 2000)

1D Surveys



GUNTHER HASINGER/ASTROPHYSICS INSTITUTE, POTSDAM

XMM-Newton, Hasinger et al., 2001,
blue: hard X-ray spectrum,
red: soft X-ray spectrum

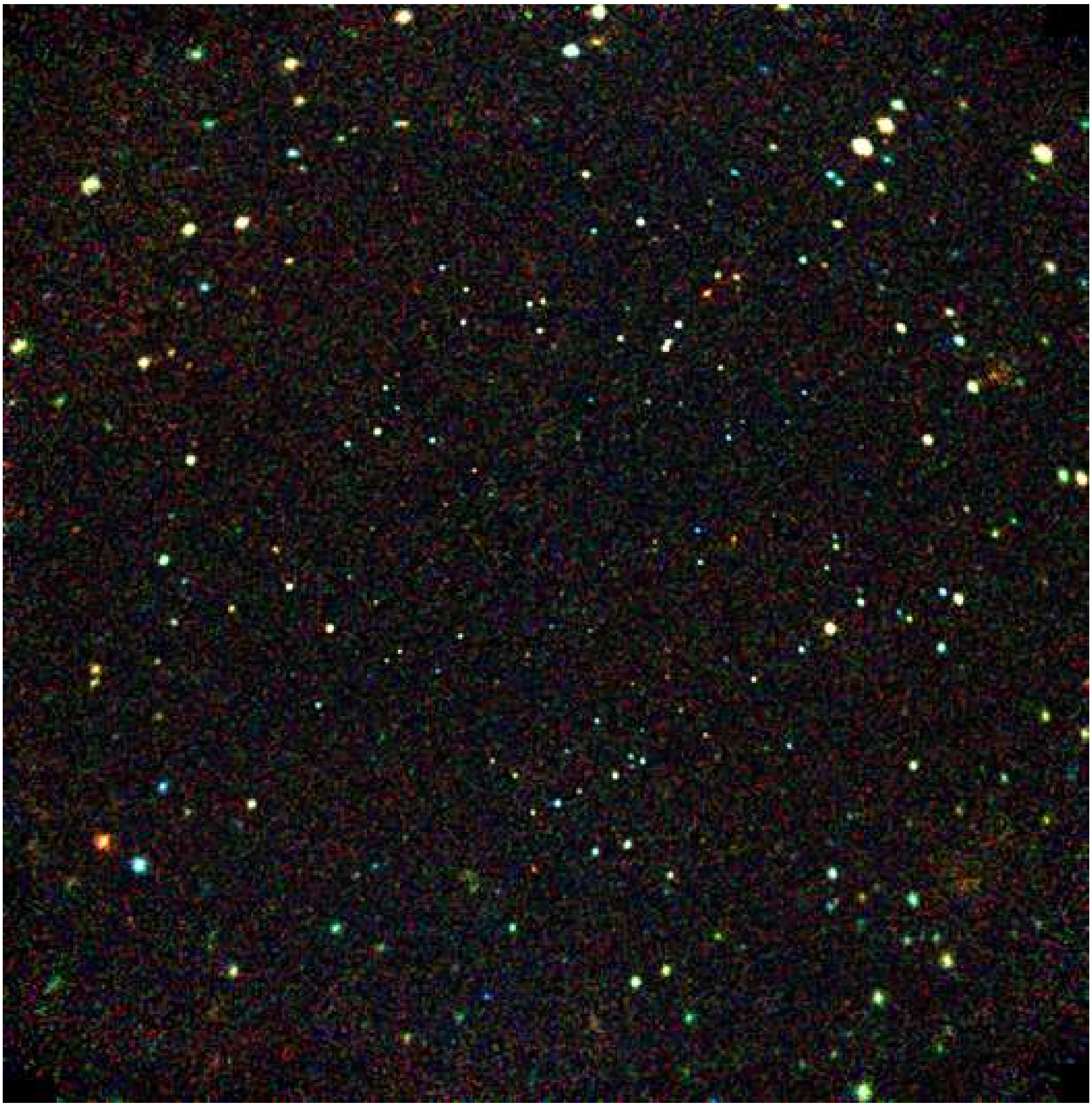
Lockman Hole: Northern Sky region with very low N_{H}

$\Longrightarrow$ low interstellar absorption

$\Longrightarrow$ “Window in the sky”

$\Longrightarrow$ X-rays: evolution of active galaxies with z!

1D Surveys

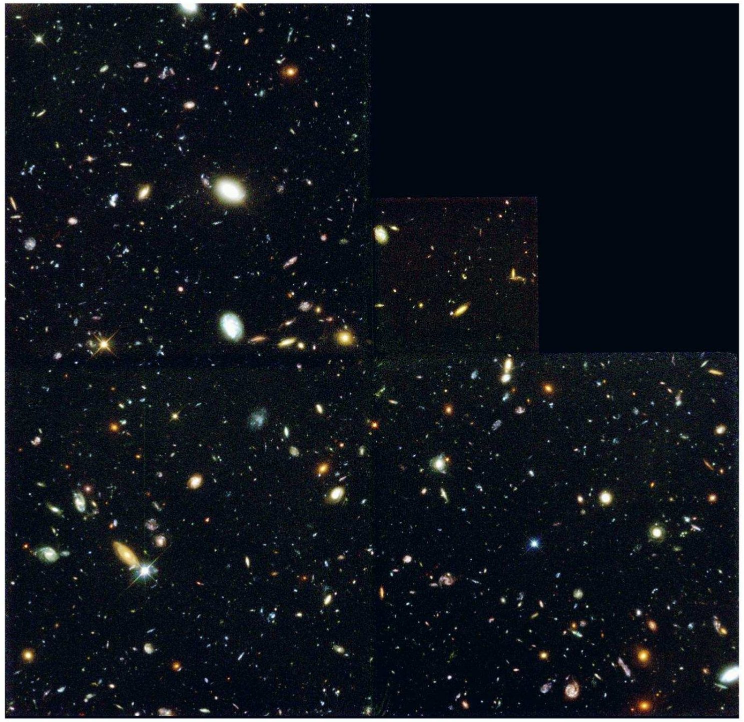


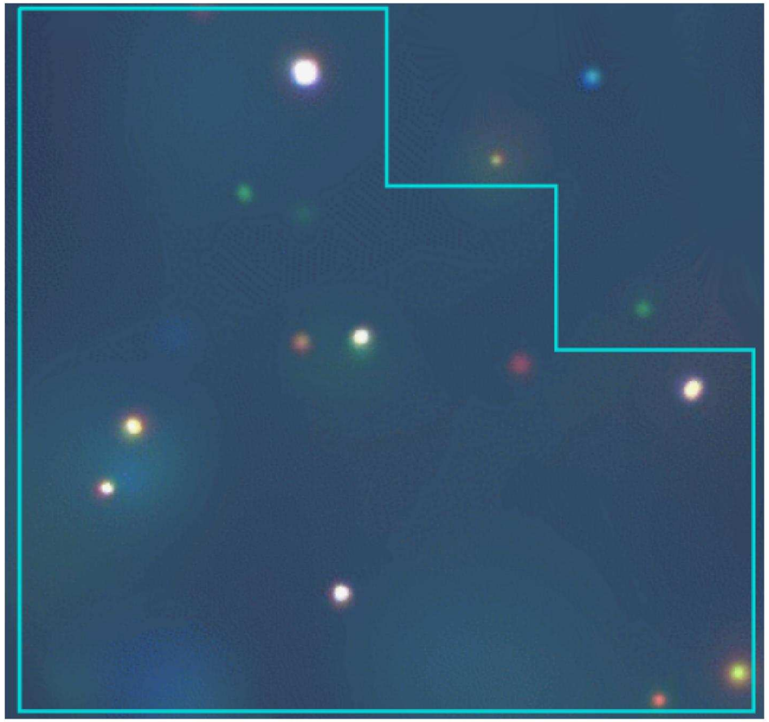
scale: $15' \times 15'$; courtesy NASA/JHU/AUI/R.Giacconi et al.

Chandra Deep Field South: 1 Msec (10.8 days) on one region in Fornax $\Longrightarrow$ Deepest X-ray field ever...

color code: spectral hardness

1D Surveys

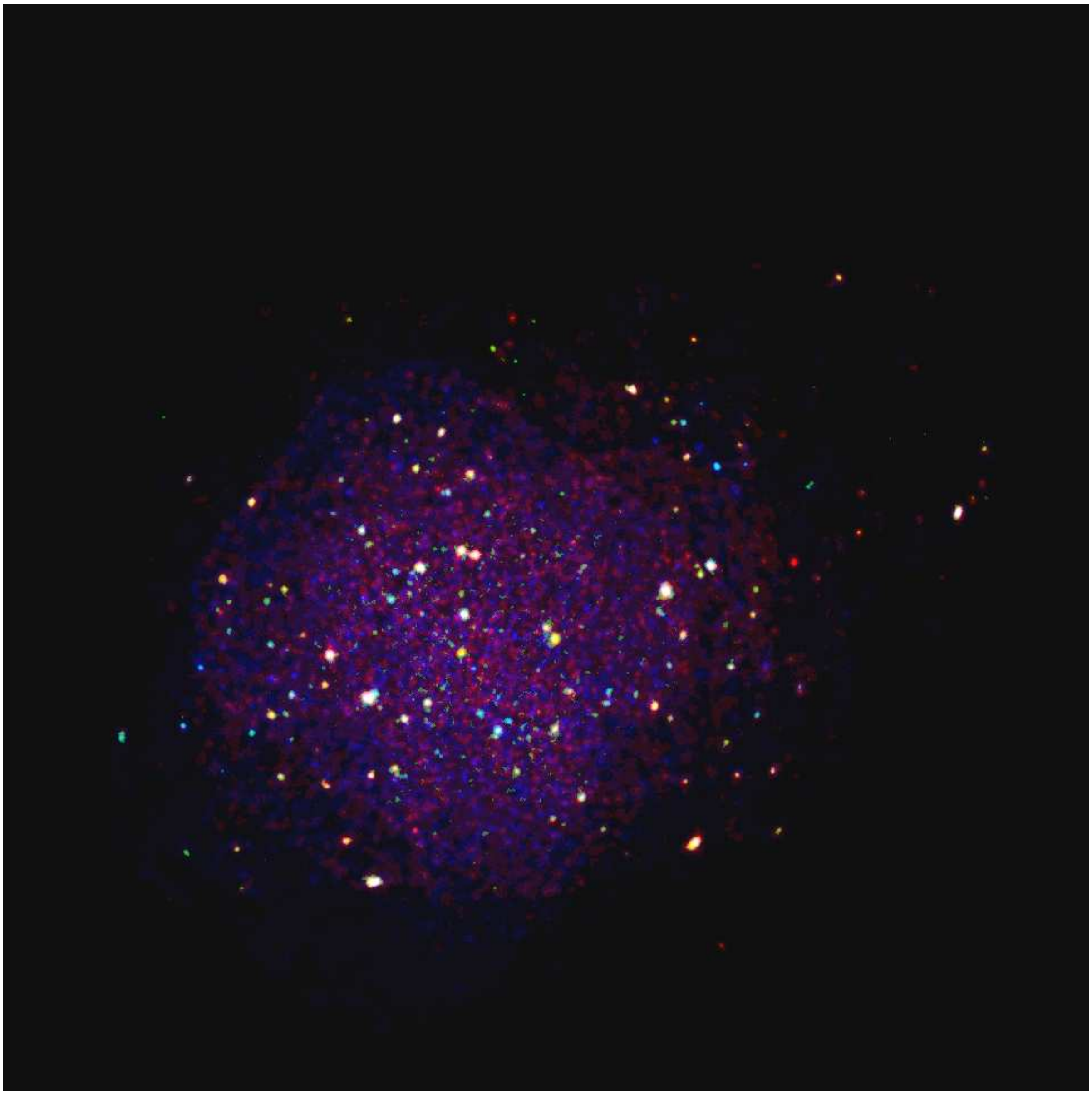




Chandra/HST Image of Hubble Deep Field North; 500 ksec

Joint multi-wavelength campaigns allow the measurement of broad-band spectra of sources in the early universe!

1D Surveys



Deep *XMM-Newton* image of the Marano Field (IAAT/AIP/MPE)

1D Surveys (“Deep Exposures”) give snapshot of evolution of galaxies over large z.

2D/3D Surveys: Technology

Future for Large Scale Structure: 2D and 3D Surveys observing large part of sky with dedicated instruments.

Currently largest surveys:

Las Campanas Redshift Survey (LCRS): 26418 redshifts in six $1.5 \times 80^\circ$ slices around NGP and SGP, out to $z = 0.2$.

CfA Redshift Survey: 30000 galaxies

APM: (Oxford University) $2 \sim 10^6$ galaxies, 10^7 stars around SGP, 10% of sky, through $B = 21\,\text{mag}$.

2MASS: IR Survey of complete sky (Mt. Hopkins/CTIO) completed 2000 October 25), 3 bands, $\sim 2 \times 10^6$ galaxies, accompanying redshift survey (8dF, CfA)

Sloan Digital Sky Survey (SDSS): dedicated 2000 October 5, Apache Point Obs., NM, 25% of whole sky, $\sim 10^8$ objects,

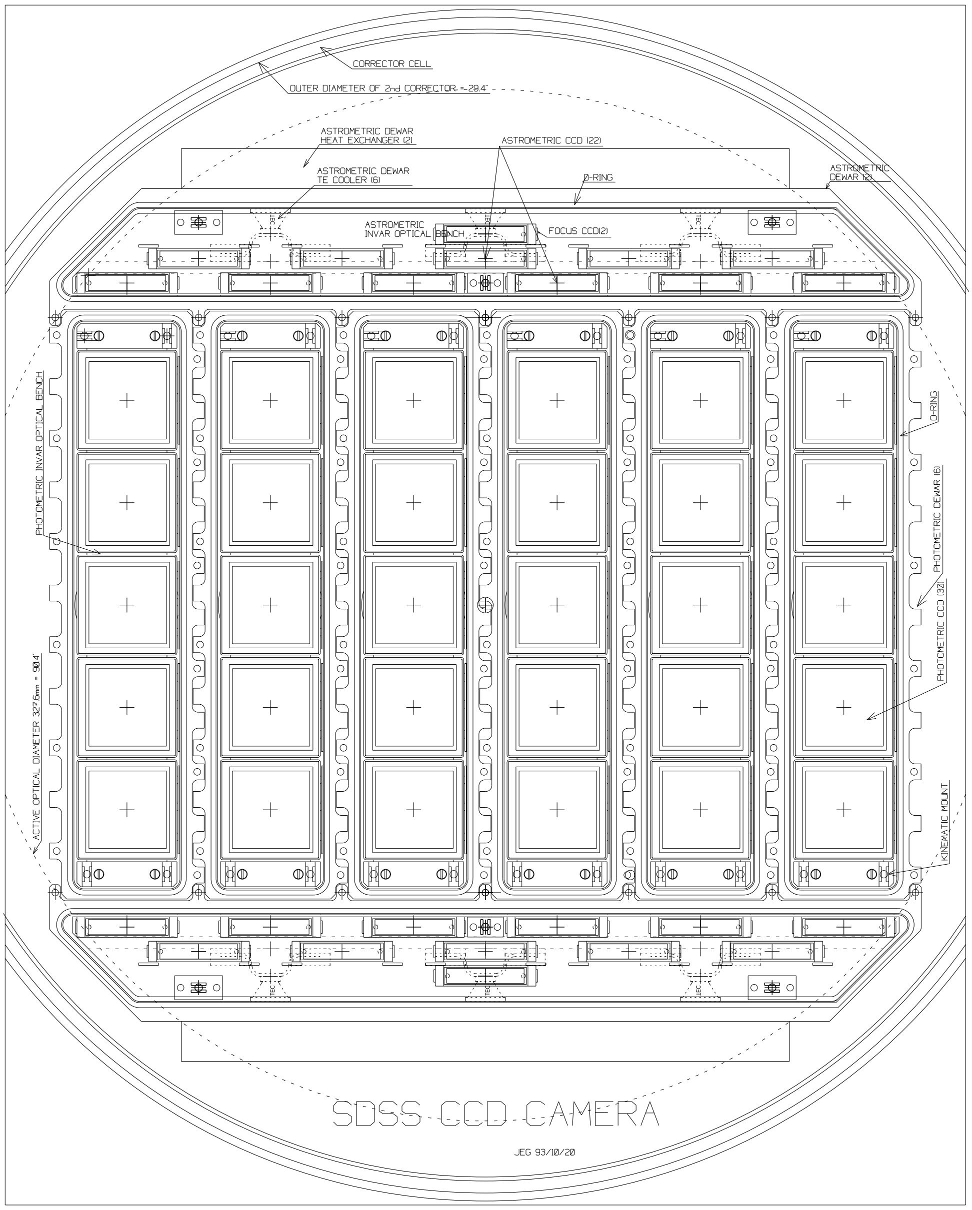
And many more (e.g., Keck, ESO,...).

