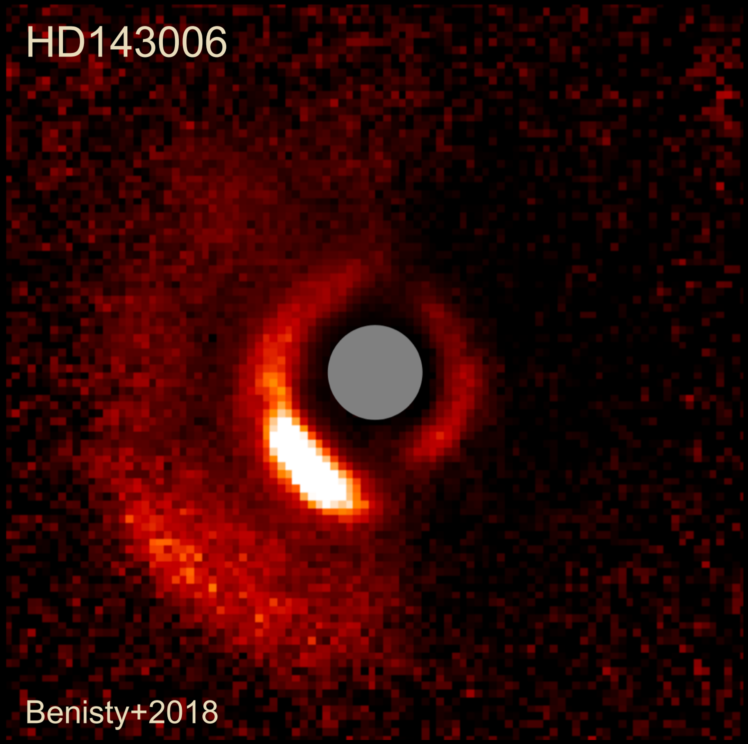
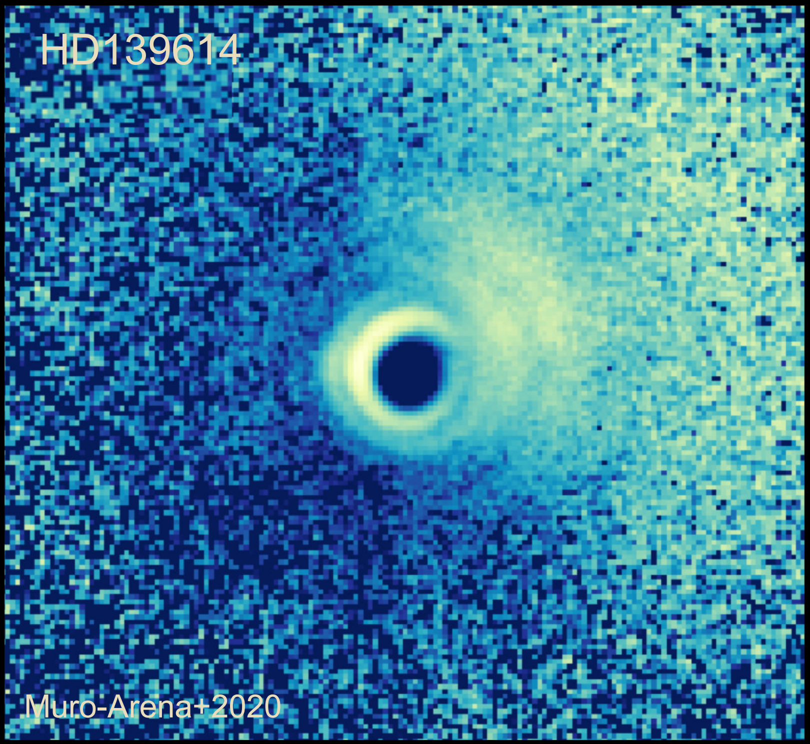
The dynamics of warped disks

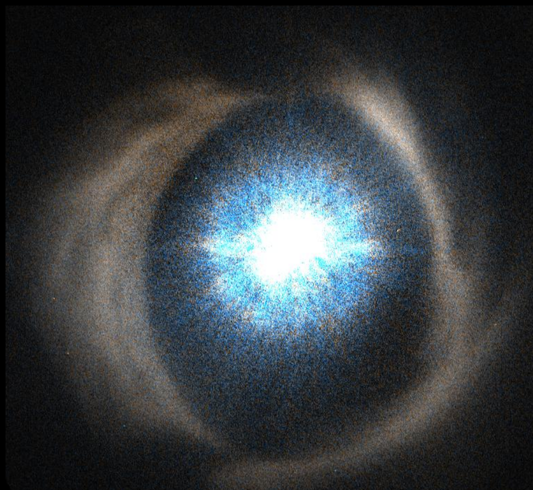


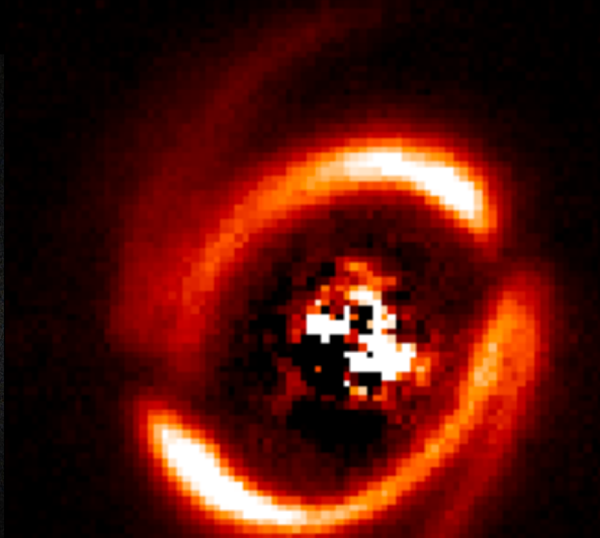


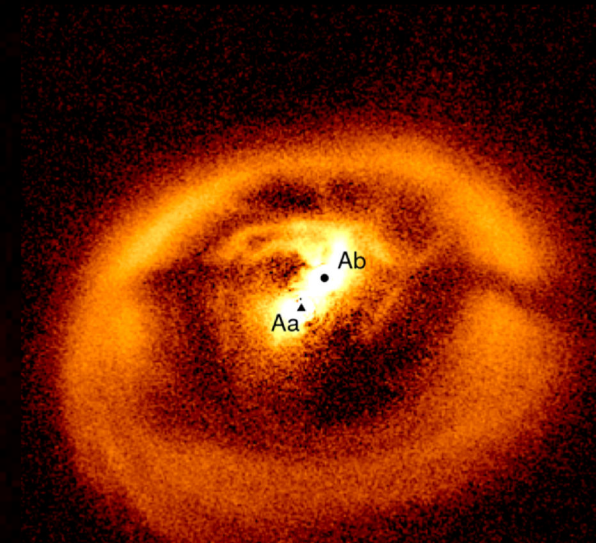
Cornelis Dullemond
with Carolin ("Lina") Kimmig and John J. Zanazzi

"Tutorial" for the MIAPP Disk Workshop

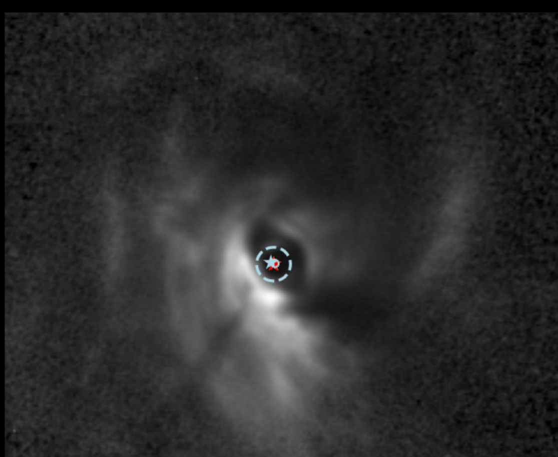
Non-planar protoplanetary disks

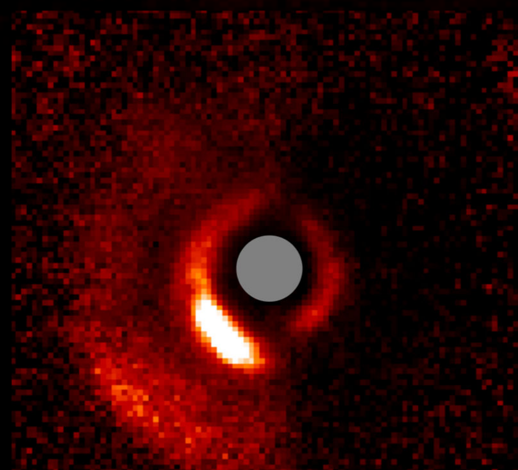

HD 142527
Avenhaus et al. (2014)

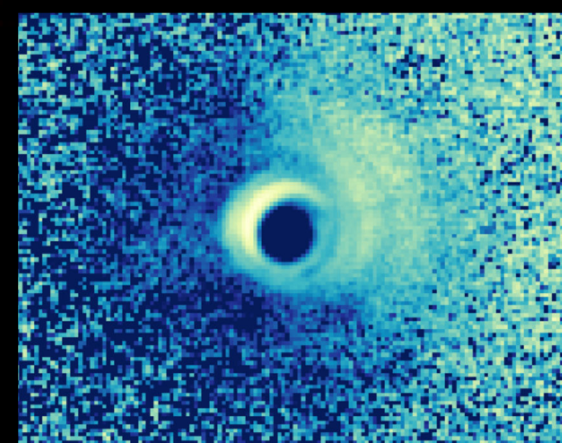

HD 100453
Benisty et al. (2017)



GG Tau
Keppler et al. (2020)


GW Orionis
Kraus et al. (2020)


HD 143006
Benisty et al. (2018)


HD 139614
Muro-Arena et al. (2020)

Warp versus twist

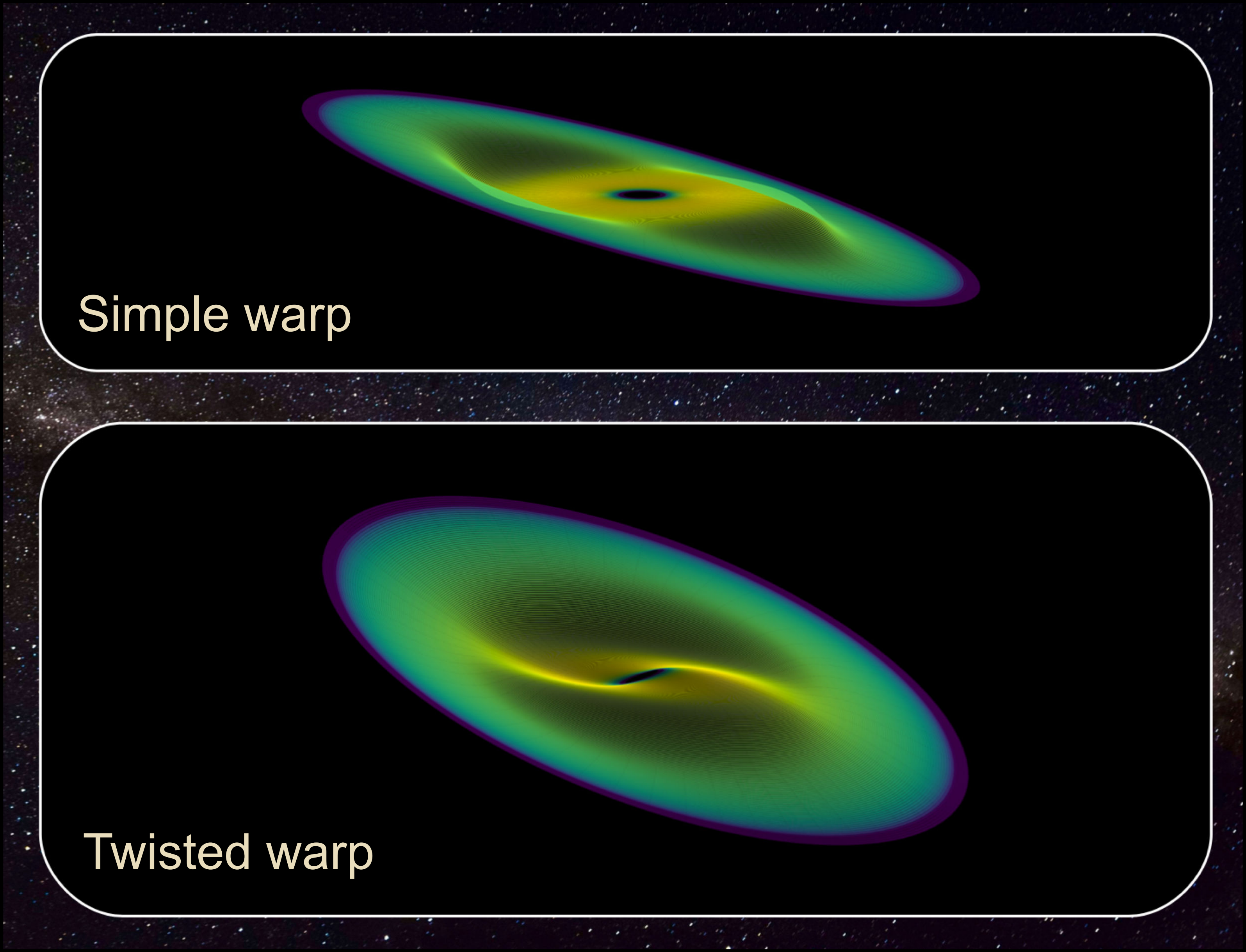
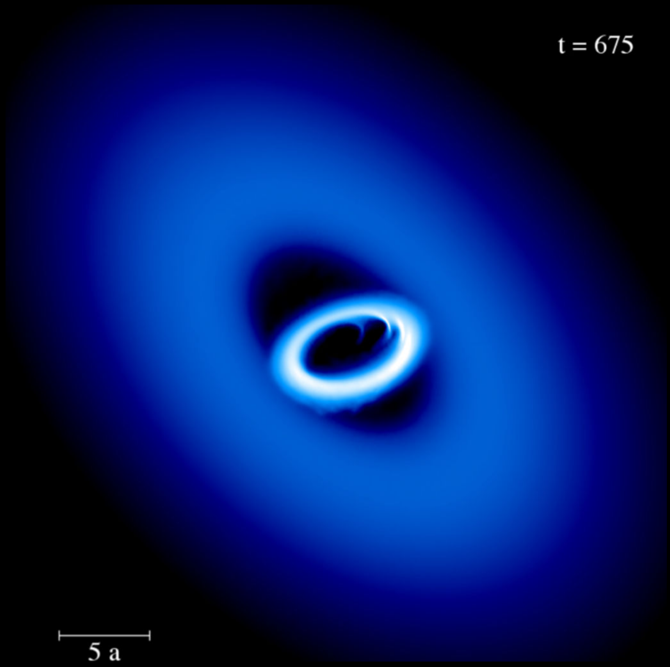