2D/3D Surveys: Technology

courtesy SDSS

SDSS 2.5 m telescope at Apache Point Observatory

2D/3D Surveys: Technology



(Strauss, 1999, Fig. 5)

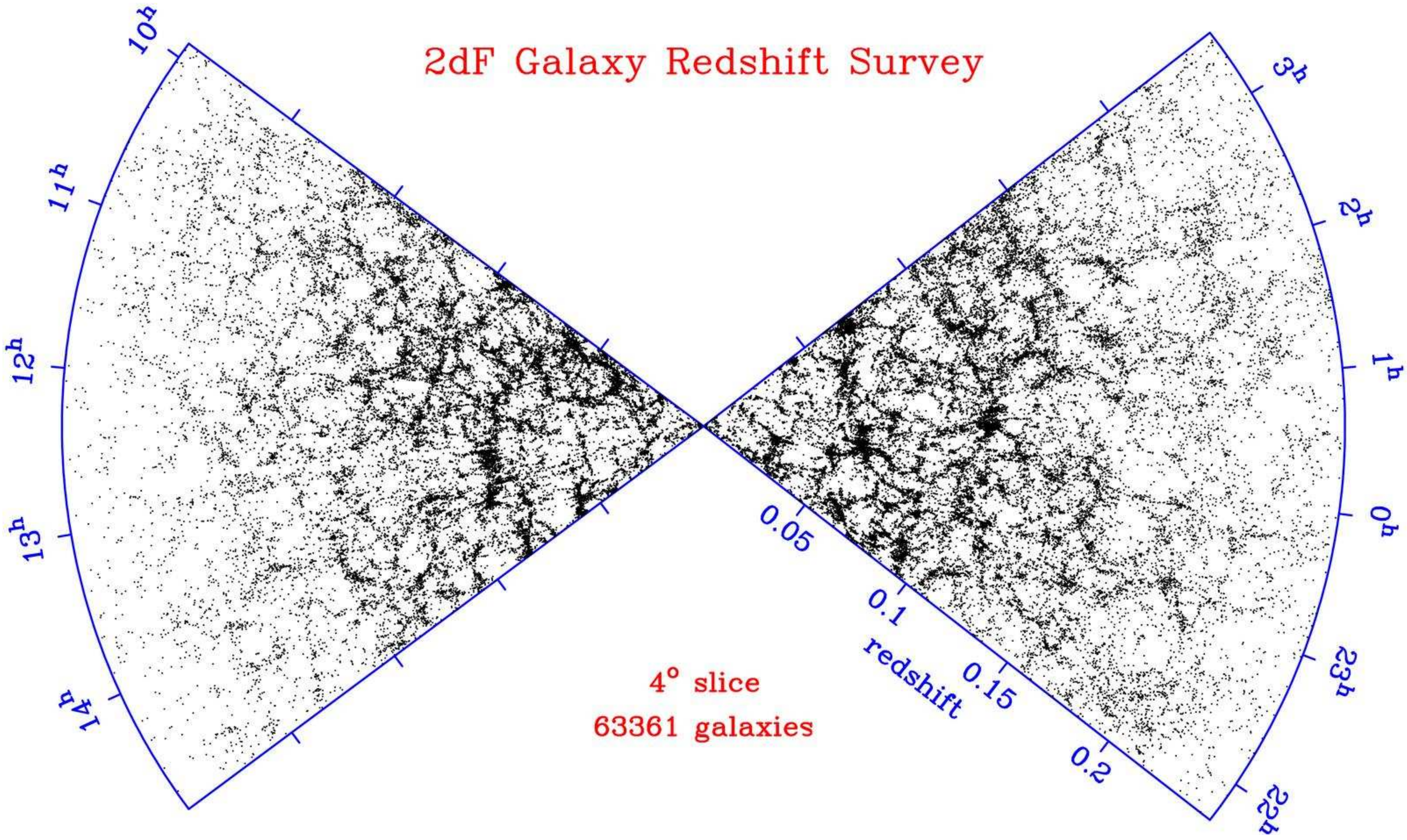
CCD alignment of SDSS:

- focal plane: 2.5°,
- 5 rows of 2048 $\times$ 2048 CCDs with r, i, u, z, g filters, saturation at $r = 14$
- 22 2048 $\times$ 400 CCD, saturation at $r = 6.6$ for astrometry

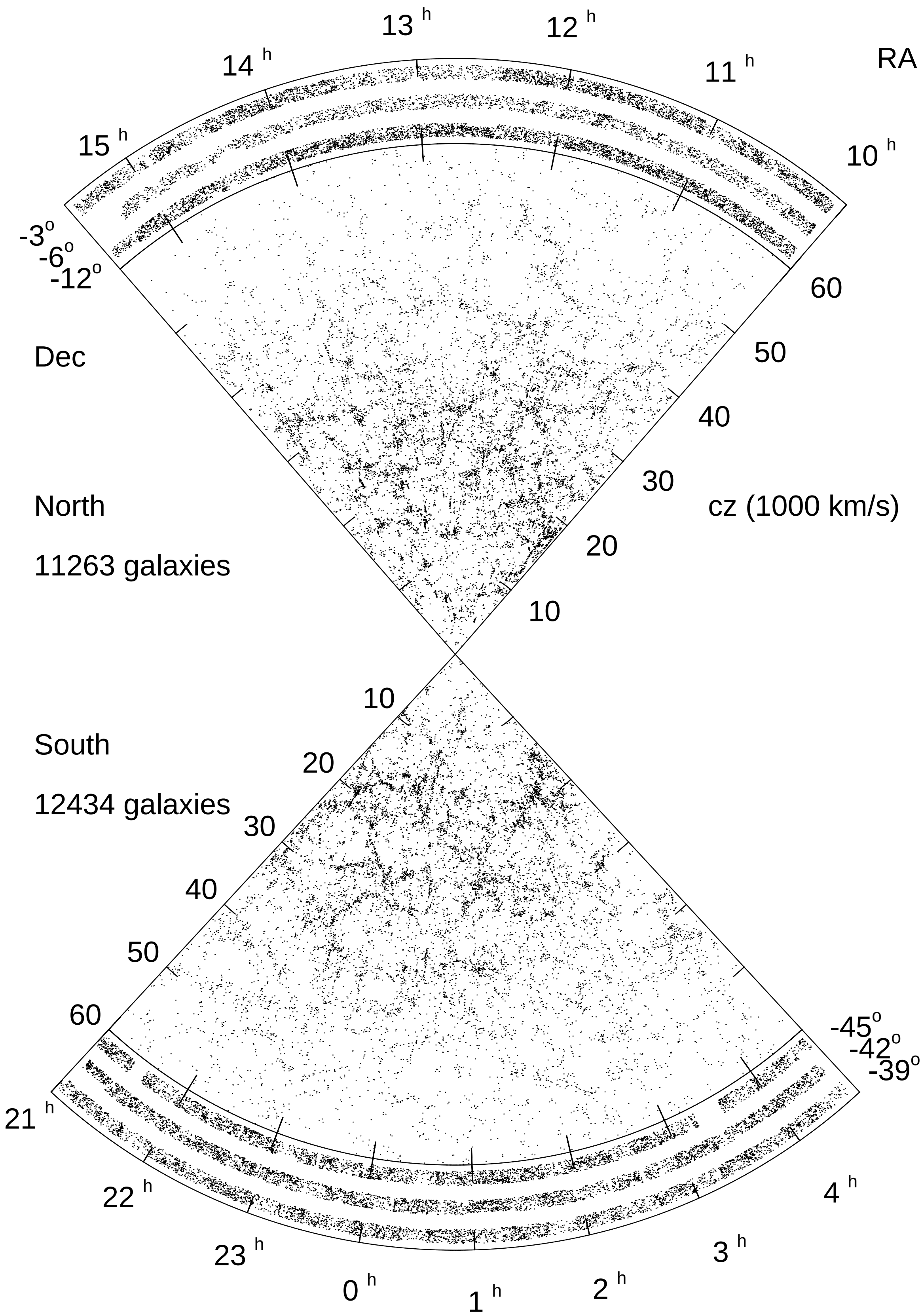
Imaging by slewing over CCD Array

2D/3D Surveys: Technology

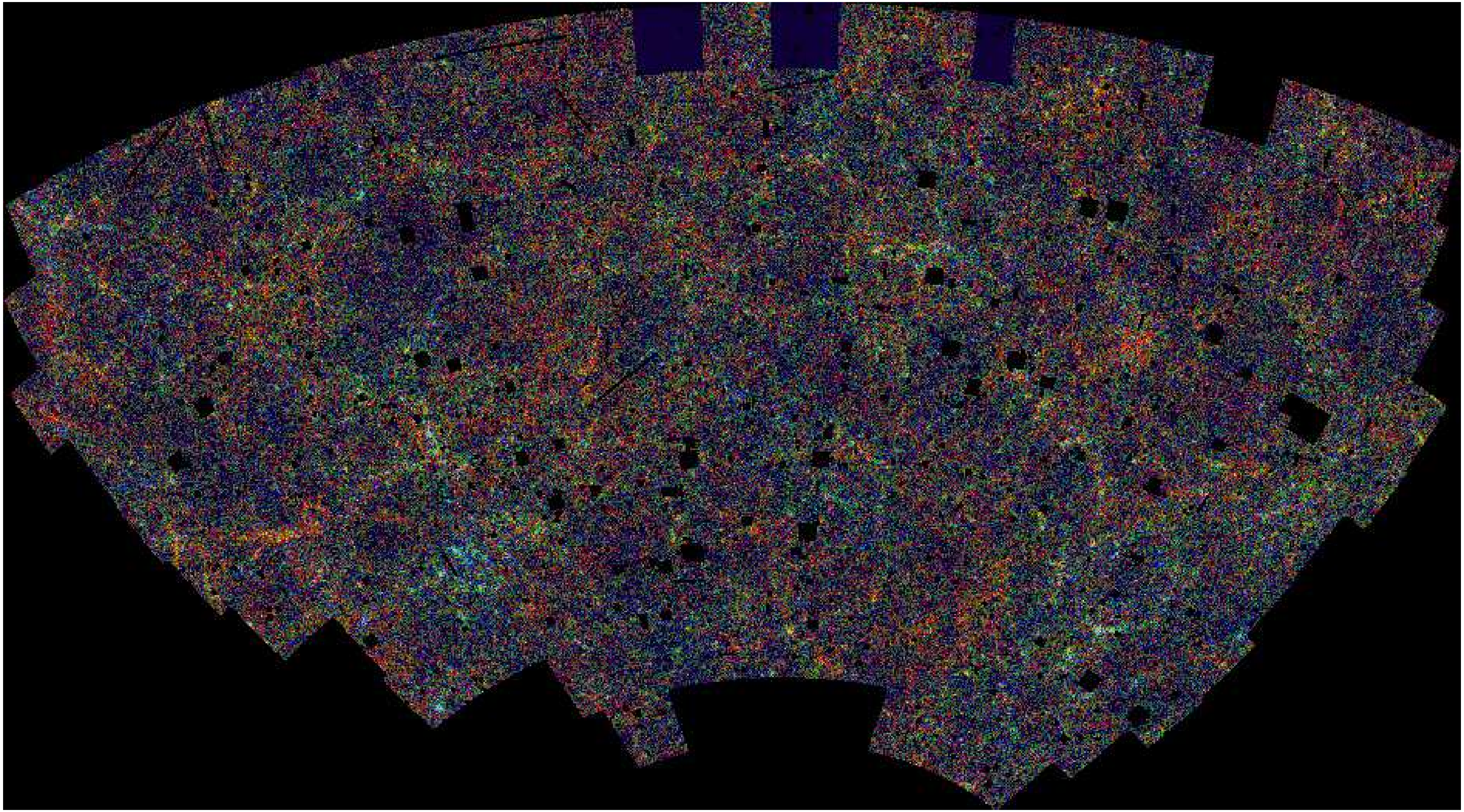
courtesy SDSS



2dF Galaxy Redshift Survey
10h
11h
12h
13h
14h
3h
2h
1h
0h
23h
22h
0.05
0.1
0.15
0.2
redshift
4° slice
63361 galaxies



The complete LCRS survey (at cz large: reach mag. limit)



Galaxies in APM catalogue, color: avg. B in pixel: blue (18) – green (19) – red (20)

Correlation Function, I

Sky surveys show:

Galaxies are *not* evenly distributed: "cosmic web"!

- Structures at scales up to several 10 Mpc
- But: Over-density even in clusters not too dramatic ($\sim$ 100$\times$ denser than average).
- Voids on scales 50 h^{-1} Mpc

$\Longrightarrow$ Need quantitative description of structures.
$\Longrightarrow$ Need physical explanation of structures.
$\Longrightarrow$ Need to understand what we see (do galaxies trace matter distribution??).

Correlation Function, II

Mathematical description of clustering:
Correlation function!

Assume *uniform* distribution of galaxies with galaxy density n (gal Mpc^{-3}).
Chance to find galaxy in volume ΔV:

$$P \propto n\Delta V \tag{8.1}$$

Probability to find galaxies in *two* volumes:

$$P = P_1 \cdot P_2 \propto n^2 \Delta V_1 \Delta V_2 \tag{8.2}$$

Universe inhomogeneous: measure (distance dependent) deviation from mean:

$$P \propto n^2 (1 + \xi(r_{12})) \Delta V_1 \Delta V_2 \tag{8.3}$$

$\xi(r_{12})$ is called the two-point correlation function.

For *small* r:

$$\xi(r) > 0 \Longrightarrow \text{clustering}$$

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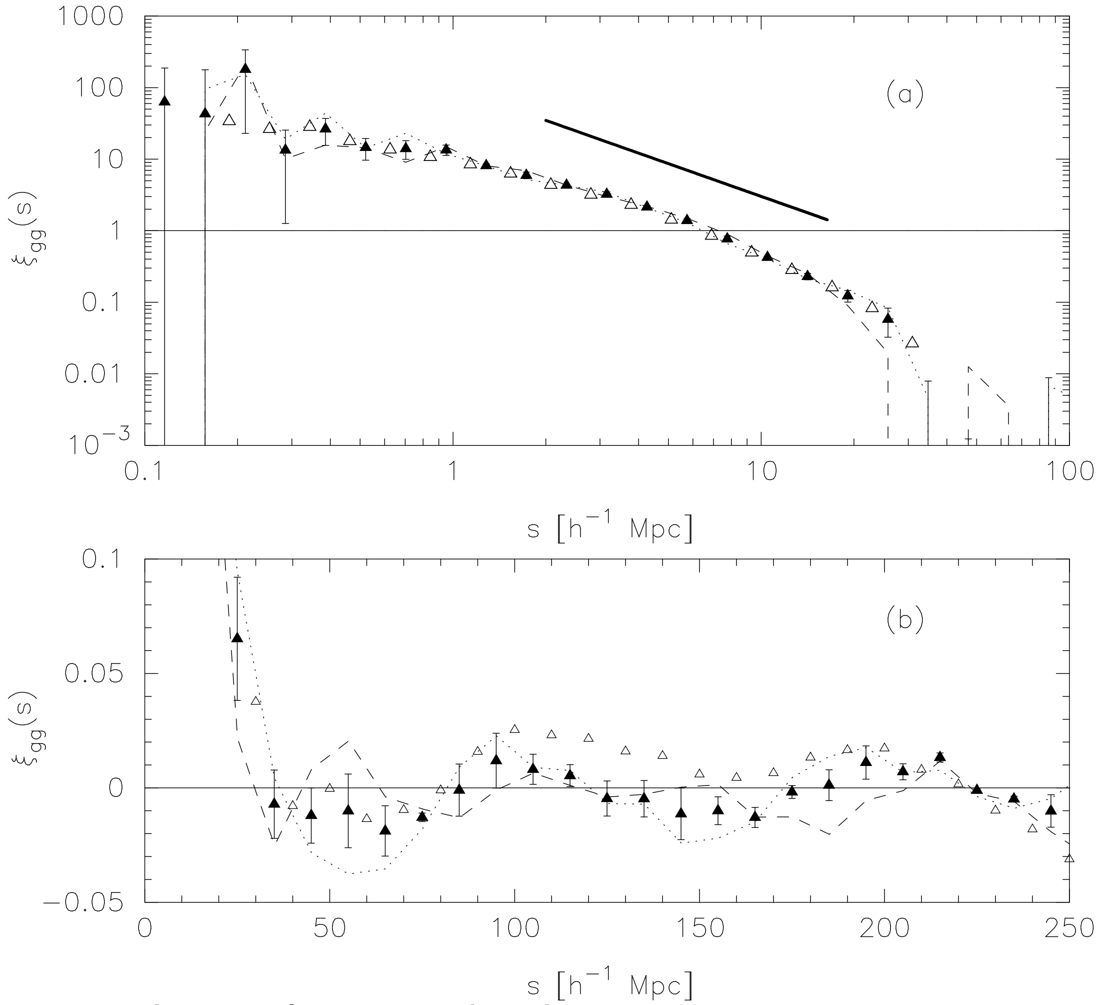
Correlation Function, III



(LCRS; Tucker et al., 1997, Fig. 1)

Rough description: power law

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma} \tag{8.4}$$

where $r_0 \sim 6\,h^{-1}$ Mpc (correlation length), and $\gamma \sim 1.5 \ldots 1.8$.

Above $r = 30\,h^{-1}$ Mpc: oscillation due to voids.

Correlation Function, IV

ξ is related to the density contrast $\Delta(x)$:
Write density n as

$$n(\mathbf{x}) = n_0(1 + \Delta(\mathbf{x})) \iff \Delta(\mathbf{x}) = \delta n/n \qquad (8.5)$$

Average joint probability to have galaxies at $\mathbf{x}$ and $\mathbf{x} + \mathbf{r}$:

$$P = \langle n(\mathbf{x})\mathrm{d}V_1 \cdot n(\mathbf{x}+\mathbf{r})\mathrm{d}V_2 \rangle \qquad (8.6)$$

$$= \left\langle n_0^2(1 + \Delta(\mathbf{x}))(1 + \Delta(\mathbf{x}+\mathbf{r}))\ \mathrm{d}V_1\mathrm{d}V_2 \right\rangle \qquad (8.7)$$

Since $\langle\Delta\rangle = 0$, only cross product survives:

$$= n_0^2\left(1 + \langle\Delta(\mathbf{x})\Delta(\mathbf{x}+\mathbf{r})\rangle\right)\ \mathrm{d}V_1\mathrm{d}V_2 \qquad (8.8)$$

where $\langle\ldots\rangle$ denotes averaging over an appropriate volume, i.e.,

$$\langle f(\mathbf{r})\rangle = \frac{1}{V}\int_V f(\mathbf{r})\,\mathrm{d}^3r \qquad (8.9)$$

Comparing Eq. (8.8) with Eq. (8.3) shows:

$$\xi(r) = \langle\Delta(\mathbf{x})\Delta(\mathbf{x}+\mathbf{r})\rangle \qquad (8.10)$$

$\xi(r)$ is a measure for the average density contrast at places separated by distance r.

Power Spectrum, I

To describe variations: more convenient to work in Fourier space than in "normal" space.

Fourier transform in spatial coordinates defined by:

$$\Delta_r(\mathbf{r}) = \frac{V}{(2\pi)^3}\int \Delta_k \exp(-i\mathbf{k}\cdot\mathbf{r})\,\mathrm{d}^3k \tag{8.11}$$

$$\Delta_k(\mathbf{k}) = \frac{1}{V}\int \Delta(\mathbf{r}) \exp(+i\mathbf{k}\cdot\mathbf{r})\,\mathrm{d}^3r \tag{8.12}$$

But note Parseval's theorem

$$\frac{1}{V}\int \Delta^2(\mathbf{r})\,\mathrm{d}^3x = \frac{V}{(2\pi)^3}\int \Delta_k^2\,\mathrm{d}^3k \tag{8.13}$$

(from signal theory: the power in a time series is the same as the power in the associated Fourier transform)

Left side: variance (mean square amplitude of fluctuations per unit volume)

$\Longrightarrow$ related to power spectrum,

$$P(k) = \Delta_k^2 \tag{8.14}$$

Therefore,

$$\left\langle \Delta^2 \right\rangle = \frac{V}{(2\pi)^3}\int P(k)\,\mathrm{d}^3k \tag{8.15}$$

where (Eq. 8.9)

$$\left\langle \Delta^2 \right\rangle = \frac{1}{V}\int \Delta^2(\mathbf{r})\,\mathrm{d}^3r \tag{8.16}$$

Power Spectrum, II

How are $\langle\Delta^2\rangle$ and ξ related?

$\Longrightarrow$ Use brute force computation or make use of the correlation theorem.

For functions g, h, the correlation theorem states that the Fourier transform of the correlation,

$$\mathrm{Corr}(g,h) = \int g(x+r)h(r)\,\mathrm{d}x \tag{8.17}$$

is given by

$$\mathrm{FT}\left(\mathrm{Corr}(g,h)\right) = G\,H^* \tag{8.18}$$

where $G = \mathrm{FT}(g)$, etc.

Therefore, setting $g = \Delta(r)$ and $h = \Delta(r)$,

$$\xi(r) = \langle\Delta(\mathbf{x})\Delta(\mathbf{x}+\mathbf{r})\rangle \tag{8.10}$$

$$= \frac{V}{(2\pi)^3}\int |\Delta_k|^2 \exp(i\,\mathbf{k}\cdot\mathbf{r})\;\mathrm{d}^3k \tag{8.19}$$

The power spectrum and ξ are Fourier transform pairs.

(remember Eq. 8.14, $P(k) = \Delta_k^2$!)

See Peebles (1980, sect. 31) for 100s of pages of the properties of ξ, P, etc.

Power Spectrum, III

To better understand ξ and P, assume isotropy for the moment...

We had

$$\xi(\mathbf{r}) \propto \int P(\mathbf{k}) \exp(i\,\mathbf{k}\cdot\mathbf{r})\,\mathrm{d}^3k \tag{8.19}$$

Spherical coordinates in k space:

$$\mathbf{k}\cdot\mathbf{r} = kr\cos\theta \tag{8.20}$$

$$\mathrm{d}V = k^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathrm{d}k \tag{8.21}$$

such that

$$\xi(r) \propto \int_0^\infty \int_0^\pi \int_0^{2\pi} P(k)\exp(ikr\cos\theta)k^2\sin\theta\,\mathrm{d}\phi\,\mathrm{d}\theta\,\mathrm{d}k \tag{8.22}$$

$$= 2\pi \int_0^\infty \int_0^\pi \xi(r)\,\exp(ikr\cos\theta)\,r^2\,\mathrm{d}(\cos\theta)\,\mathrm{d}r \tag{8.23}$$

$$= \frac{V}{2\pi^2}\int_0^\infty P(k)\frac{\sin kr}{kr}\,\mathrm{d}r \tag{8.24}$$

(the last eq. is exact).

For $kr < \pi$: $\sin kr/kr > 0$, while oscillation for $kr > \pi$

$\Longrightarrow$ only wavenumbers $k \lesssim r^{-1}$ contribute to amplitude on scale r.

Since P and ξ are FT pairs, a similar relation holds in the other direction.

Power Spectrum, IV

For a power law spectrum,

$$P(k) \propto k^n \tag{8.25}$$

the correlation function is

$$\begin{aligned} \xi(r) &\propto \int_0^\infty \frac{\sin kr}{kr} k^{n+2}\,\mathrm{d}k \\ &\sim \int_0^{1/r} k^{n+2}\,\mathrm{d}k \\ &\propto r^{-(n+3)} \end{aligned} \tag{8.26}$$

Mass within fluctuation is $M \sim \rho r^3$, i.e., the mass fluctuation spectrum is

$$\xi(M) \propto M^{-(n+3)/3} \tag{8.27}$$

and the rms density fluctuation at mass scale M is

$$\frac{\delta\rho}{\rho} = \xi(M)^{1/2} \propto M^{-(n+3)/6} \tag{8.28}$$

For $n > -3$, the rms mass fluctuations decrease with $M \Longrightarrow$ isotropic universe on largest scales

Power Spectrum, V

What spectra would we expect?

Two simple cases:

Poisson noise: Random statistical fluctuations in number of particles on scale r:

$$\frac{\delta N}{N} = \frac{1}{N} \quad \Longrightarrow \quad \frac{\delta M}{M} = \frac{1}{M} \tag{8.29}$$

and therefore $n = 0$ ($\rho \propto M$!) ("white noise").

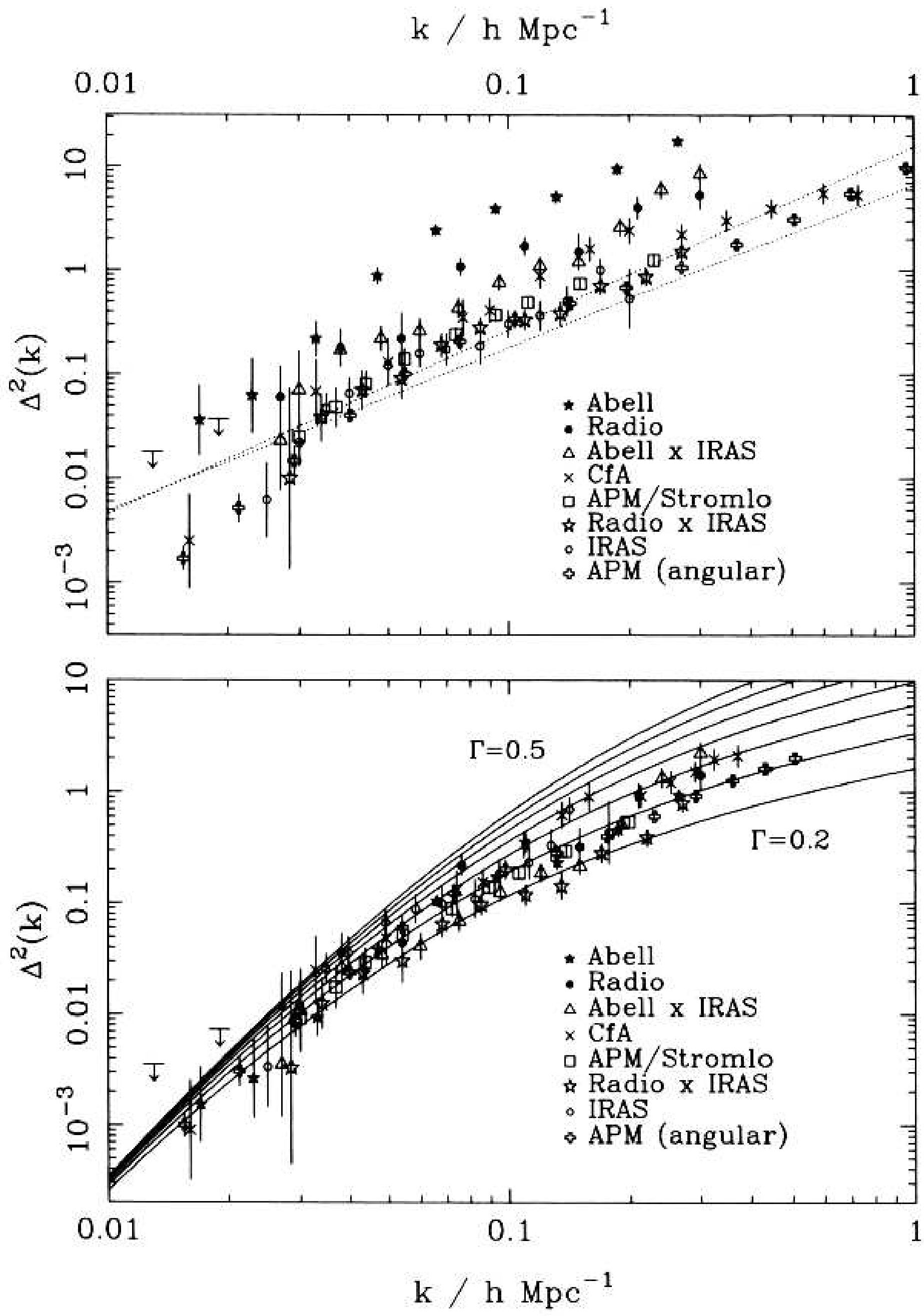
Zeldovich spectrum: defined by $n = 1$. Thus

$$\frac{\delta\rho}{\rho} \propto M^{-2/3} \tag{8.30}$$

. . . will be important later

The Zeldovich spectrum is the spectrum expected for the case when initial density fluctuations coming through the horizon had the same amplitude.

Power Spectrum, VI



(Peacock, 1999, Fig. 16.4)

Measured power spectrum is more complicated

$\Longrightarrow$ Structure formation to understand details!

Structure formation: Linear Theory, I

Structure formation = evolution of overdensity in universe with time.

Describe density and scale factor wrt normal expansion:

$$\rho(t) = \bar{\rho}(t) \cdot (1 + \delta(t)) \tag{8.31}$$
$$a(t) = \bar{a}(t) \cdot (1 - \epsilon(t)) \tag{8.32}$$

Sign:
$\delta > 0 \Longrightarrow$ *Overdensity*
$\epsilon > 0 \Longrightarrow$ *collapse*

Seek mathematical model for collapse of gravitating material in expanding universe
$\Longrightarrow$ identical to Friedmann equation!
$\Longrightarrow$ Equation describing structure formation:

$$\dot{a}(t) = \frac{8\pi G}{3} \rho(t) a^2(t) + H_0^2 (1 - \Omega_0) \tag{8.33}$$

Drop explicit t dependency in the following

Structure formation: Linear Theory, II

Onset of structure formation:

linear regime: $\delta(t)$, $\epsilon(t) \ll 1$

$\Longrightarrow$ Ignore all higher combinations of δ and ϵ.

Left side of Friedmann:

$$\dot{a}^2 = (\dot{\bar{a}} - \dot{\bar{a}}\epsilon - \bar{a}\dot{\epsilon})^2 \tag{8.34}$$

$$= \dot{\bar{a}}^2 - 2\dot{\bar{a}}^2\epsilon - 2\bar{a}\dot{\bar{a}}\,\dot{\epsilon} \tag{8.35}$$

$$= \dot{\bar{a}}^2 - 2\dot{\bar{a}}\frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\epsilon) \tag{8.36}$$

Right side of Friedmann:

$$\frac{8\pi G}{3}\bar{\rho}(1+\delta)\bar{a}^2(1-\epsilon)^2 + H_0^2(1-\Omega_0) \tag{8.37}$$

$$= \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(1+\delta)(1-2\epsilon) + H_0^2(1-\Omega_0) \tag{8.38}$$

$$= \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(1+\delta-2\epsilon) + H_0^2(1-\Omega_0) \tag{8.39}$$

Now Eq. (8.36)=Eq. (8.39), and subtract terms from Friedmann Equation (eq. 8.33):

$$2\dot{\bar{a}}\cdot\frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\epsilon) = \frac{8\pi G}{3}\bar{\rho}\bar{a}^2(\delta-2\epsilon) \tag{8.40}$$

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Structure formation: Linear Theory, III

To solve Eq. (8.40): Assume for simplicity $\Omega = 1$, matter-dominated universe.

Matter domination $\Longrightarrow$ $\rho a^3 =$ const. $\Longrightarrow$

$$\bar{\rho}(1+\delta)\bar{a}^3(1-\epsilon)^3 \sim \bar{\rho}\bar{a}^3(1-3\epsilon+\delta) \overset{!}{=} \text{const.} \tag{8.41}$$

and therefore

$$\epsilon = \delta/3 \tag{8.42}$$

$\Longrightarrow$ Eq. (8.40) becomes

$$2\dot{\bar{a}} \cdot \frac{\mathrm{d}}{\mathrm{d}t}(\bar{a}\delta) = \frac{8\pi G}{3}\bar{\rho}\bar{a}^2\delta \tag{8.43}$$

In a $k = 0$ universe,

$$\bar{a}(t) = \left(\frac{3H_0}{2}\,t\right)^{2/3} =: a_0 t^{2/3} \tag{4.77}$$

and because of $\rho a^3 =$ const.,

$$\bar{\rho}(t) \propto t^{-2} =: \rho_0 t^{-2} \tag{8.44}$$

Structure formation: Linear Theory, IV

Insert $\bar{a}$, $\bar{\rho}$ into Eq. (8.43):

$$\frac{4a_0}{3}t^{-1/3}\left(\frac{2a_0}{3}t^{-1/3}\delta + a_0 t^{2/3}\dot{\delta}\right) = \frac{8\pi G}{3}\rho_0 t^{-2} a_0^2 t^{4/3}\delta \tag{8.45}$$

and simplify

$$t^{-2/3}\delta + t^{1/3}\dot{\delta} = 2\pi G\rho_0 t^{-2/3}\delta \tag{8.46}$$

$$t\dot{\delta} + (1 - 2\pi G\rho_0)\delta = 0 \tag{8.47}$$

The general solution of Eq. (8.47) is a power-law
$\Longrightarrow$ Growth of structure!

Since also *negative* PL indexes possible $\Longrightarrow$ Some initial perturbations are damped out!

Need better theory to do that in detail. . .

Structure formation: Linear Theory, V

Better linear theory: Use linearized equations of motion from hydrodynamics to compute gravitational collapse

Detailed theory very difficult

see handout for a few ideas of what is going on...

Classical approach:

Consider sphere of material:

Potential energy of sphere:

$$U = -\frac{1}{2}\int \rho(x)\Phi(x)\,\mathrm{d}^3x \sim -\frac{16\pi^2}{15}G\rho^2 r^5 \tag{8.48}$$

Total kinetic energy content:

$$T \sim \frac{c_\mathrm{s}^2}{2}\frac{4\pi r^3\rho}{3} \tag{8.49}$$

c_s: speed of sound; for neutral Hydrogen, $c_\mathrm{s} = \sqrt{5T/3m_\mathrm{p}}$.