Image credit: Carolin ("Lina") Kimmig

Warped disks: Two modeling methods

Full 3D Hydro Models

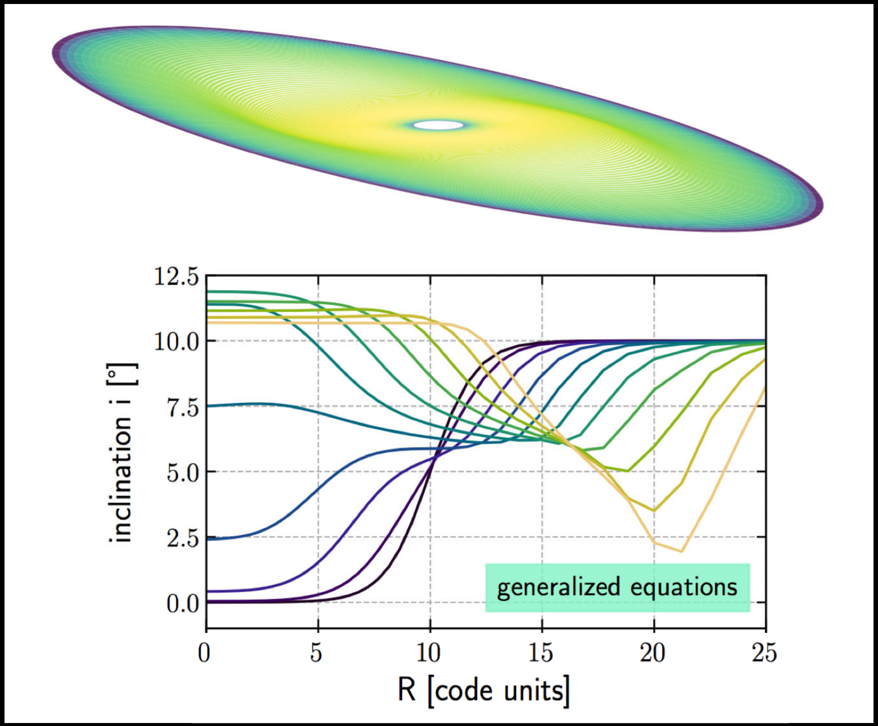
Usually done with SPH



Here: Facchini, Juhasz & Lodato 2018

Detailed, but costly

1D Multi-Ring Models



Here: Masterthesis Carolin ("Lina") Kimmig

Fast, but simplified

1D Multi-Ring Models

General principle

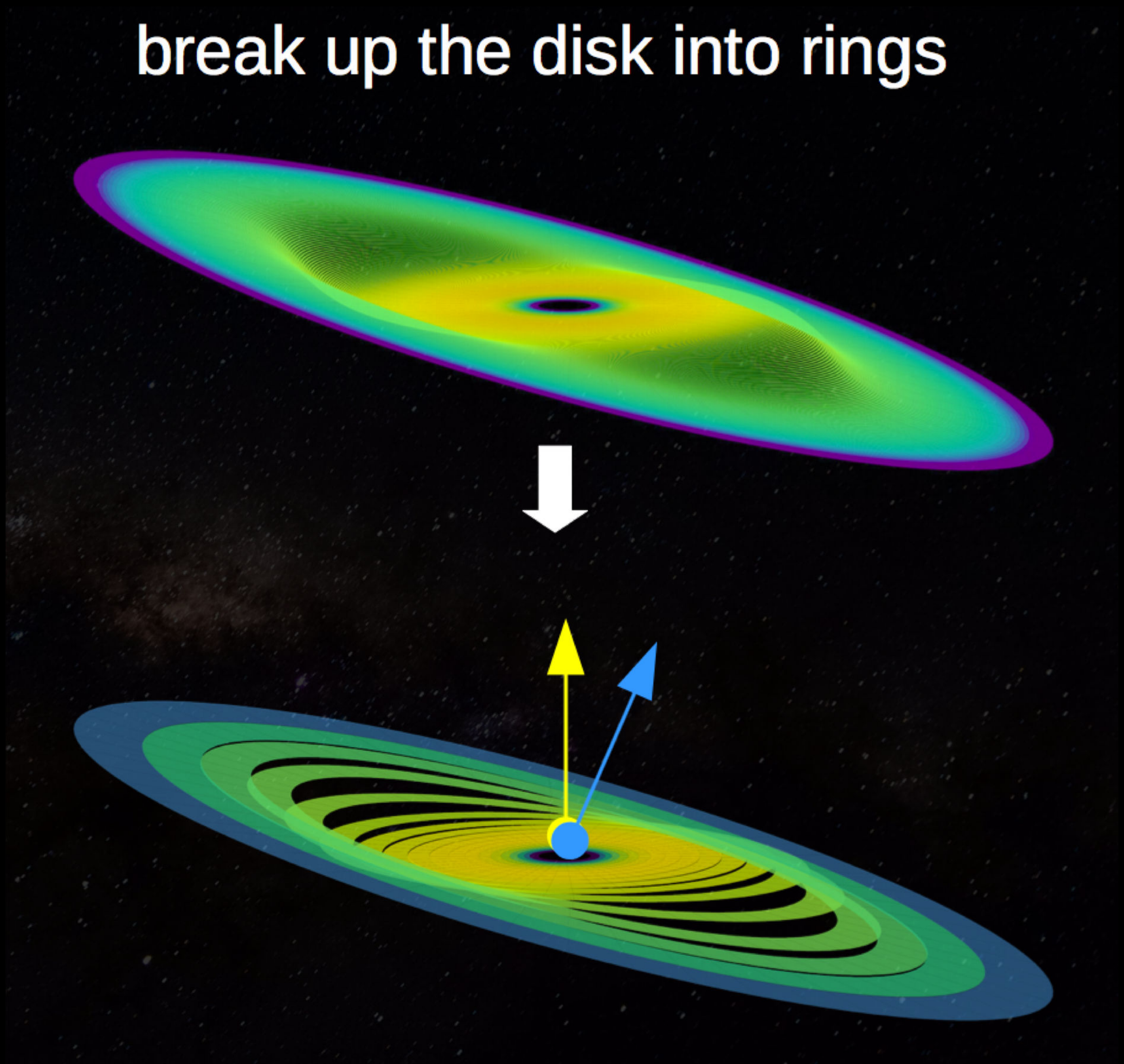
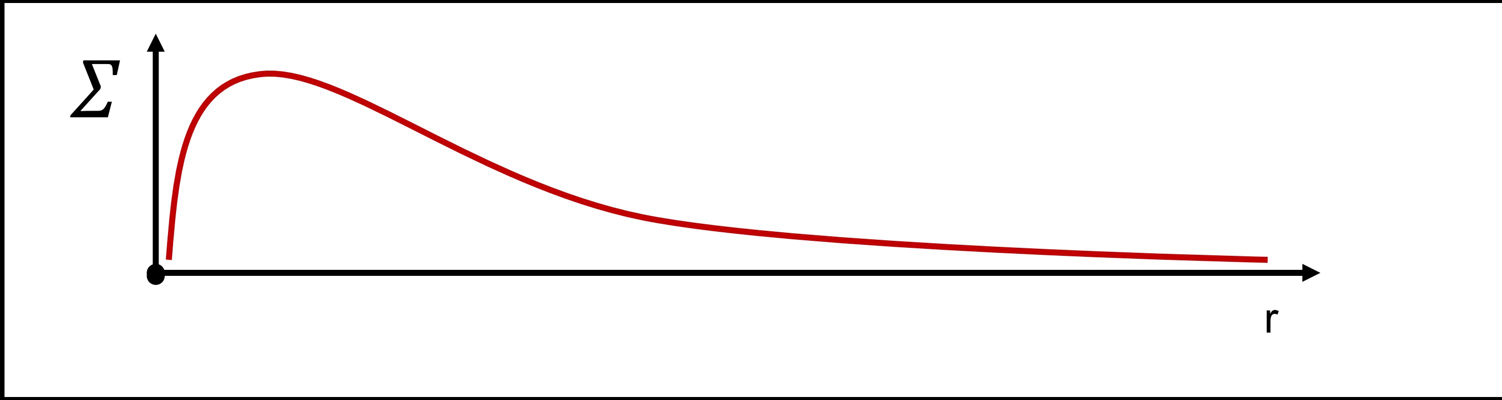


Image credit: C. Kimmig 2021

1D Multi-Ring Models

General principle



$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \Sigma v_r\right) = 0$$

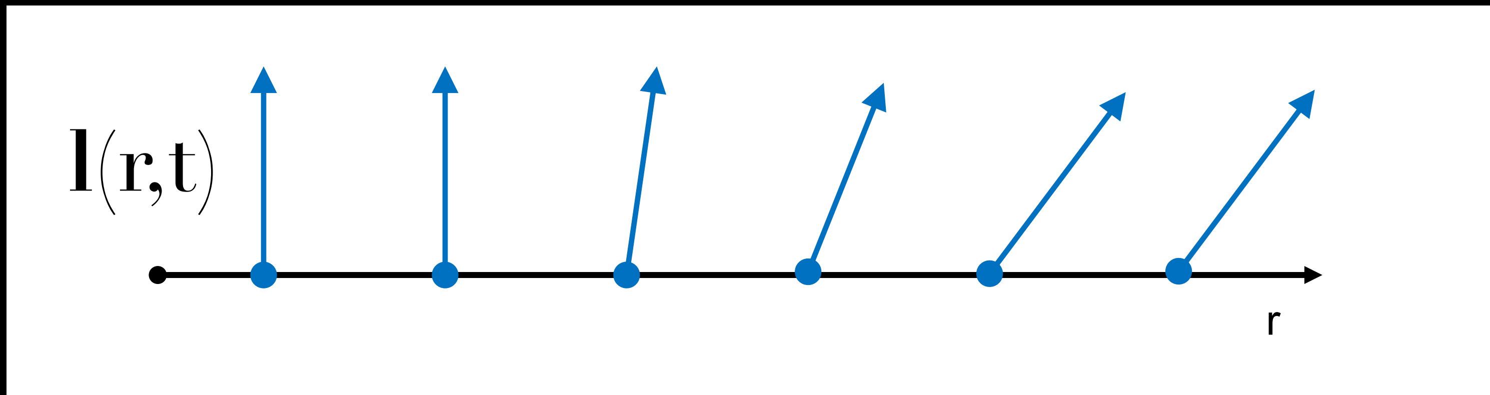
$$v_r = -\frac{\partial (r\mathbf{G})/\partial r \cdot \mathbf{l}}{r\Sigma\, \partial(\Omega r^2)/\partial r}$$