Sphere collapses if $|U| > T$, i.e., when

$$2r \gtrsim \sqrt{\frac{5}{2\pi}}\sqrt{\frac{c_\mathrm{s}^2}{G\rho}} \sim c_\mathrm{s}\sqrt{\frac{\pi}{G\rho_0}} =: \lambda_\mathrm{J} \tag{8.50}$$

λ_J is called the Jeans length, the corresponding mass is the Jeans mass,

$$M_\mathrm{J} = \frac{\pi}{6}\rho\lambda_\mathrm{J}^3 \tag{8.51}$$

Structures with $m < M_\mathrm{J}$ cannot grow.

Note that c_s is time dependent $\Longrightarrow M_\mathrm{J}$ can change with time!

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A better derivation of the Jeans length comes from considering the evolution of a fluid in an expanding universe. Assuming that the initial density perturbations were small, we can use perturbation theory for obtaining deviations from homogeneity (=structures).

In a Friedmann universe, for length scales $< 1/H$, dynamical equations are Newtonian to first order, but we need to still use the scale factor, $a(t)$ in the fluid equations.

Continuity equation:

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{8.52}$$

Euler's equation:

$$\dot{\mathbf{v}} + (v \cdot \nabla)v = -\nabla \left(\Phi + \frac{\rho}{c} \right) \tag{8.53}$$

Poisson's equation:

$$\nabla^2 \Phi = 4\pi G \rho \tag{8.54}$$

Without perturbations (i.e., the zeroth order solution) is given by the normal Friedmann solutions:

$$\rho_0(t, \mathbf{r}) = \frac{\rho_0}{a^3(t)} \quad \text{(dilution by expansion)} \tag{8.55}$$

$$\mathbf{v}_0(t, \mathbf{r}) = \frac{\dot{a}(t)}{a(t)} \mathbf{r} \quad \text{(Hubble law)} \tag{8.56}$$

$$\Phi_0(t, \mathbf{r}) = \frac{2\pi G \rho_0 r^2}{3} \quad \text{(soln. of Poisson with } \rho = \text{const.)} \tag{8.57}$$

Convert into comoving coordinates ($\mathbf{x} = \mathbf{r}/a(t)$) to get rid of the $a(t)$'s and write down perturbation equations:

$$\rho(t, \mathbf{x}) = \rho_0(t) + \rho_1(t) =: \rho_0(t) \left(1 + \delta(t, \mathbf{x})\right) \tag{8.58}$$

$$\mathbf{v}(t, \mathbf{x}) = \mathbf{v}_0(t, \mathbf{x}) + \mathbf{v}_1(t, \mathbf{x}) \tag{8.59}$$

$$\Phi(t, \mathbf{x}) = \Phi_0(t, \mathbf{x}) + \Phi_1(t, \mathbf{x}) \tag{8.60}$$

where $|\delta|$, $|\mathbf{v}_1|$, $|\Phi_1|$ small (δ is called density perturbation field).

Since the equations are spatially homogeneous, we can Fourier transform them to search for plane wave solutions. The general perturbation solution can then later be found by performing linear combinations of these plane waves.

$$\delta(t, \mathbf{x}) = \frac{1}{(2\pi)^3} \int e^{i\mathbf{k}\cdot\mathbf{x}} \delta(t, \mathbf{k}) d^3k \iff \delta(t, \mathbf{k}) = \int e^{-i\mathbf{k}\cdot\mathbf{x}} \delta(t, \mathbf{x}) d^3x \tag{8.61}$$

Inserting into hydro equations gives

$$\ddot{\delta}(t, \mathbf{k}) + 2\frac{\dot{a}(t)}{a(t)} \dot{\delta}(t, \mathbf{k}) + \left(\frac{k^2 c_s^2}{a^2(t)} - 4\pi G \rho_0 \right) \delta(t, \mathbf{k}) = 0 \tag{8.62}$$

where the sound speed is $c_s^2 = (\partial p / \partial \rho)_{\text{adiabatic}}$.

Solutions to eq. 8.62 grow or decrease depending on sign of

$$\kappa_J = \left(\frac{k^2 c_s^2}{a^2(t)} - 4\pi G \rho_0 \right) \tag{8.63}$$

Thus, growth is only possible for $k > k_\mathrm{J}$ where

$$k_\mathrm{J} = \sqrt{\frac{4\pi G\rho_0 a^2(t)}{c_\mathrm{s}^2}} \tag{8.64}$$

or, in terms of physical wavelengths,

$$\lambda_\mathrm{J} = \frac{2\pi a(t)}{k_\mathrm{J}} = c_\mathrm{s}\sqrt{\frac{\pi}{G\rho_0}} \tag{8.65}$$

the Jeans length.

Stages of Structure Formation

Early universe: radiation dominates:

$$c_s = c/\sqrt{3} \quad \text{and} \quad \rho_r c^2 = \sigma T^4 \tag{8.66}$$

and therefore

$$\lambda_{J,rad} = c^2\sqrt{\pi}3G\sigma T^4 \propto a^2 \quad \text{and} \quad M_J \propto \rho_m \lambda_{J,rad}^3 \propto a^3 \tag{8.67}$$

In the early universe, the Jeans mass grows quickly.

At time of radiation – matter equilibrium,

$$\rho_m = \rho_{rad} = \sigma T_{eq}^4/c^2 \tag{8.68}$$

and

$$M_J(t_{eq}) = \frac{\pi^{5/2}}{18\sqrt{3}} \frac{c^4}{G^{3/2}\sigma^{1/2}} \frac{1}{T_{eq}} \sim \frac{3.6 \times 10^{16}(\Omega_0 h^2)^{-2}\, M_\odot}{(T/T_{eq})^3} \tag{8.69}$$

assuming $1 + z_{eq} = 24000\,\Omega_0 h^2$.
$\Longrightarrow$ much larger than mass in galaxy cluster (about mass in cube with 50 Mpc side length $\Longrightarrow$ size of voids!)

Overdense regions with $m < M_{J,rad}$ are smoothed out by the radiation coupling to matter.

Much larger structures also cannot grow since λ is larger than horizon radius $\Longrightarrow$ Mass spectrum of possible structures.

Stages of Structure Formation

After t_{eq} not much happens until $T_{\text{rec}} \sim 3000\,\text{K}$

$\Longrightarrow$ recombination

$\Longrightarrow$ Sound speed drops dramatically (radiation and matter decouple):

$$c_{\text{s}} \sim \frac{kT}{m_{\text{p}}} \sim 5\,\text{km}\,\text{s}^{-1} \tag{8.70}$$

$\Longrightarrow$ M_{J} drops by 10^{11}:

$$M_{\text{J,eq}} = \frac{\pi\bar{\rho}}{6}\left(\frac{\pi k T_{\text{rec}}}{G\bar{\rho}m_{\text{p}}}\right)^{1/2} \sim 5 \times 10^5 (\Omega_0 h^2)^{-1/2}\, M_\odot \tag{8.71}$$

after that, M_{J} drops because of expansion.

So, in pure matter universe:

- at begin: huge structures form (Zeldovich pancakes)
- suddenly at recombination: fragmentation

$\Longrightarrow$ top-down model

Problem: Not really what has been observed

Solution: Dark matter

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Stages of Structure Formation

Structure formation with dark matter:
DM unaffected by radiation pressure $\Longrightarrow$ collapse of smaller structures possible $\Longrightarrow$ bottom-up model

As long as DM relativistic:

$$M_{\text{J,HDM}} = \frac{\pi \rho_{\text{DM}}}{6} \left(\frac{\pi c_{\text{DM}}}{G \rho_{\text{DM}}} \right)^{3/2} \tag{8.72}$$

Hot Dark Matter: $c_{\text{HDM}} \sim c/\sqrt{3}$
Cold Dark Matter: $c_{\text{CDM}} \ll c/\sqrt{3}$
Standard CDM Scenario:

- DM cools long before t_{rec}
- CDM structures form, M_{J} about galaxy mass, while baryons coupled to radiation $\Longrightarrow$ stays smooth
- t_{rec}: matter decouples, falls in DM gravity wells

CDM "seeds" structures!

Gives not exactly observed power spectrum $\Longrightarrow$ Currently preferred: combination of CDM and ΛDM

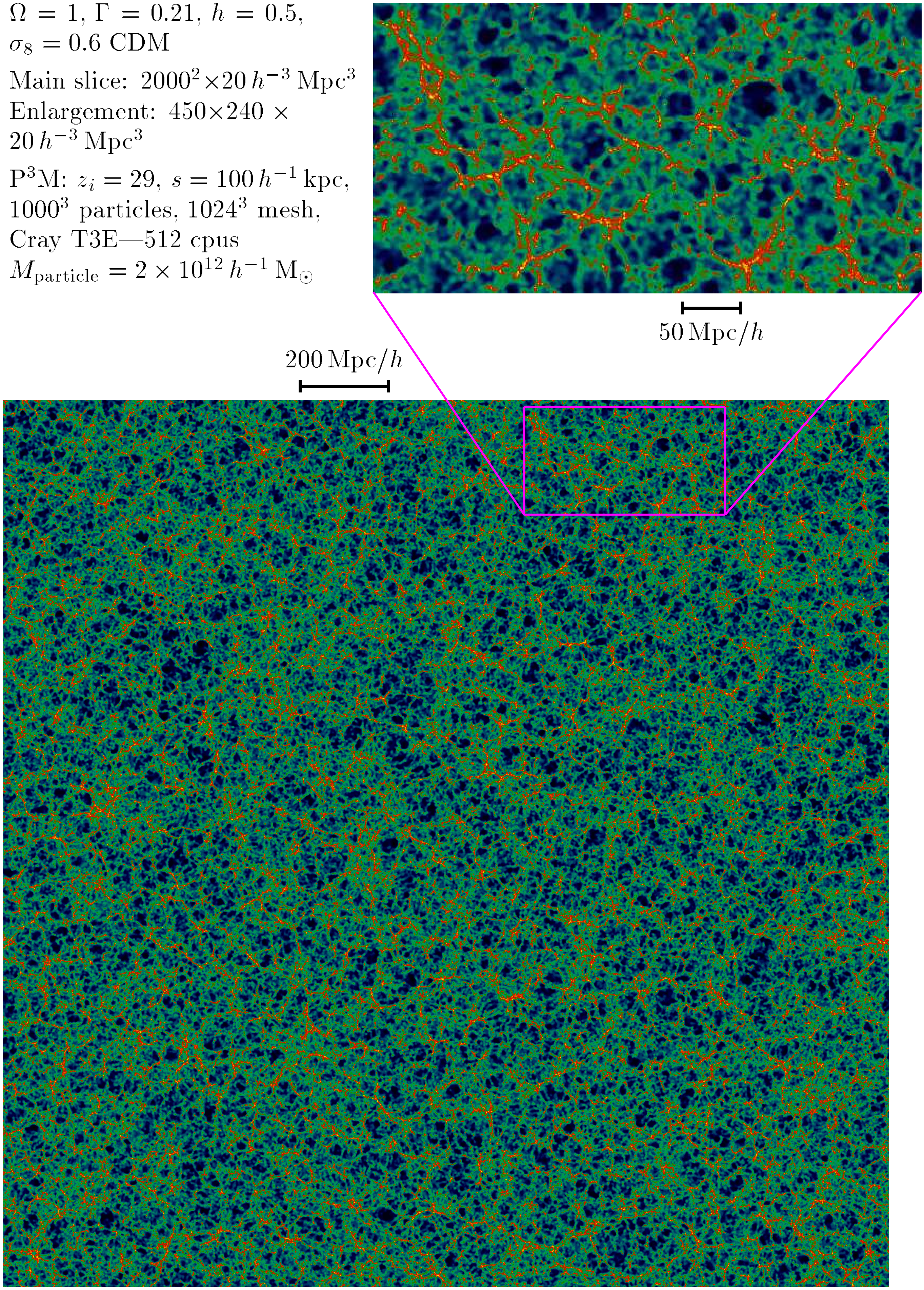
Stages of Structure Formation

Finally, the *real* linear theory has to be done in linearized or even full general relativity
$\Longrightarrow$ very, very complicated.

Full fledged, detailed structure formation is mainly done numerically.

N-body codes: describe particles (=galaxies) as points, compute mutual interactions in expanding universe
Requires massive computing power.
VIRGO consortium: U.S.A., Canada, Germany, UK
Hubble Volume Simulation: Garching T3E (512 processors), 70 h CPU time
Show some results on following slides and movies.

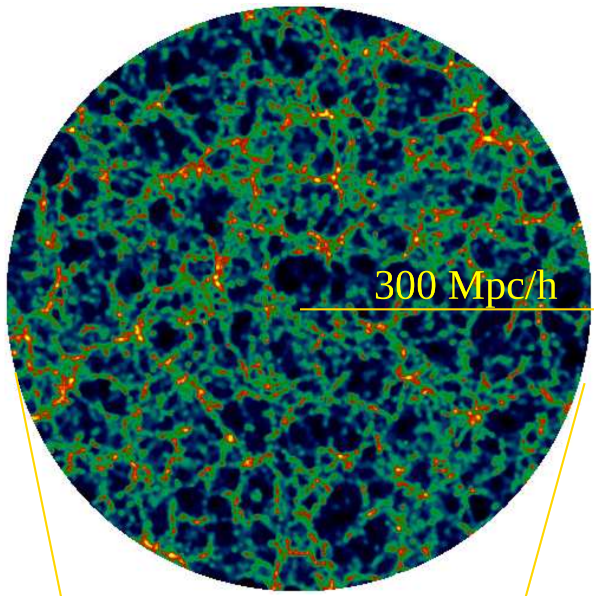
`http://www.mpa-garching.mpg.de/~virgo/virgo/`

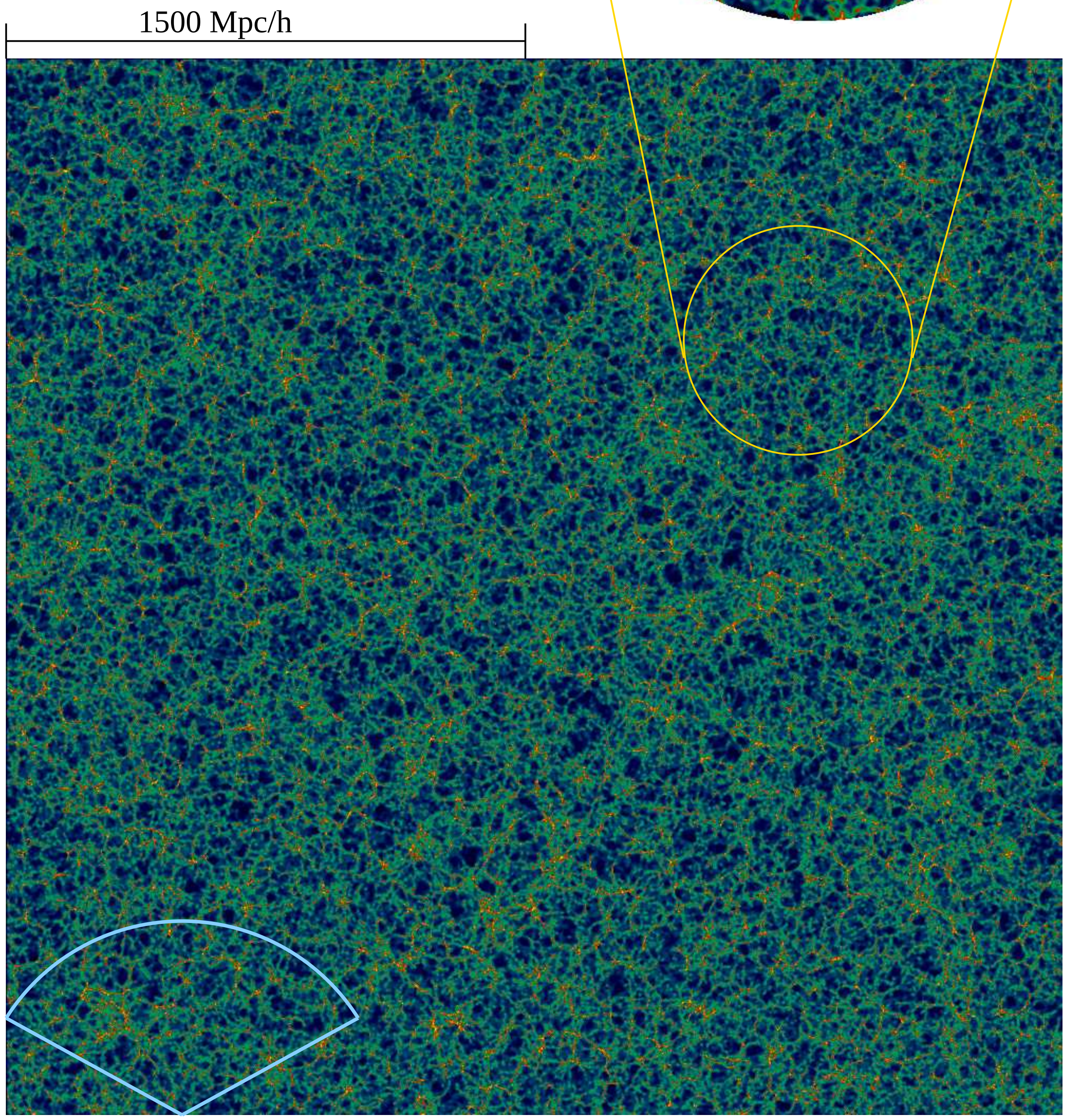


Standard CDM

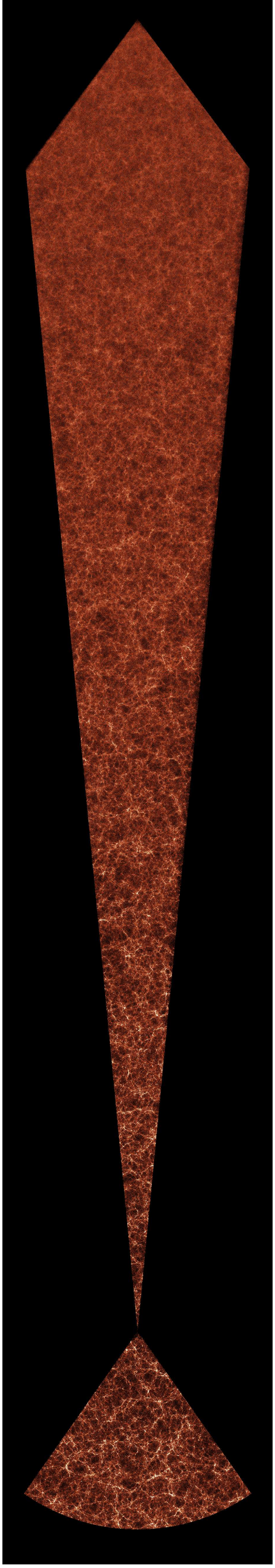
The Hubble Volume Simulation

$\Omega=0.3, \Lambda=0.7, h=0.7,$
$\sigma_8=0.9$ (ΛCDM)
3000 x 3000 x 30 h^{-3}Mpc^3
P^3M: $z_i=35$, $s=100\ h^{-1}$kpc
1000^3 particles, 1024^3 mesh
T3E(Garching) - 512cpus
$M_{\mathbf{particle}} = 2.2 \times 10^{12} h^{-1} M_{\mathbf{sol}}$