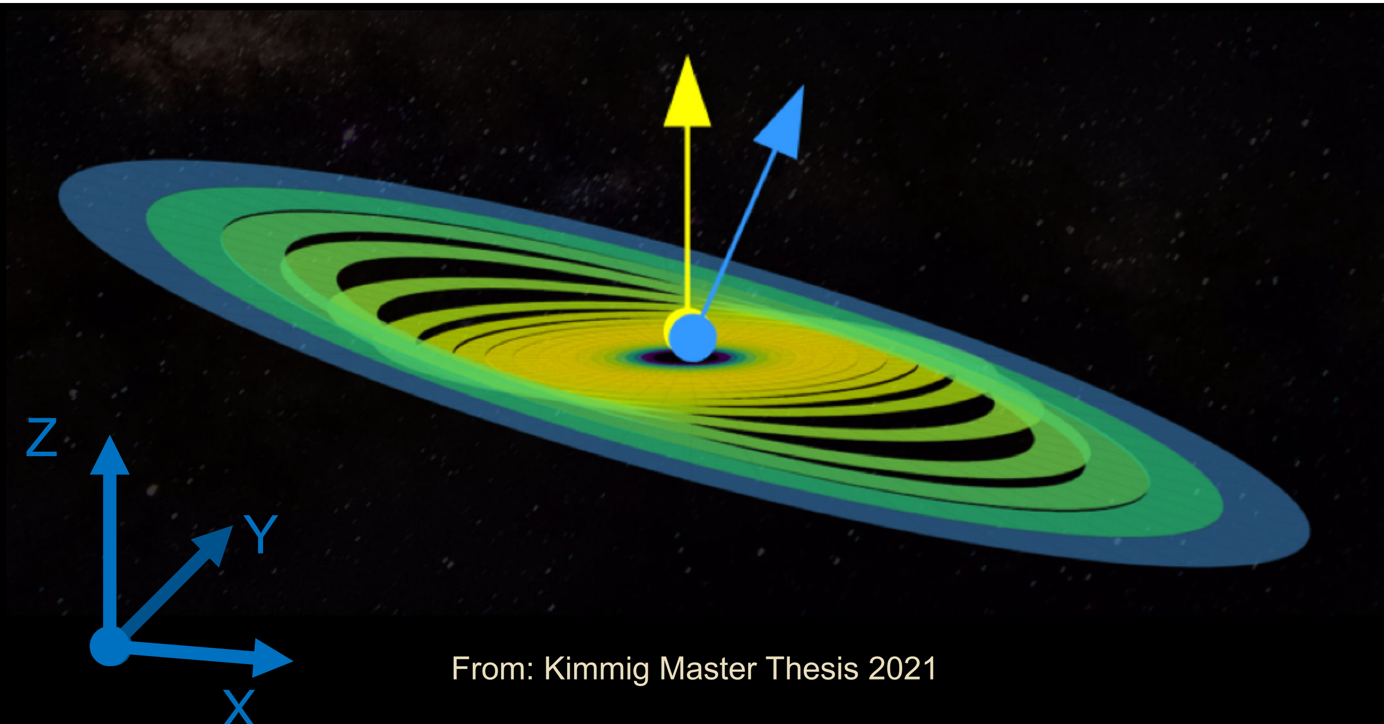
G is the internal torque vector in the disk, which for flat disks is simply the turbulent viscosity times the unit vector **l**.

So far nothing new: the usual viscous disk evolution equations

1D Multi-Ring Models

General principle

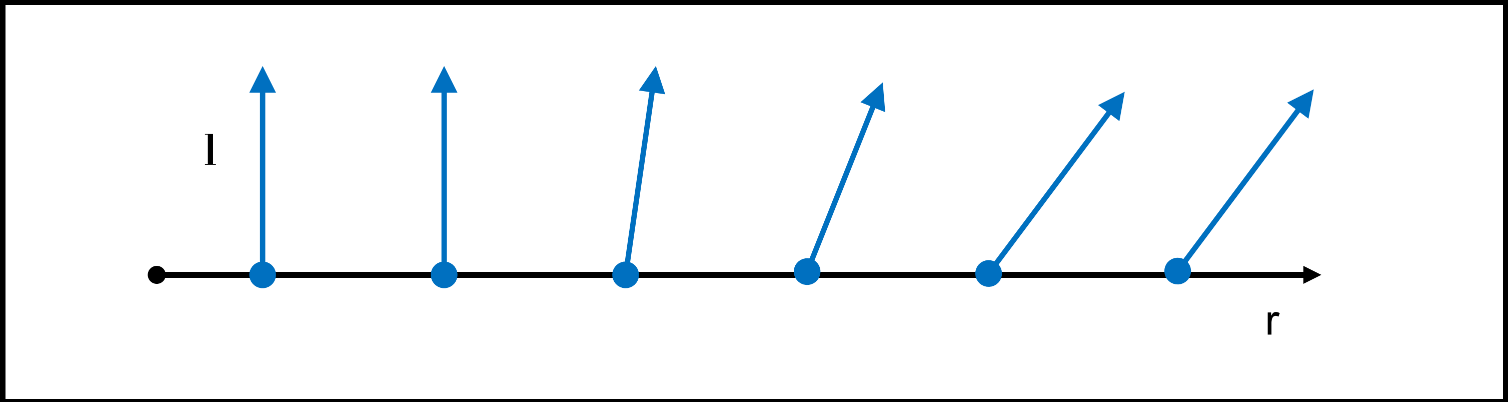




From: Kimmig Master Thesis 2021

1D Multi-Ring Models

General principle



$$\mathbf{L}(r,t) \equiv \Sigma(r,t)\,\Omega(r)\,r^2\mathbf{l}$$

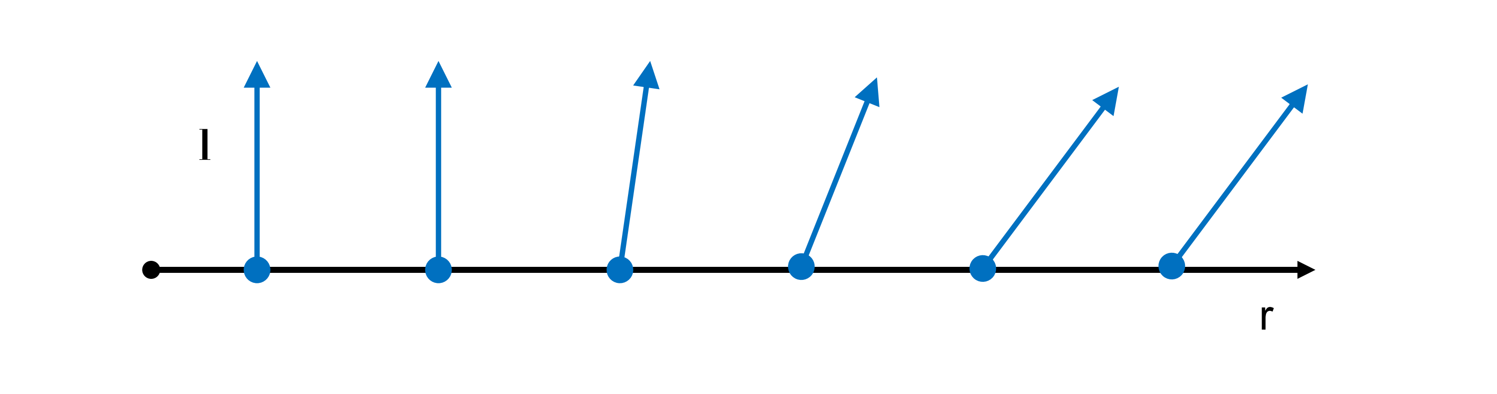
Local angular momentum vector

$$\frac{\partial \mathbf{L}}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mathbf{L}v_r + r\mathbf{G}\right) = \mathbf{T}$$

Angular momentum conservation is now a vectorial equation

1D Multi-Ring Models

General principle



$$\mathbf{L}(r,t) \equiv \Sigma(r,t)\,\Omega(r)\,r^2\mathbf{l}$$

Local angular momentum vector

$$\frac{\partial \mathbf{L}}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mathbf{L}v_r + r\mathbf{G}\right) = \mathbf{T}$$