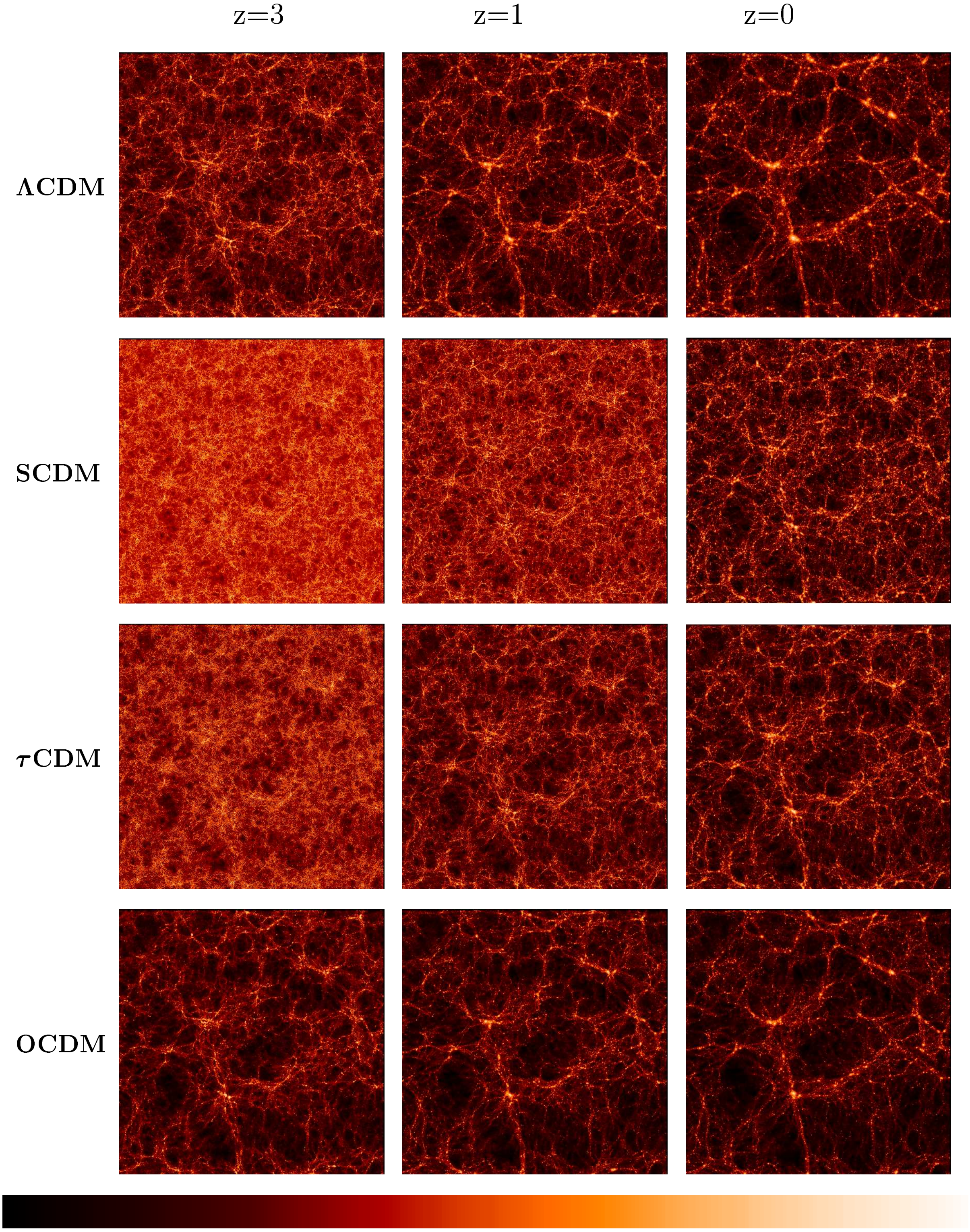




ΛDM

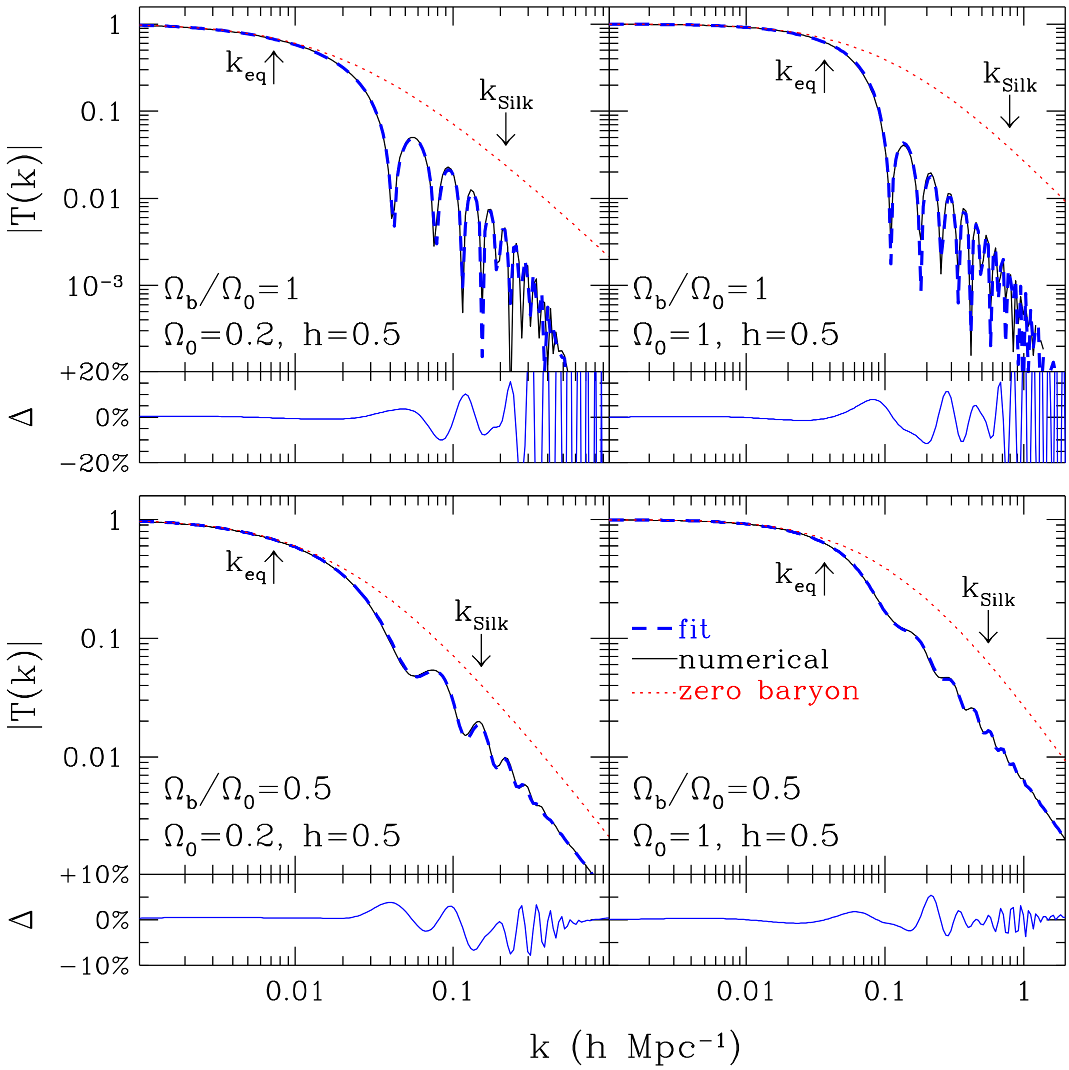


Evolution of clustering along light cone



The VIRGO Collaboration 1996

Formal Structure Formation



Eisenstein & Hu, 1997

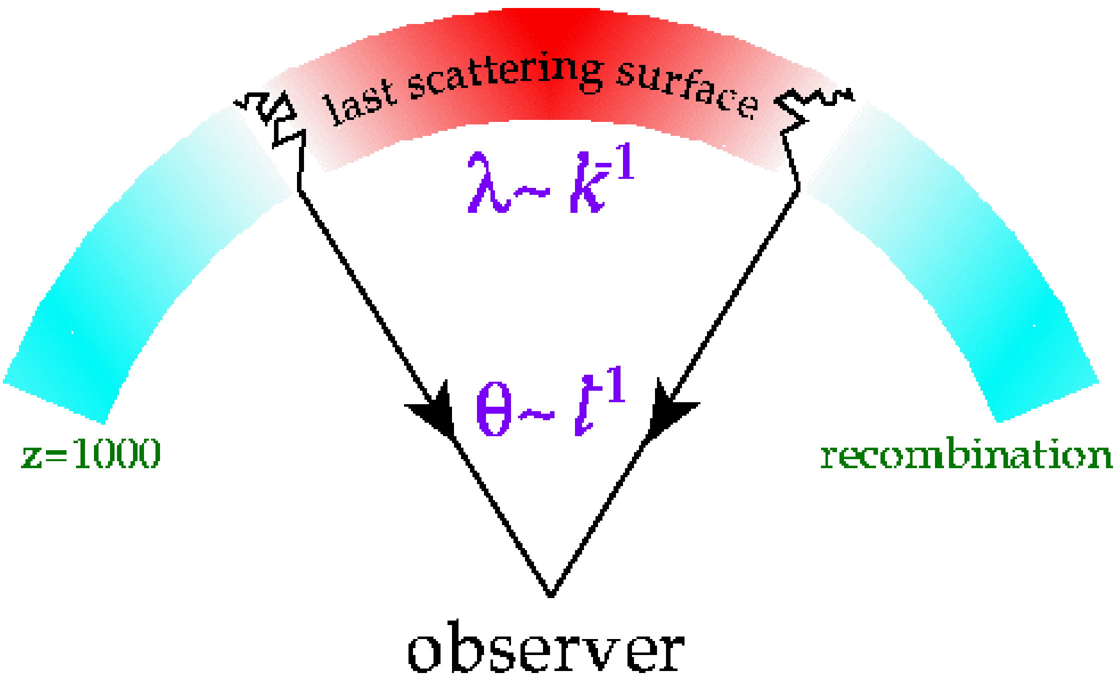
Computation of real power spectra difficult: growth under self-gravitation pressure effects dissipation.

To predict observations from today: define transfer function

$$\delta_k(z=0) = \delta_k(z) D(z) T_k \tag{8.73}$$

But: need initial conditions, $\delta_k(z)$!

CMBR



courtesy Wayne Hu

Matter and Radiation are coupled, i.e., large mass density $=$ high photon density.

Photons from overdense regions: gravitational redshift $\Longrightarrow$ observable!

(Sachs Wolfe Effect)

CMBR: Radiation from surface of last scattering

CMBR Fluctuations trace gravitational potential at $z \sim 1100$!

CMBR

Temperature fluctuations:

$$\frac{\Delta T}{T} \sim \frac{\Delta\Phi_{\mathrm{g}}}{c^2} \tag{8.74}$$

where

$$\Delta\Phi_{\mathrm{g}} \sim -\frac{2G\Delta M}{R} = \frac{8\pi G}{3}\bar{\rho}R^2\delta \tag{8.75}$$

$$= -\delta(t)\,(H(t)R)^2 \tag{8.76}$$

Current angle of region on sky:

$$\alpha \sim R/d_{\mathrm{A}} \tag{8.77}$$

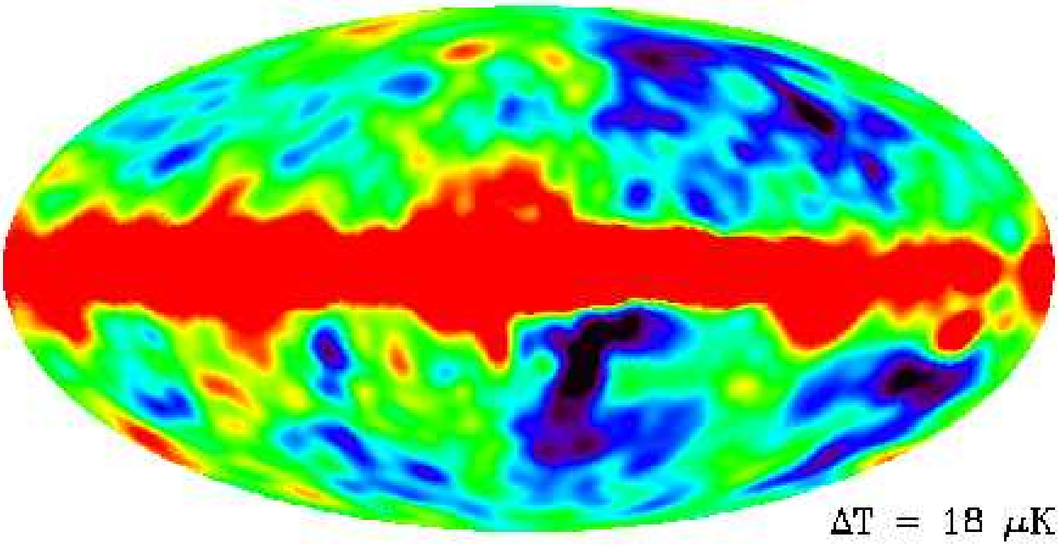
where the angular diameter distance

$$d_{\mathrm{A}} = d_{\mathrm{L}}/(1+z)^2 \tag{8.78}$$

Therefore:

$$\boxed{\frac{\Delta T}{T} \sim \frac{\Delta\Phi_{\mathrm{g}}}{c^2} \propto -\delta\alpha^2} \tag{8.79}$$

CMBR



Eqs. (8.76) and (8.79) imply

$$\frac{\Delta T}{T} \sim -\frac{\delta\alpha^2}{3} \qquad (8.80)$$

Quotient 3 from more detailed theory, "Integrated Sachs Wolfe effect"

COBE: Resolution $\alpha \sim 7^\circ$ (corresponds to $\sim 10^{20}\,M_\odot$ at recombination)

COBE results imply $\delta \sim 10^{-3}$ at recombination

This is small for pure matter dominated universe

$\Longrightarrow$ Implies existence of dark matter!

CMBR

Expand CMB fluctuations on sky in spherical harmonics:

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell,m} a_{\ell,m} Y_{\ell,m}(\theta, \phi) \tag{8.81}$$

Since rotationally symmetric, can express variation in terms of multipole coefficients, C_ℓ:

$$C(\theta) = \frac{1}{4\pi} \sum_{\ell} \sum_{m=-\ell}^{+\ell} |a_{\ell,m}| P_\ell(\cos\theta) \tag{8.82}$$

$$=: \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_\ell P_\ell(\cos\theta) \tag{8.83}$$

where $C(\theta) = \langle \Delta T/T \rangle$ and where the P_ℓ are the Legendre polynomials.