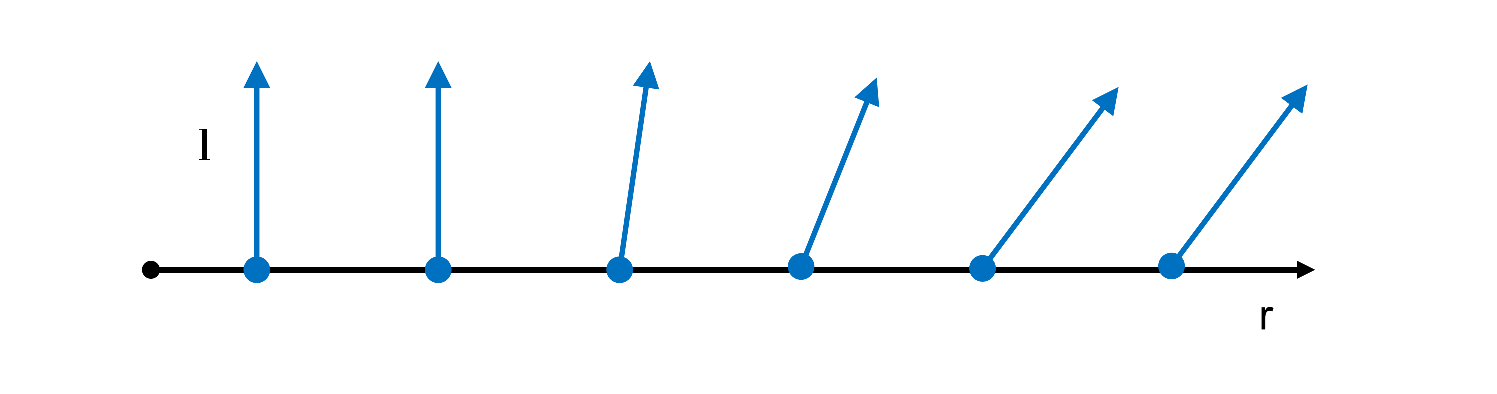
Internal torque

External torque

Angular momentum conservation is now a vectorial equation

1D Multi-Ring Models

General principle



$$\mathbf{L}(r,t) \equiv \Sigma(r,t)\,\Omega(r)\,r^2\mathbf{l}$$

Local angular momentum vector

$$\frac{\partial \mathbf{L}}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mathbf{L}v_r + r\mathbf{G}\right) = \mathbf{T}$$

Internal torque

Angular momentum conservation is now a vectorial equation

History of 1D Multi-Ring Models

for warped disks

- Papaloizou & Pringle 1983; Pringle 1992; Papaloizou & Lin 1995; Lubow & Pringle 1993
- Ogilvie 1999; Lubow & Ogilvie 2000; Ogilvie & Latter 2013; Ogilvie 2018
- Martin et al. 2019; Zanazzi & Lai 2018
- Many more...

The conservation equations are simple and always the same. But the big question is: What is the correct equation for the internal torque vector **G**(r,t)?

Equation for **G** (internal torque vector)

Two regimes, treated separately:

- Viscous/diffusive regime (α>>h/r):

$$\mathbf{G}(r,t) = ...$$

G is a direct function of the disk conditions

Pringle 1992; Lodato & Price 2010; Ogilvie & Latter 2013

- Wavelike regime (α<<h/r):

$$\frac{\partial \mathbf{G}(r,t)}{\partial t} = ...$$

G is a dynamic quantity

Ogilvie 1999; Lubow & Ogilvie 2000

Equation for **G** (internal torque vector)

Martin et al. 2019 proposed a unified equation:

$$\frac{\partial \mathbf{G}}{\partial t} + \alpha \Omega \mathbf{G} = \frac{3\, g_0}{2} \alpha^2 \mathbf{l} - \frac{g_0}{4} \frac{d\mathbf{l}}{d \ln r}$$

g_0 contains stuff such as Σ and h

Martin, Lubow, Pringle et al. 2019

Note: Strongly simplified here

Equation for **G** (internal torque vector)

Martin et al. 2019 proposed a unified equation:

$$\frac{\partial \mathbf{G}}{\partial t} + \alpha \Omega \mathbf{G} = \frac{3}{2}\frac{g_0}{2}\alpha^2 \mathbf{l} - \frac{g_0}{4}\frac{d\mathbf{l}}{d\ln r}$$

g_0 contains stuff such as Σ and h

Responsible for the viscous evolution of Σ (Shakura & Sunyaev style)

Responsible for the dynamic evolution of the warp

Martin, Lubow, Pringle et al. 2019

Note: Strongly simplified here

Equation for **G** (internal torque vector)

Viscous/diffusive regime (α>>h/r):

$$\frac{\partial \mathbf{G}}{\partial t} + \alpha\Omega\mathbf{G} = \frac{3\,g_0}{2}\alpha^2\mathbf{l} - \frac{g_0}{4}\frac{d\mathbf{l}}{d\ln r}$$

g_0 contains stuff such as Σ and h

$$\mathbf{G} = \frac{3\,g_0}{2\alpha\Omega}\alpha^2\mathbf{l} - \frac{g_0}{4\alpha\Omega}\frac{d\mathbf{l}}{d\ln r}$$

Martin, Lubow, Pringle et al. 2019

Note: Strongly simplified here

Equation for **G** (internal torque vector)

Viscous/diffusive regime (α>>h/r):

$$\cancel{\frac{\partial \mathbf{G}}{\partial t}} + \alpha\Omega\mathbf{G} = \frac{3\,g_0}{2}\alpha^2\mathbf{l} - \frac{g_0}{4}\frac{d\mathbf{l}}{d\ln r}$$

g_0 contains stuff such as Σ and h

Matches the equations of Pringle 1992; Lodato & Price 2010; Ogilvie & Latter 2013

✓

Martin, Lubow, Pringle et al. 2019

Note: Strongly simplified here

Equation for **G** (internal torque vector)

Wavelike regime (α<<h/r):

$$\frac{\partial \mathbf{G}}{\partial t} + \alpha\Omega\mathbf{G} = \cancel{\frac{3\, g_0}{2}\alpha^2\mathbf{l}} - \frac{g_0}{4}\frac{d\mathbf{l}}{d\ln r}$$

g_0 contains stuff such as Σ and h

Matches the equations of Ogilvie 1999 and Lubow & Ogilvie 2000

✓

Martin, Lubow, Pringle et al. 2019

Note: Strongly simplified here

Equation for **G** (internal torque vector)

Wavelike regime (α<<h/r):

$$\frac{\partial \mathbf{G}}{\partial t} + \alpha \Omega \mathbf{G} = \frac{3\, g_0}{2} \alpha^2 \mathbf{l} - \frac{g_0}{4} \frac{d\mathbf{l}}{d \ln r}$$

g_0 contains stuff such as Σ and h

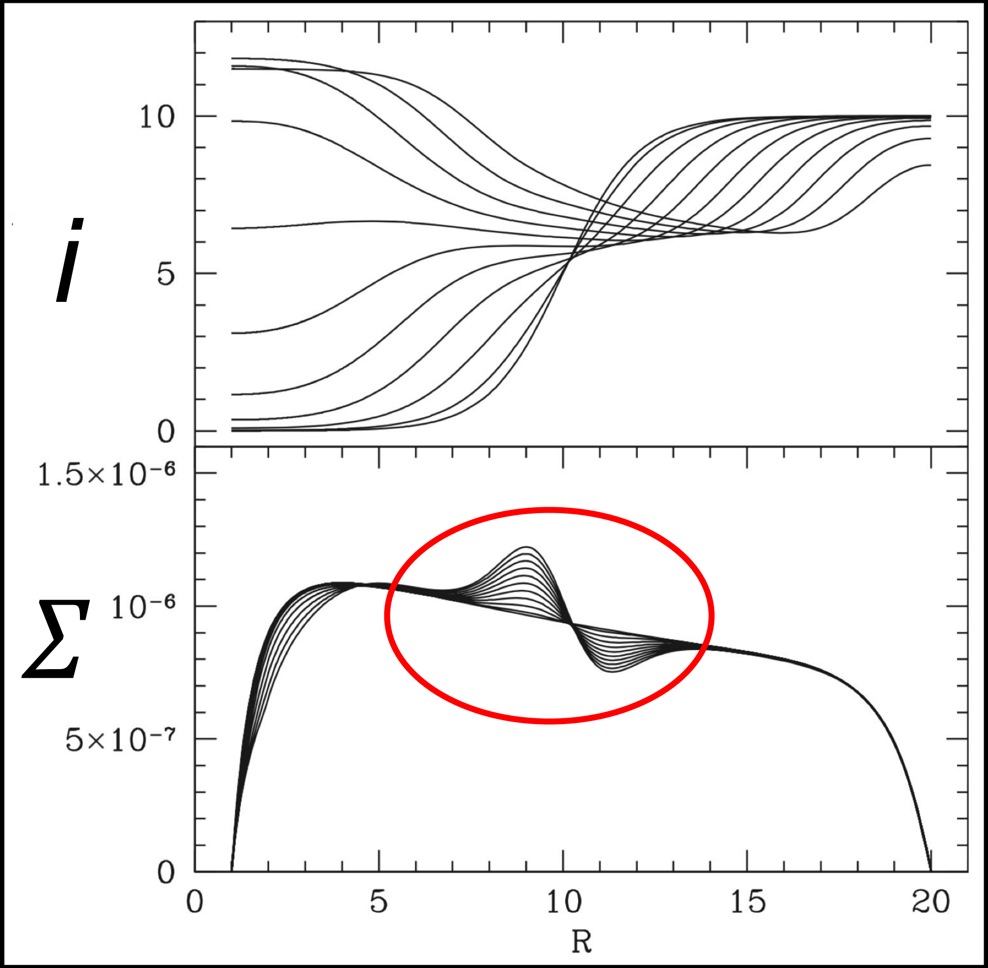
However, for long evolution times, we do not want to ignore the viscous evolution of Σ(r,t)

Martin, Lubow, Pringle et al. 2019

Note: Strongly simplified here

Equation for **G** (internal torque vector)

Strange behavior, including negative viscosity!