CMBR

Expect following features:

Large angle anisotropy: (small ℓ, scales $\gtrsim$ horizon at decoupling): Flat part due to Sachs-Wolfe effect

Smaller angular scales: (larger ℓ): Influenced by photon-baryon interactions: Matter falls in potential well

$\Longrightarrow$ Pressure resists

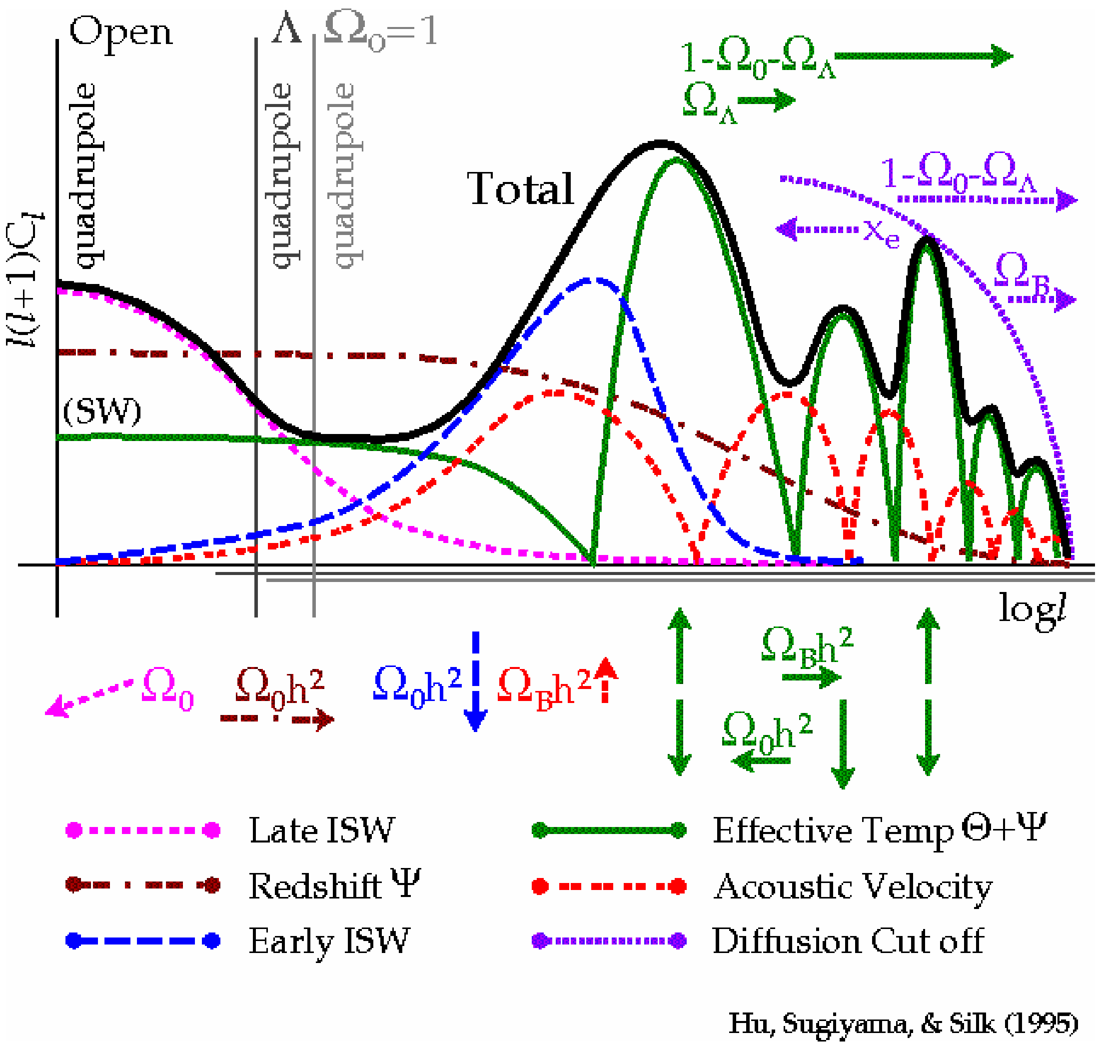
$\Longrightarrow$ acoustic oscillations

$\Longrightarrow$ Power at selected scales!

Power from those density fluctuations which had their maximum amplitude at time of last scattering dominates $\Longrightarrow$ acoustic peak

Also damping from photon diffusion (Compton scattering; Silk damping [after Joseph Silk])

CMBR



Location and strength of acoustic peaks dependent on

$$\Omega_b \quad H_0 \quad \Omega_0$$

Position of acoustic peak not observed with COBE (at smaller scale than 7°)

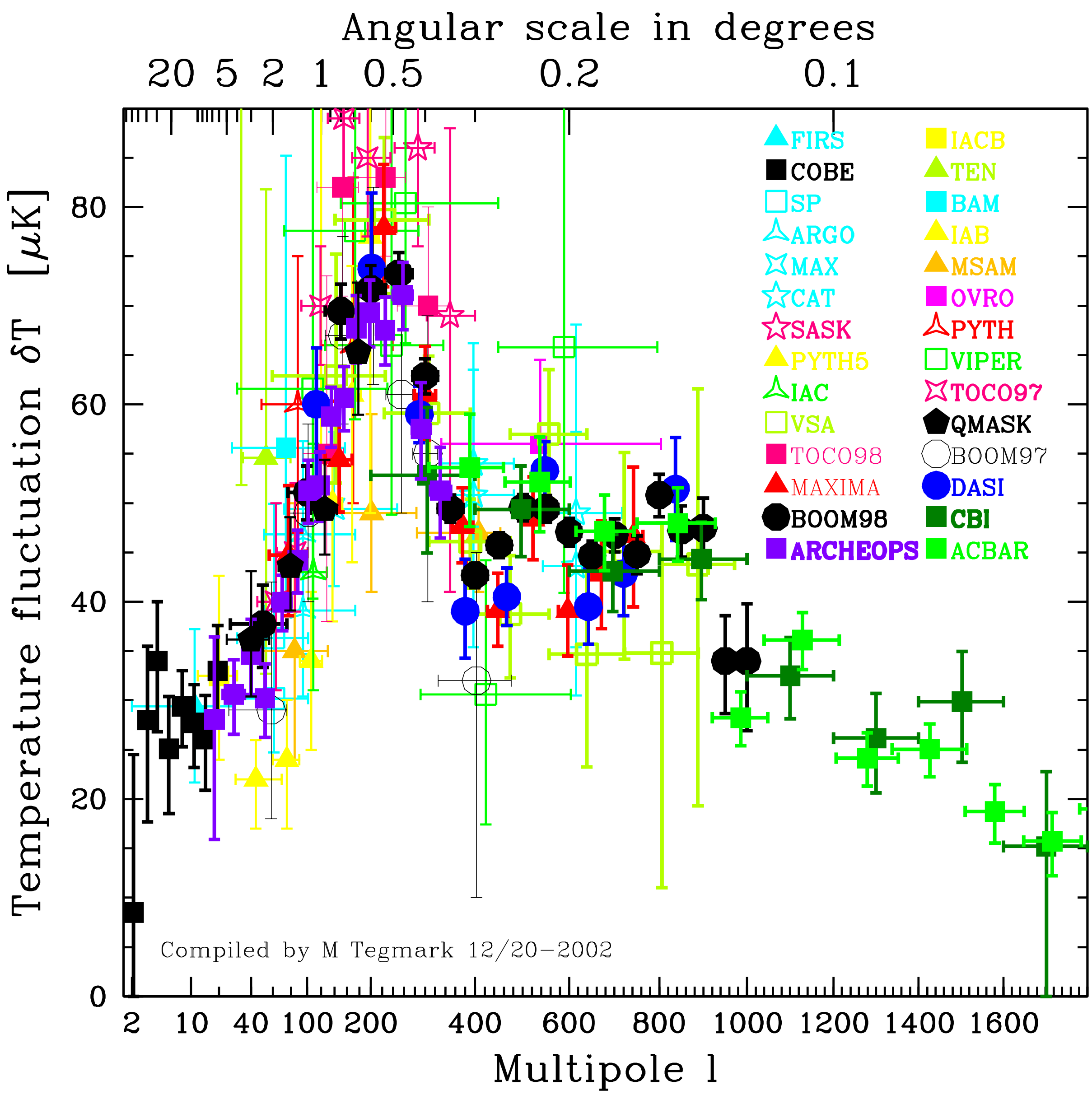
courtesy BOOMERANG team

Enter: BOOMERANG (Balloon Observations of Milimetric Extragalactic Radiation and Geophysics), Flight in Antarctica 1998 December 29 – 1999 January 9

BOOMERANG before Mt. Erebus; courtesy BOOMERANG team

Other balloon missions: MAXIMA-1,. . .

Summary: Pre-WMAP



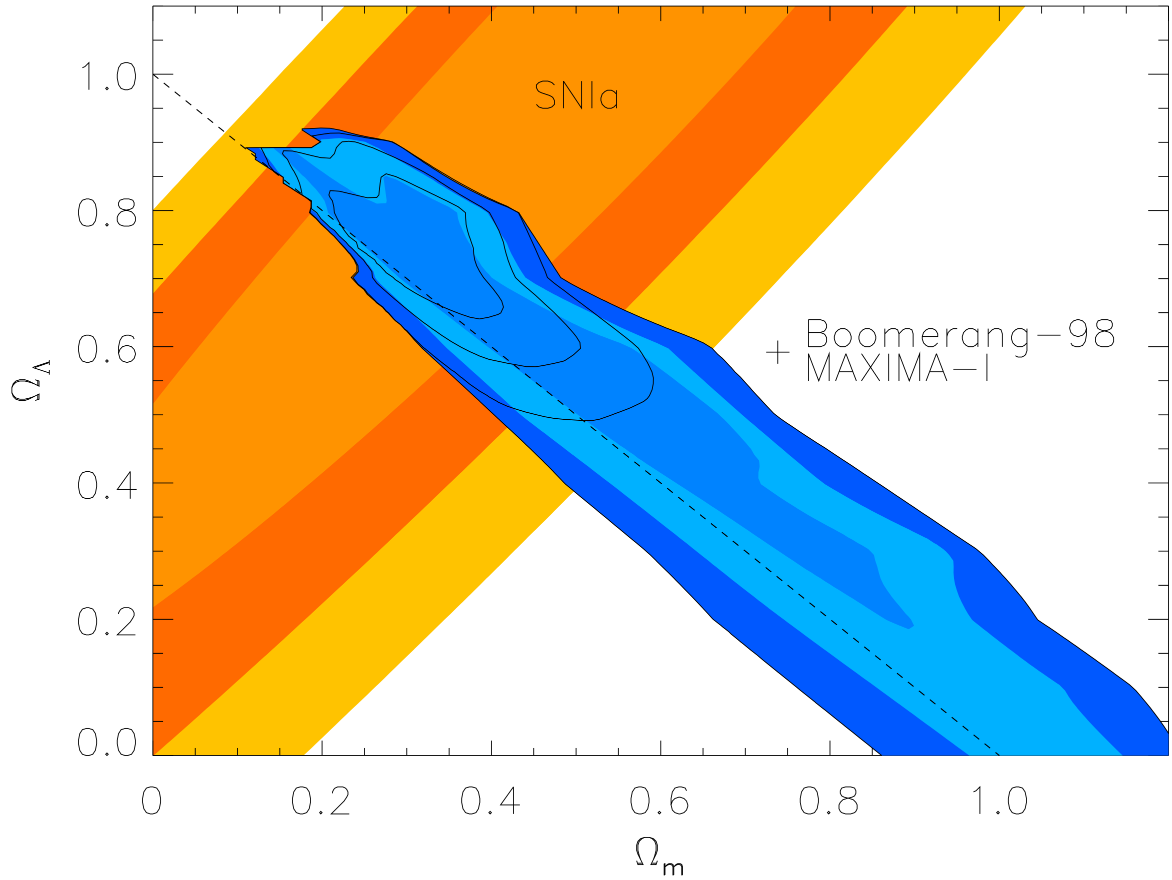
Courtesy M. Tegmark

1st acoustic peak found by BOOMERANG in 1999

(Jaffe et al., 2000)

. . . confirmed by many experiments since then

Summary: Pre-WMAP



(Jaffe et al., 2000, black contours: incl. Large Scale Structure)

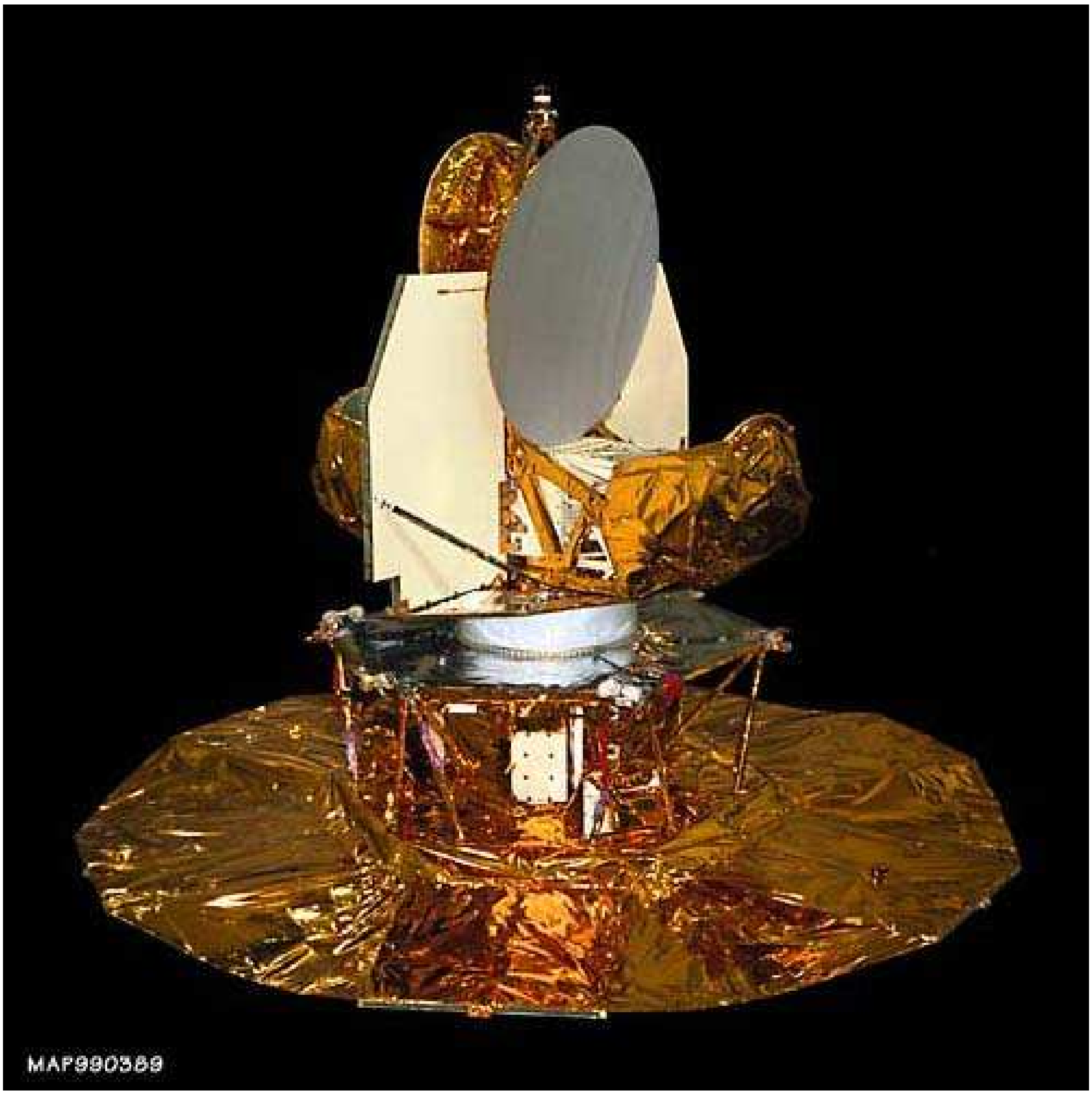
General summary of CMB fluctuations (COBE, BOOMERANG, MAXIMA):

$$\Omega_{\mathrm{tot}} \simeq 1.11 \pm 0.07 \left({}^{+0.13}_{-0.12}\right) \tag{8.84}$$

and

$$\Omega_b h^2 \simeq 0.032^{+0.005}_{-0.004} \left({}^{+0.009}_{-0.008}\right) \tag{8.85}$$

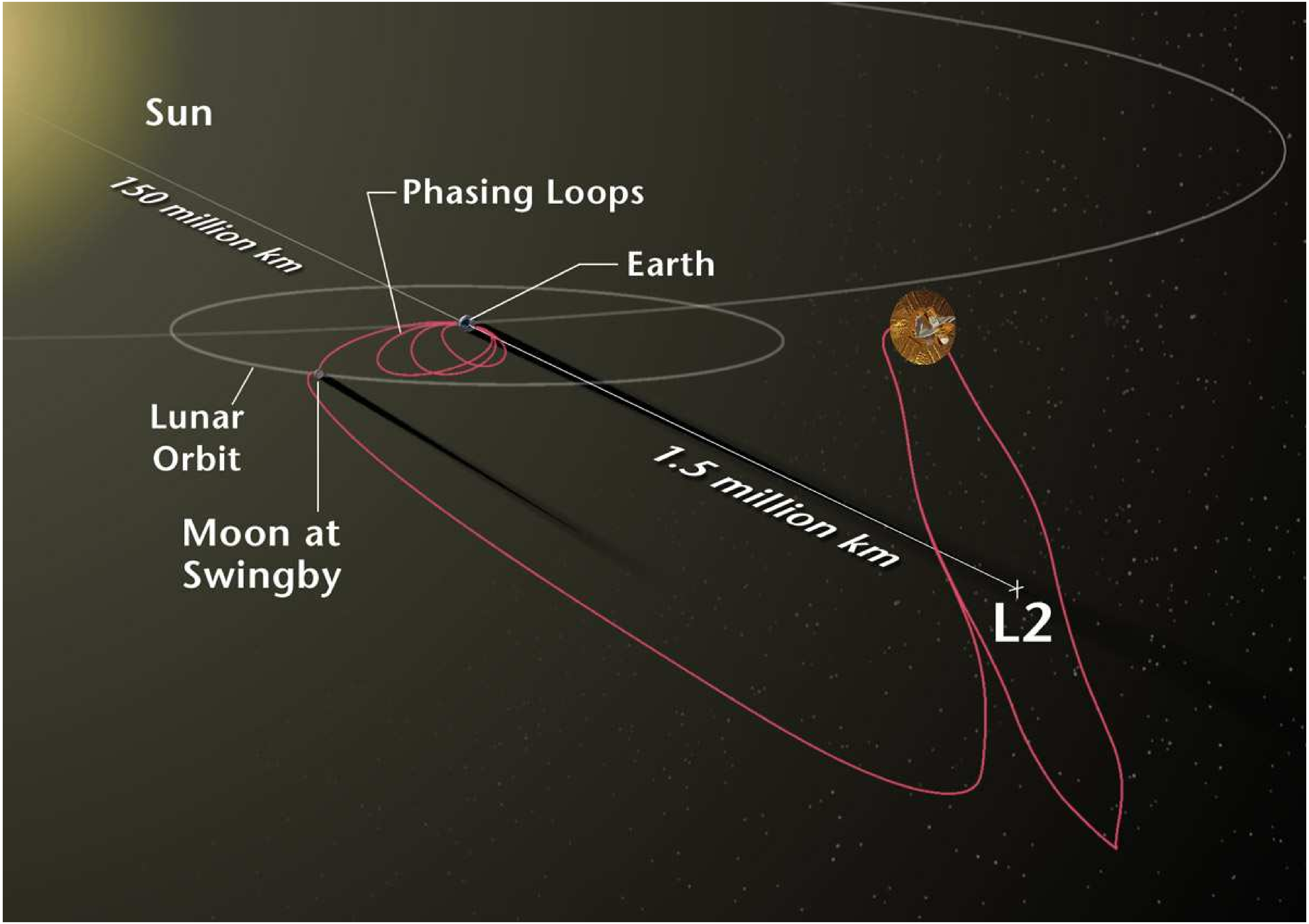
WMAP

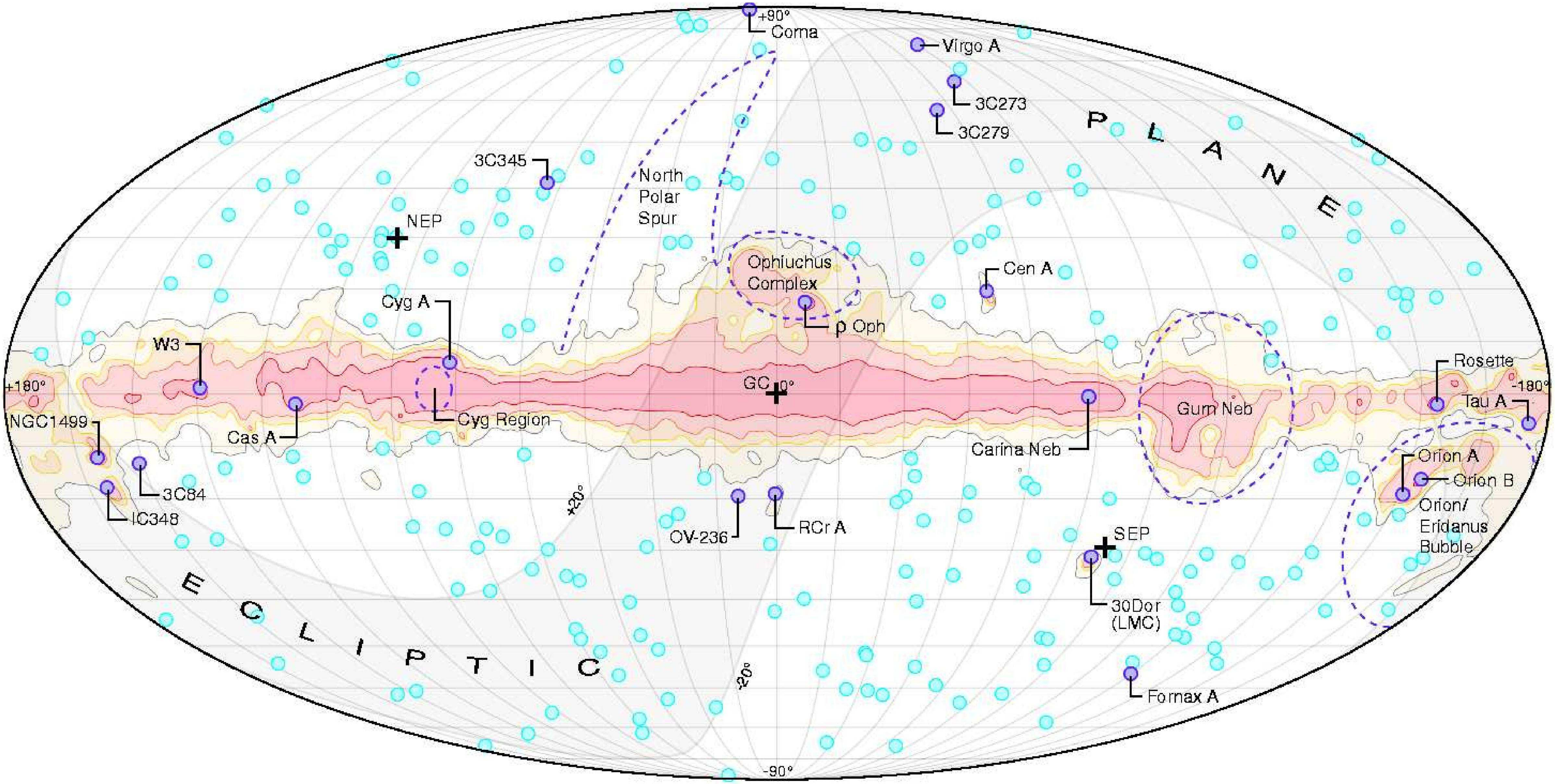


Wilkinson Microwave Anisotropy Probe (WMAP):

- Launched 2001 June 30, measurements began 2001 August 10
- Orbit around 2nd Lagrange Point of Sun-Earth System
- Highly precise radiometers of high spatial resolution (best: 0.21° FWHM in W-Band at 3.2 mm) in five wavebands

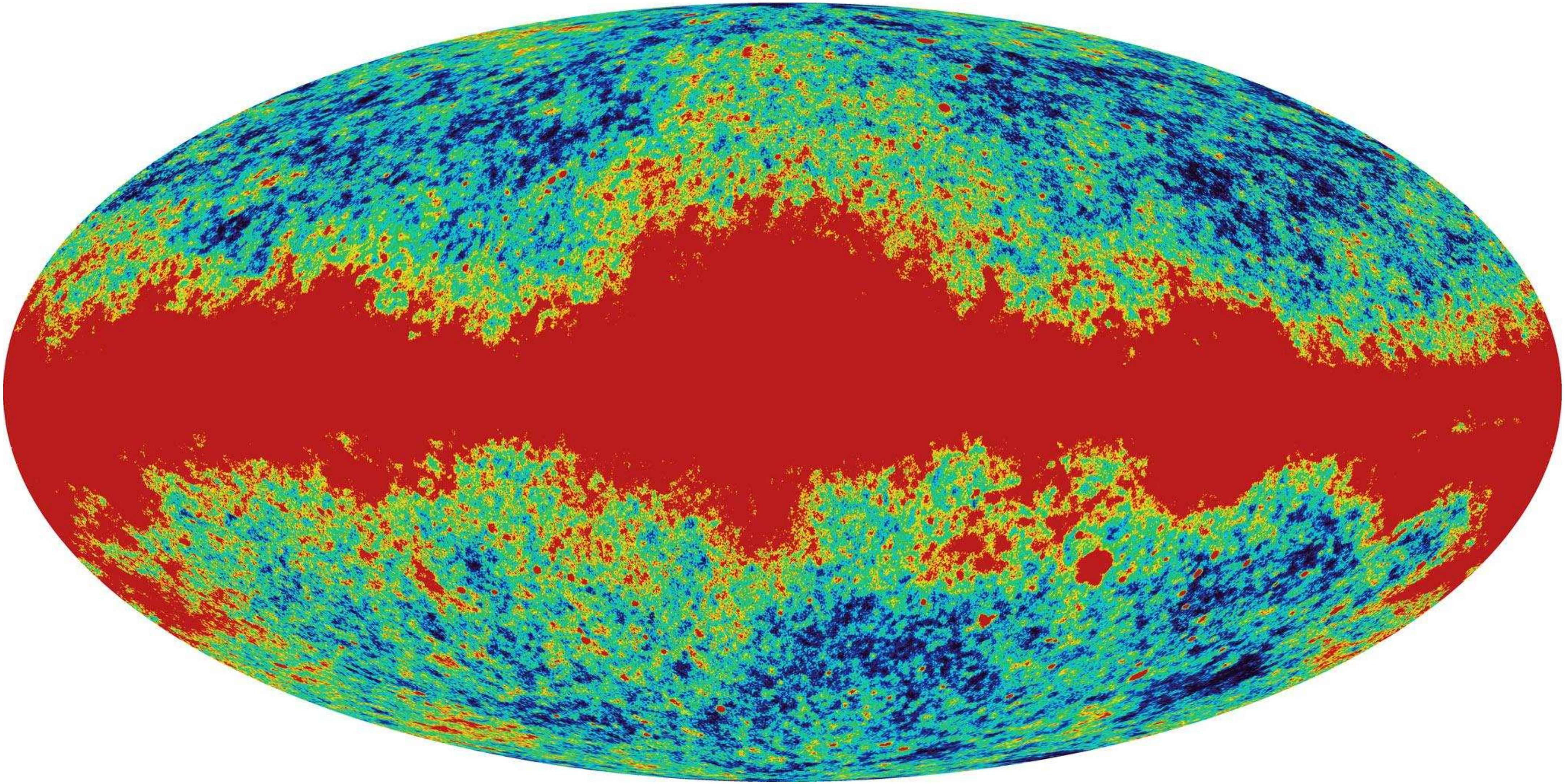
(see Bennett et al. 2003 for an overview).


Sun
150 million km
Phasing Loops
Earth
Lunar Orbit
Moon at Swingby
1.5 million km
L2

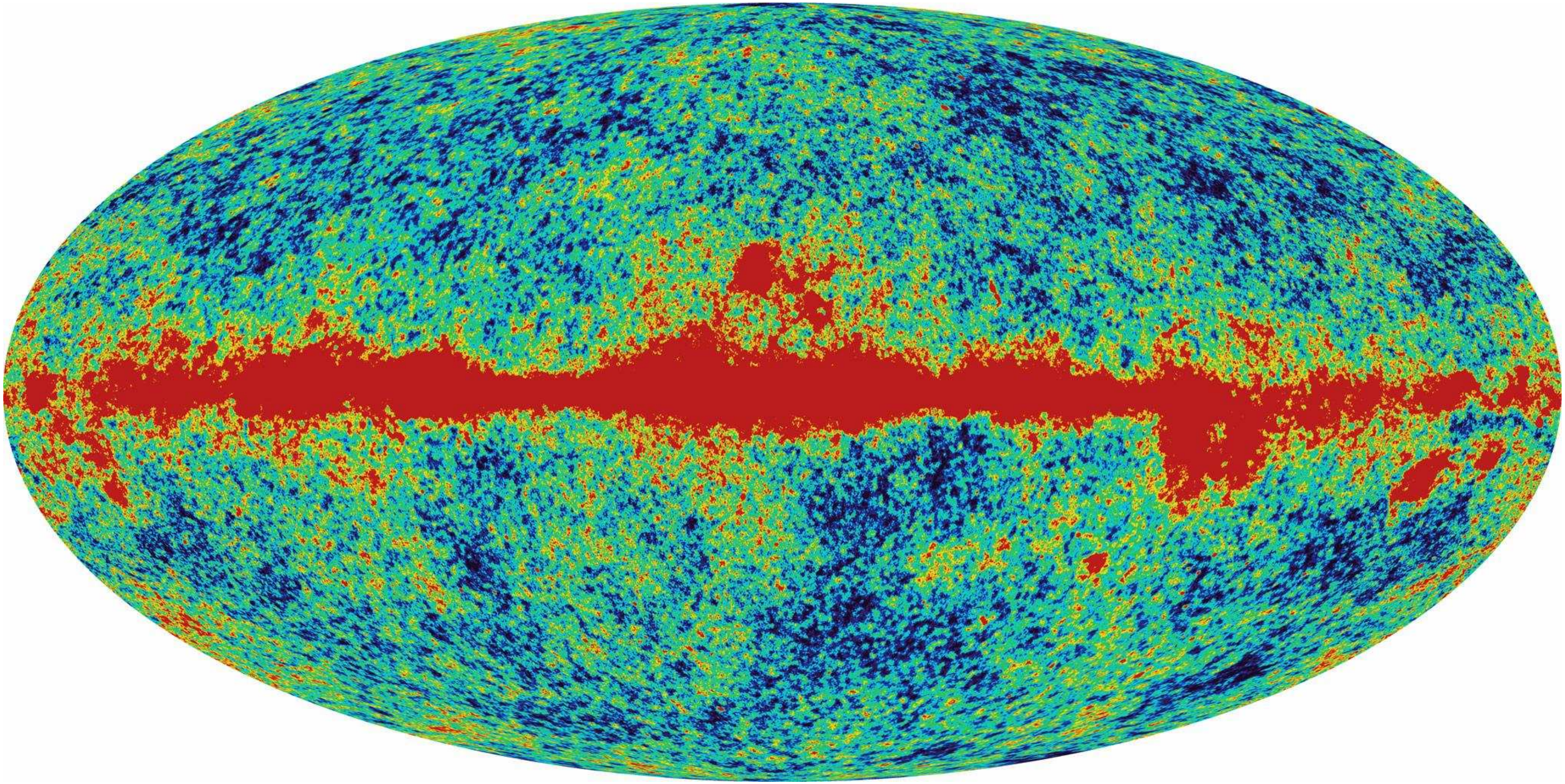


Foreground features of the microwave sky (Bennett et al., 2003).

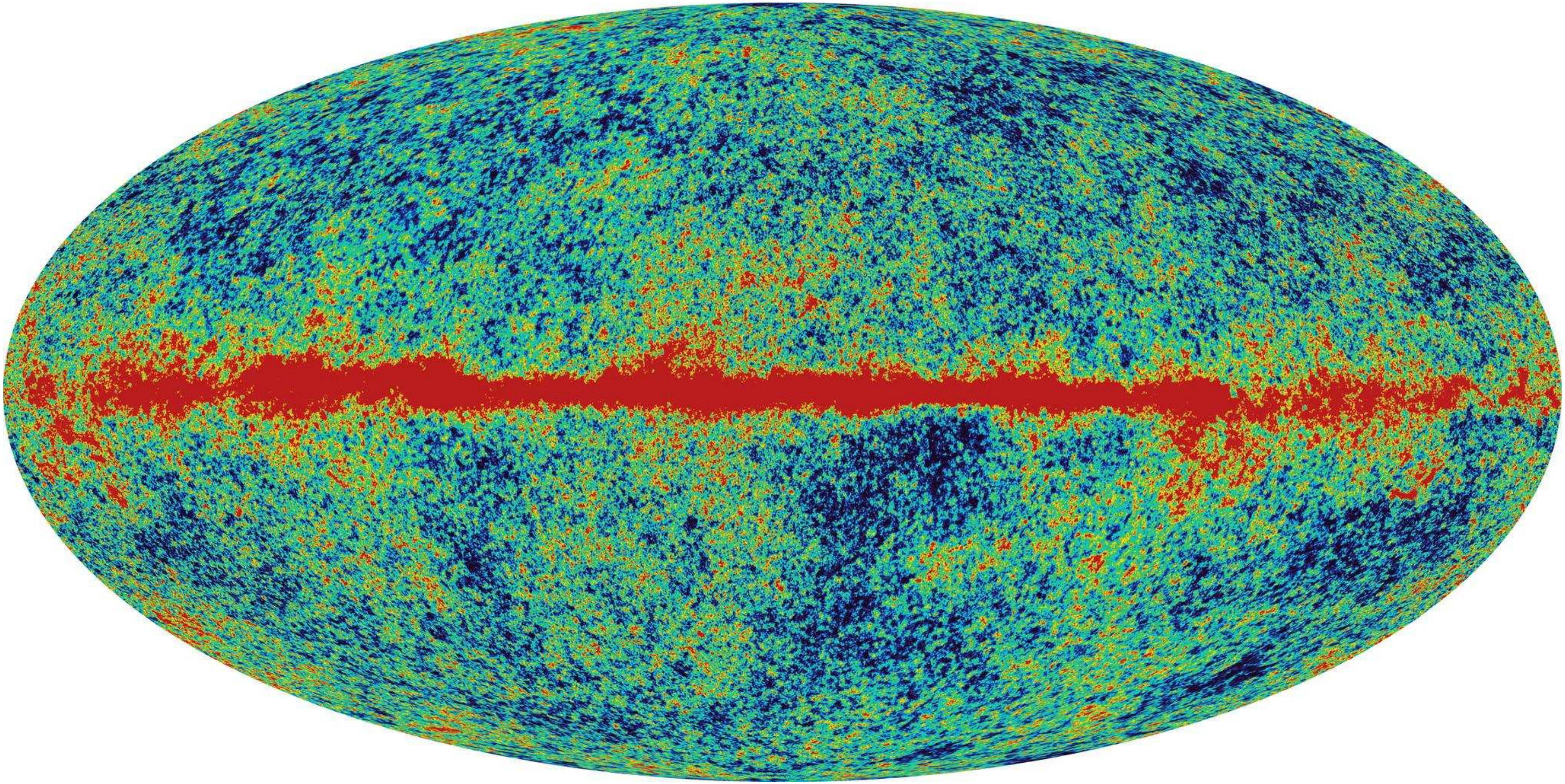
Sunyaev Zeldovich effect is expected to be strongest in Coma cluster, temperatures of -0.34 ± 0.18 mK in W and -0.24 ± 0.18 mK in K-band; barely detectable with WMAP, does not contaminate maps.