Martin, Lubow, Pringle et al. 2019

Equation for **G** (internal torque vector)

Martin's solution: add a damping coefficient β

$$\frac{\partial \mathbf{G}}{\partial t} + \alpha \Omega \mathbf{G} = \frac{3\, g_0}{2} \alpha^2 \mathbf{l} - \frac{g_0}{4} \frac{d\mathbf{l}}{d \ln r}$$

Martin, Lubow, Pringle et al. 2019

Equation for **G** (internal torque vector)

Martin's solution: add a damping coefficient β

$$\frac{\partial \mathbf{G}}{\partial t} + \alpha\Omega\mathbf{G} + \beta(\mathbf{G}\cdot\mathbf{l})\mathbf{l} = \frac{3\,g_0}{2}\alpha(\alpha+\beta)\mathbf{l} - \frac{g_0}{4}\frac{d\mathbf{l}}{d\ln r}$$

Works well, as long as β is large enough (e.g. β=100)

But what is the physical meaning of this?

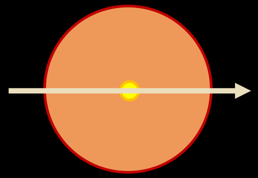
We wanted to find out
(Dullemond, Kimmig & Zanazzi 2021 MNRAS in press)

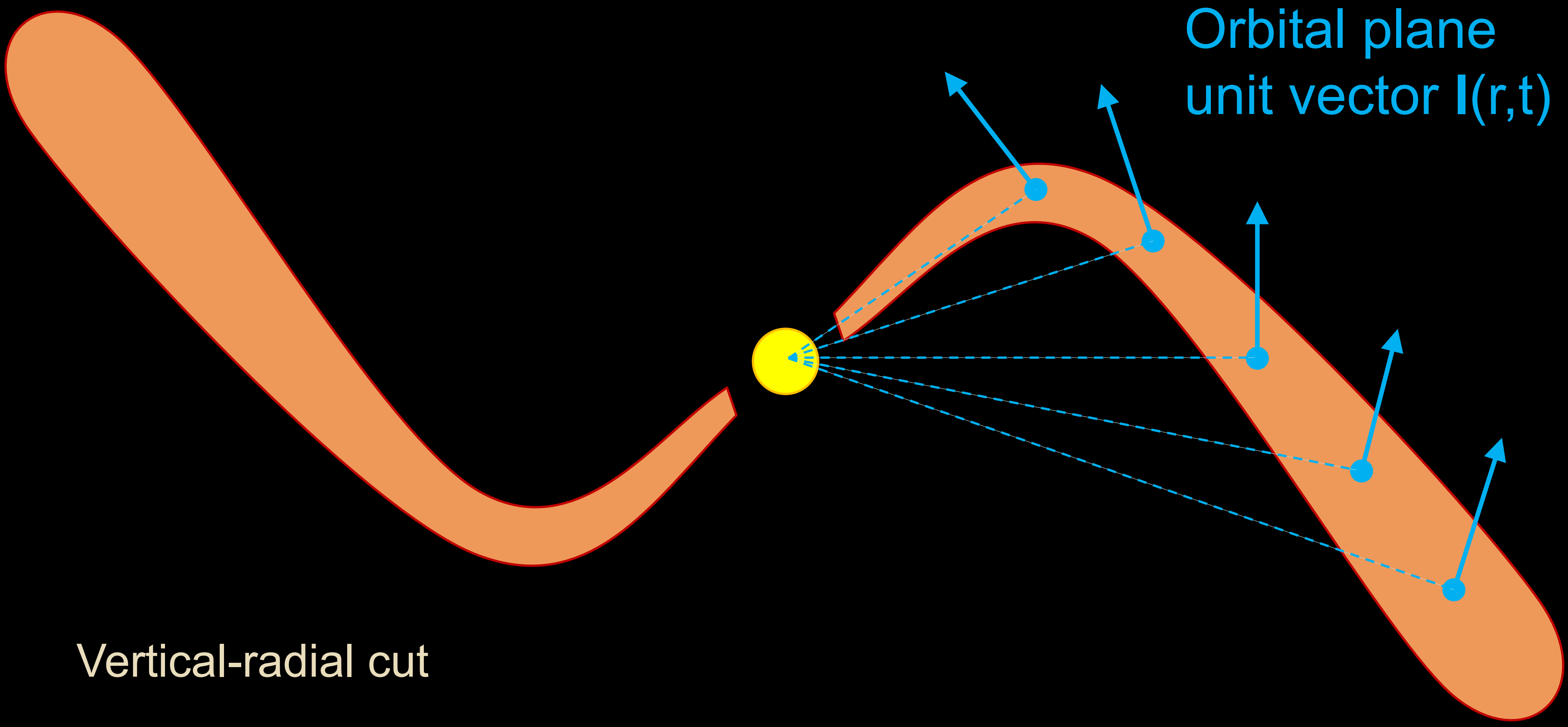
Martin, Lubow, Pringle et al. 2019

https://www.onlygfx.com/band-aid-png-transparent/

Warped disk geometry

Top view:

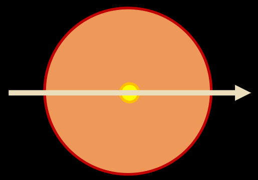




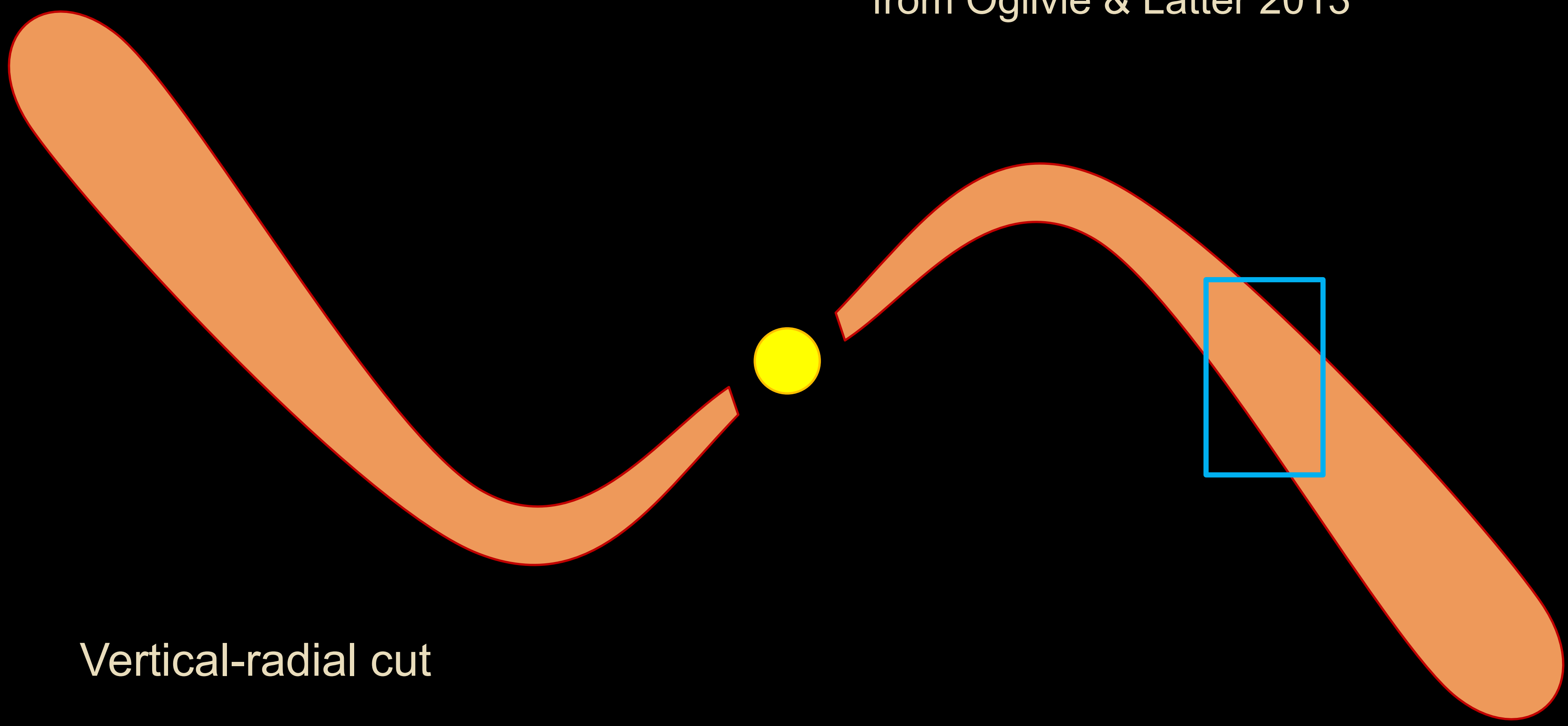
Vertical-radial cut

Warped disk geometry & shearing box

Top view:



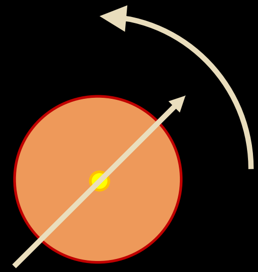
Warped shearing box formalism from Ogilvie & Latter 2013



Vertical-radial cut

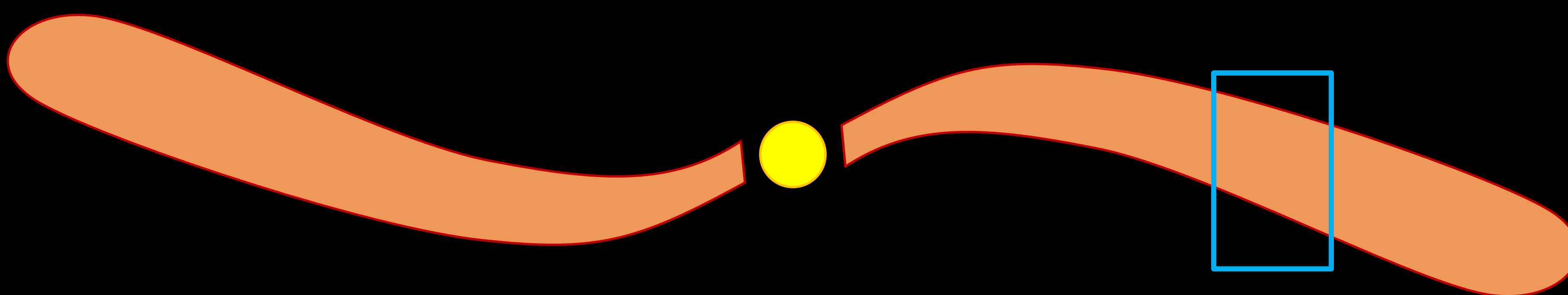
Warped disk geometry & shearing box

Top view:



Orbital rotation direction

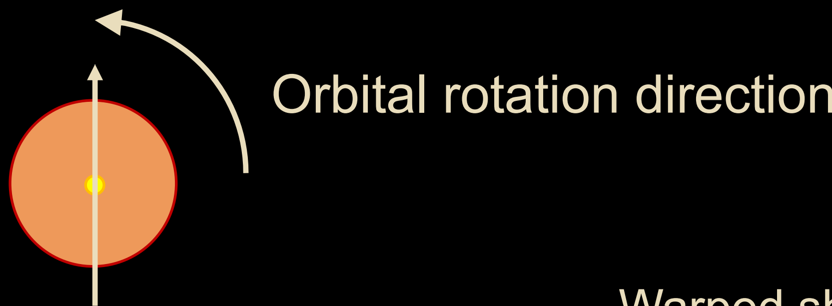
Warped shearing box formalism
from Ogilvie & Latter 2013



Vertical-radial cut

Warped disk geometry & shearing box

Top view:



Warped shearing box formalism
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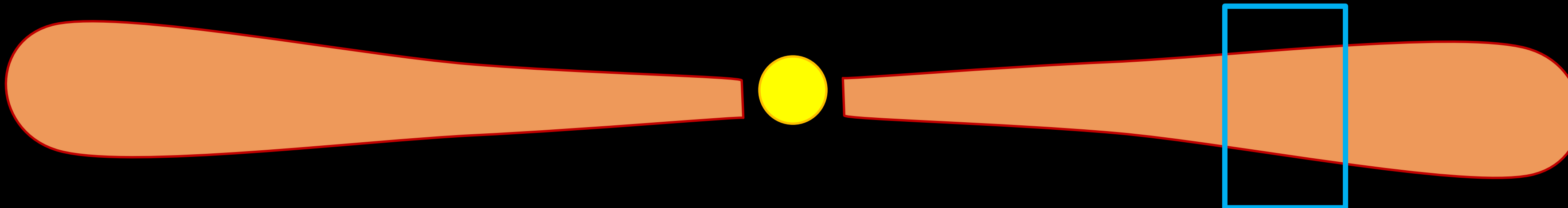
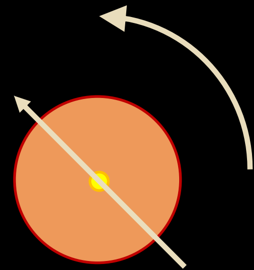


Vertical-radial cut

Warped disk geometry & shearing box

Top view:



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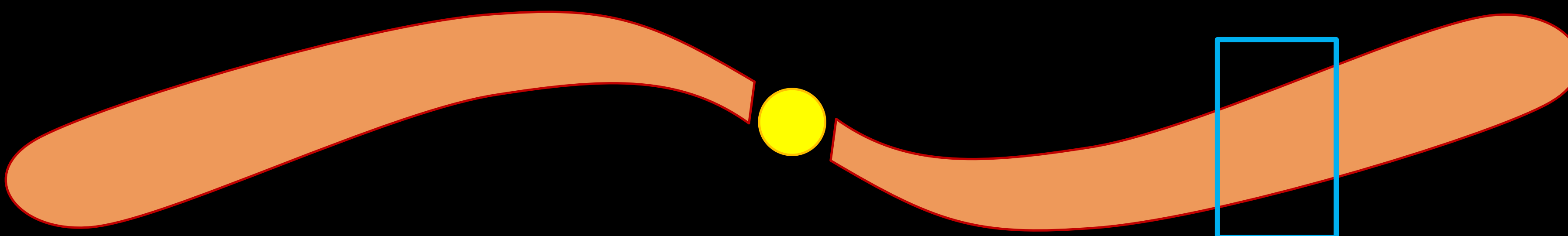
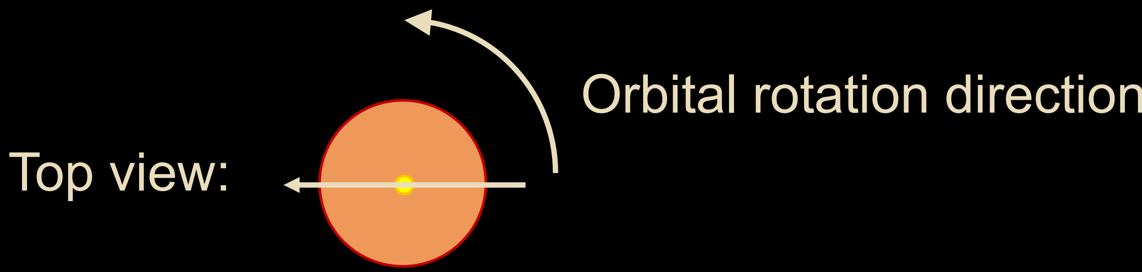