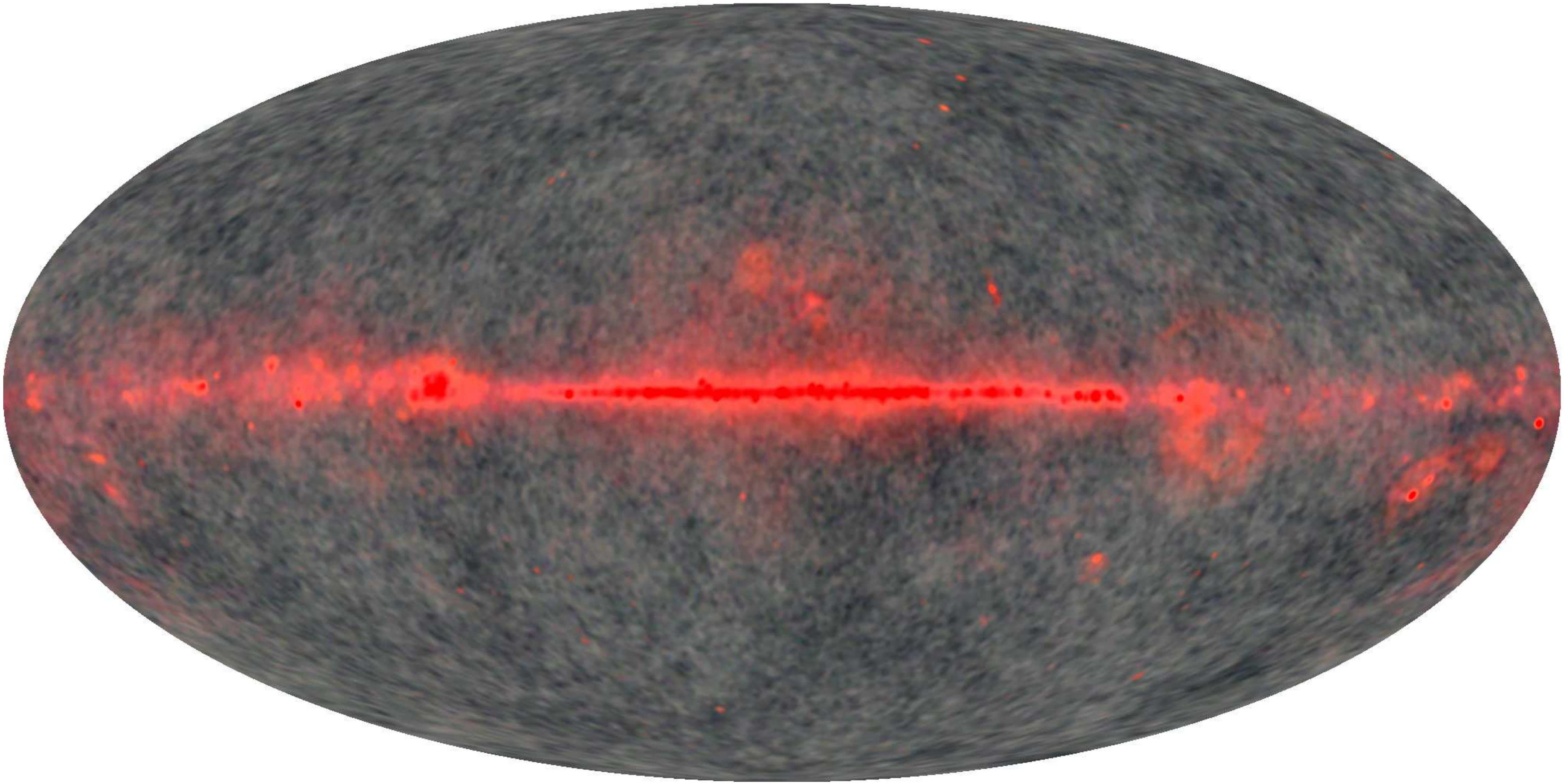
WMAP, K-Band, $\lambda = 13\,\text{mm}$, $\nu = 22.8\,\text{GHz}$, $\theta = 0.83^\circ$ FWHM



WMAP, Q-Band, $\lambda = 7.3\,\mathrm{mm}$, $\nu = 40.7\,\mathrm{GHz}$, $\theta = 0.49^\circ$ FWHM

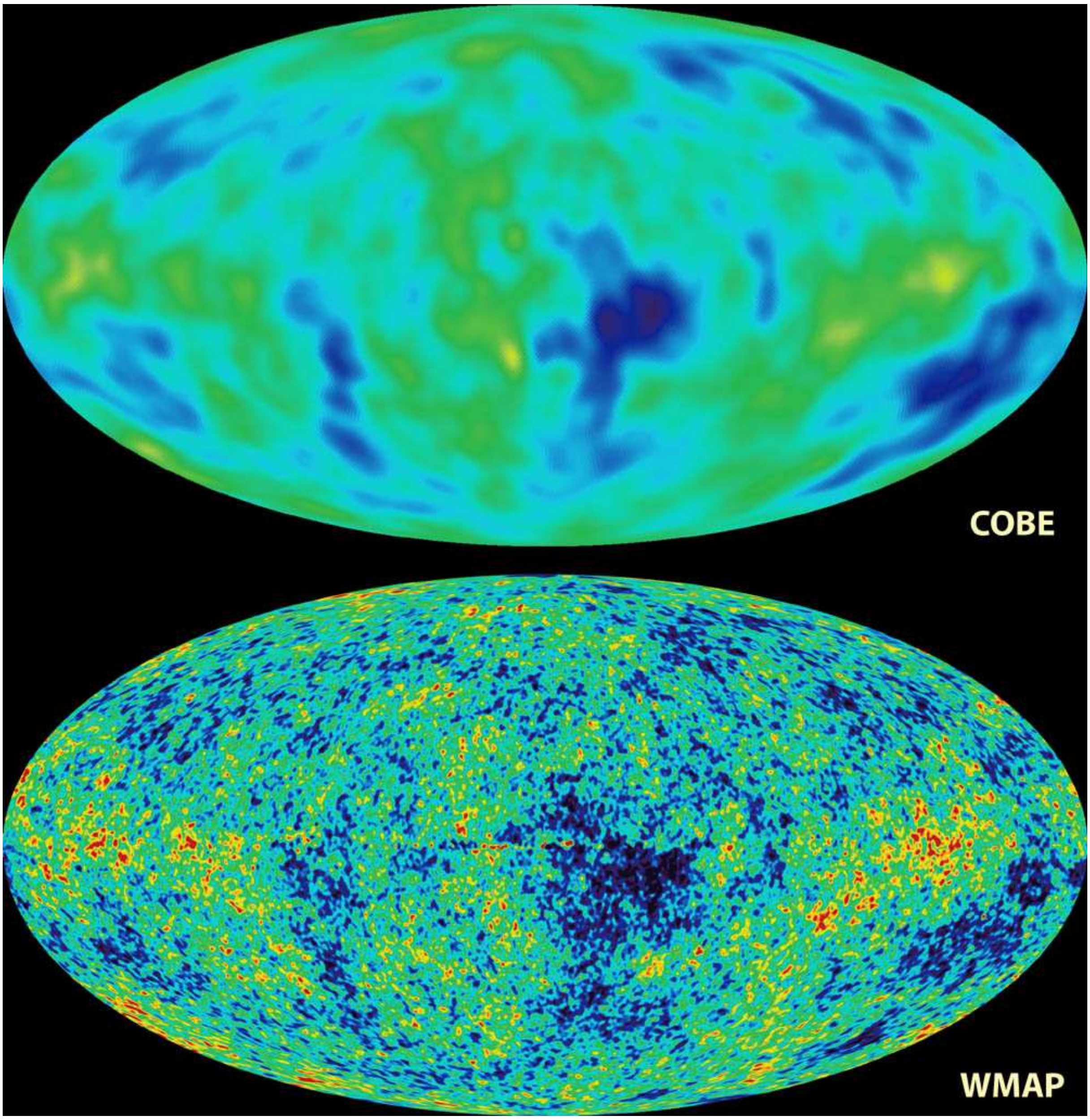


WMAP, W-Band, $\lambda = 3.2\,\mathrm{mm}$, $\nu = 93.5\,\mathrm{GHz}$, $\theta = 0.21^\circ$ FWHM



Different spectral signature enables identification of Galaxy foreground radiation

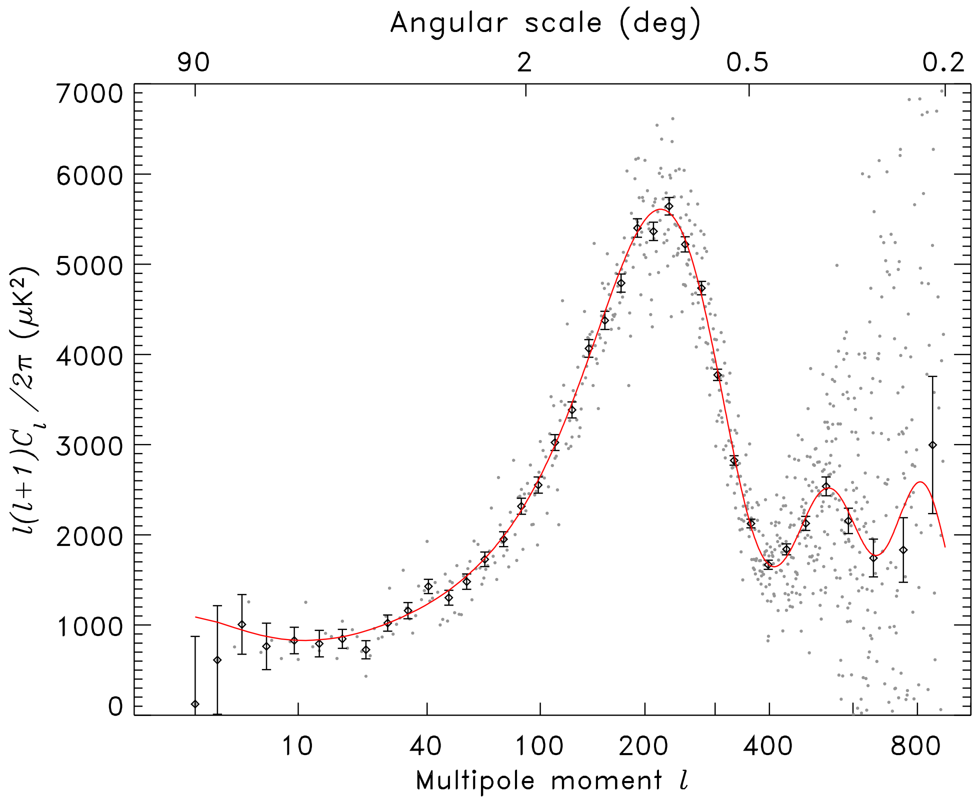
WMAP



After correction for foreground emission determine map of structure of the CMB.

WMAP data are best image of the CMB available

WMAP



(Spergel et al., 2003, Fig. 1)

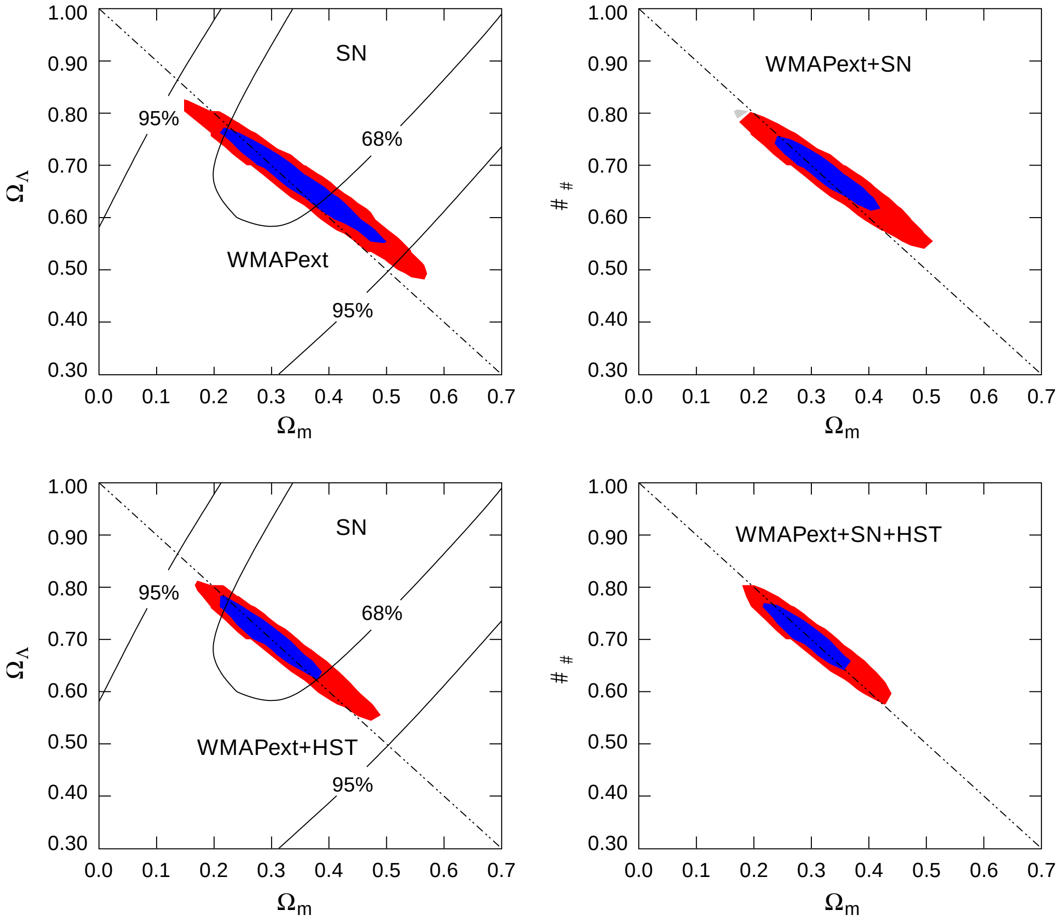
Best fit power-law ΛCDM to WMAP power spectrum $\Longrightarrow$ Very good agreement between data and theory

Best fit parameters for WMAP data:

$$\begin{aligned} h &= 0.72 \pm 0.05 \\ \Omega_{\mathrm{m}} h^2 &= 0.14 \pm 0.02 \\ \Omega_{\mathrm{b}} h^2 &= 0.024 \pm 0.01 \end{aligned} \tag{8.86}$$

(and assuming $\Omega = 1$)

WMAP



(Spergel et al., 2003, Fig. 13, SN contours are only given where they are not a prior in the analysis)

Removing constraint $\Omega = 1$

$\Longrightarrow$ Test how "flat" universe really is.

Using H_0 from HST and SN Iae results as priors into Bayesian analysis results in

$$\Omega = 1.02 \pm 0.02 \quad (1\sigma) \tag{8.87}$$

A model with $\Omega_\Lambda = 0$ is found to be consistent with the WMAP data only if $H_0 = 32.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\Omega_{\mathrm{tot}} = 1.28$

$\Longrightarrow$ Ruled out by other measurements.

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