Vertical-radial cut

Warped disk geometry & shearing box



Warped shearing box formalism
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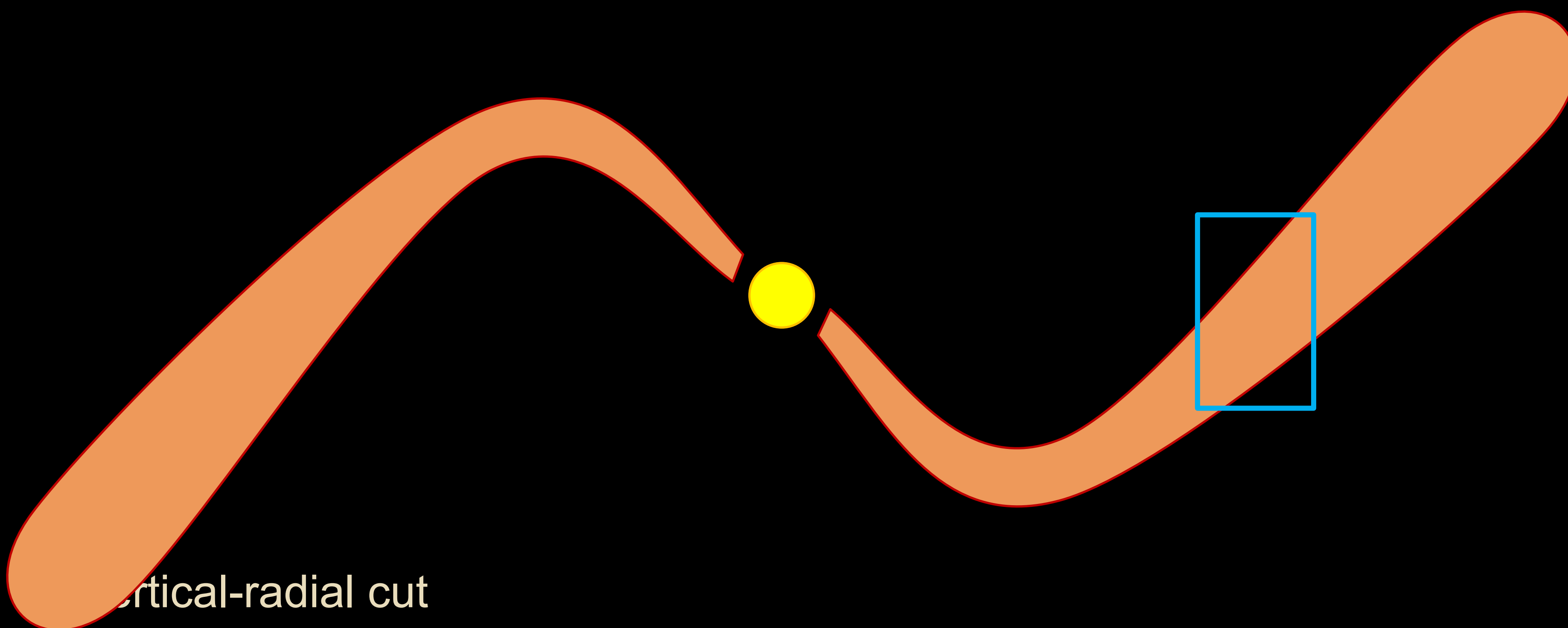
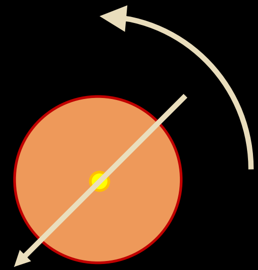


Warped disk geometry & shearing box

Orbital rotation direction

Top view:



Warped shearing box formalism
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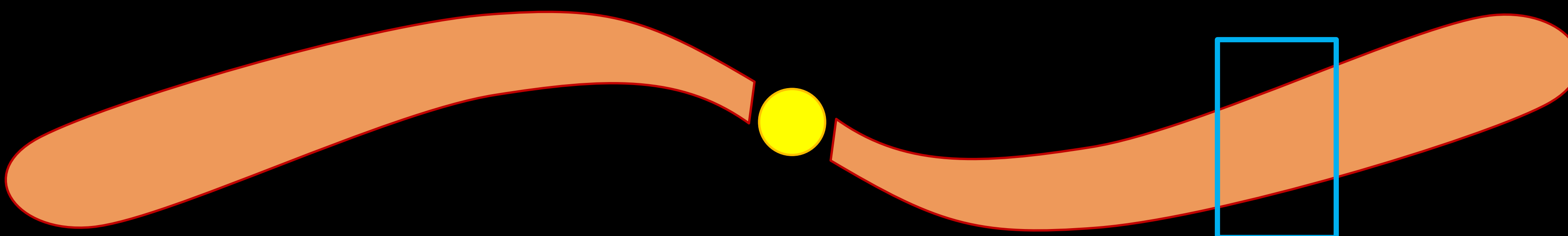
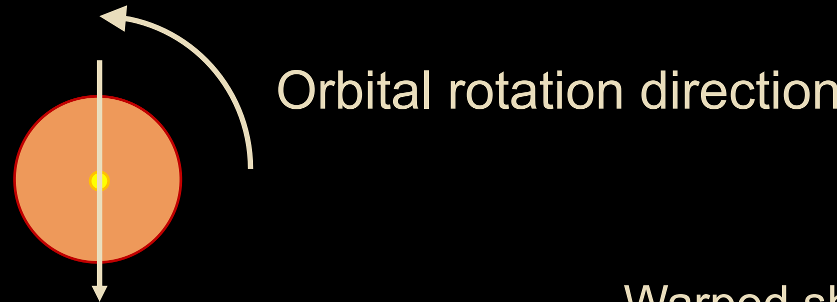


Vertical-radial cut

Warped disk geometry & shearing box

Top view:



Warped shearing box formalism
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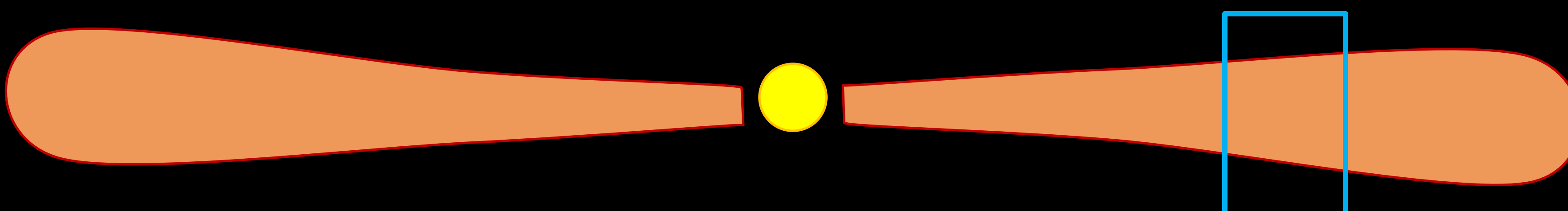
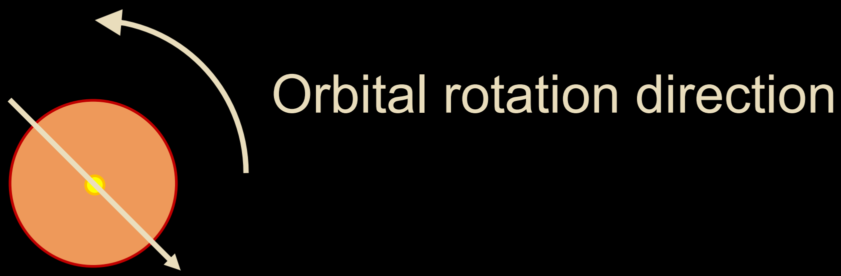


Vertical-radial cut

Warped disk geometry & shearing box

Top view:



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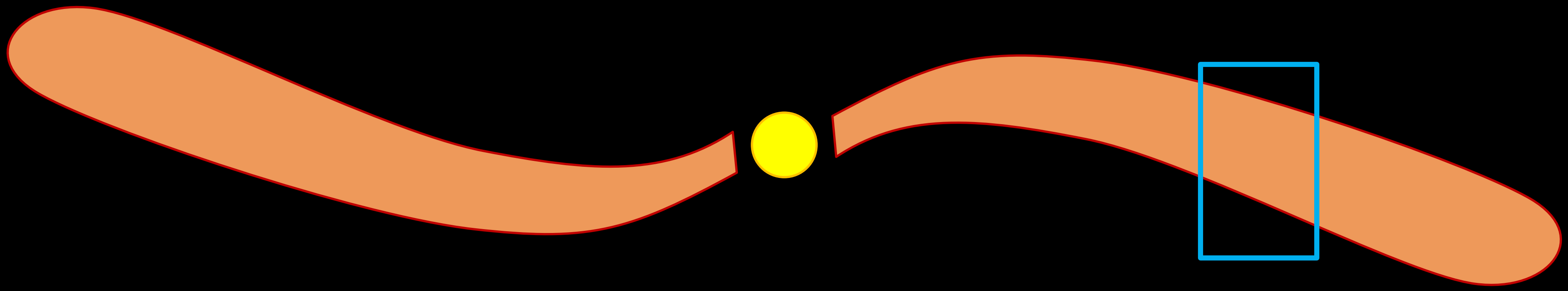
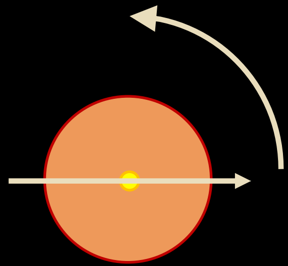


Vertical-radial cut

Warped disk geometry & shearing box

Orbital rotation direction

Top view:



Warped shearing box formalism
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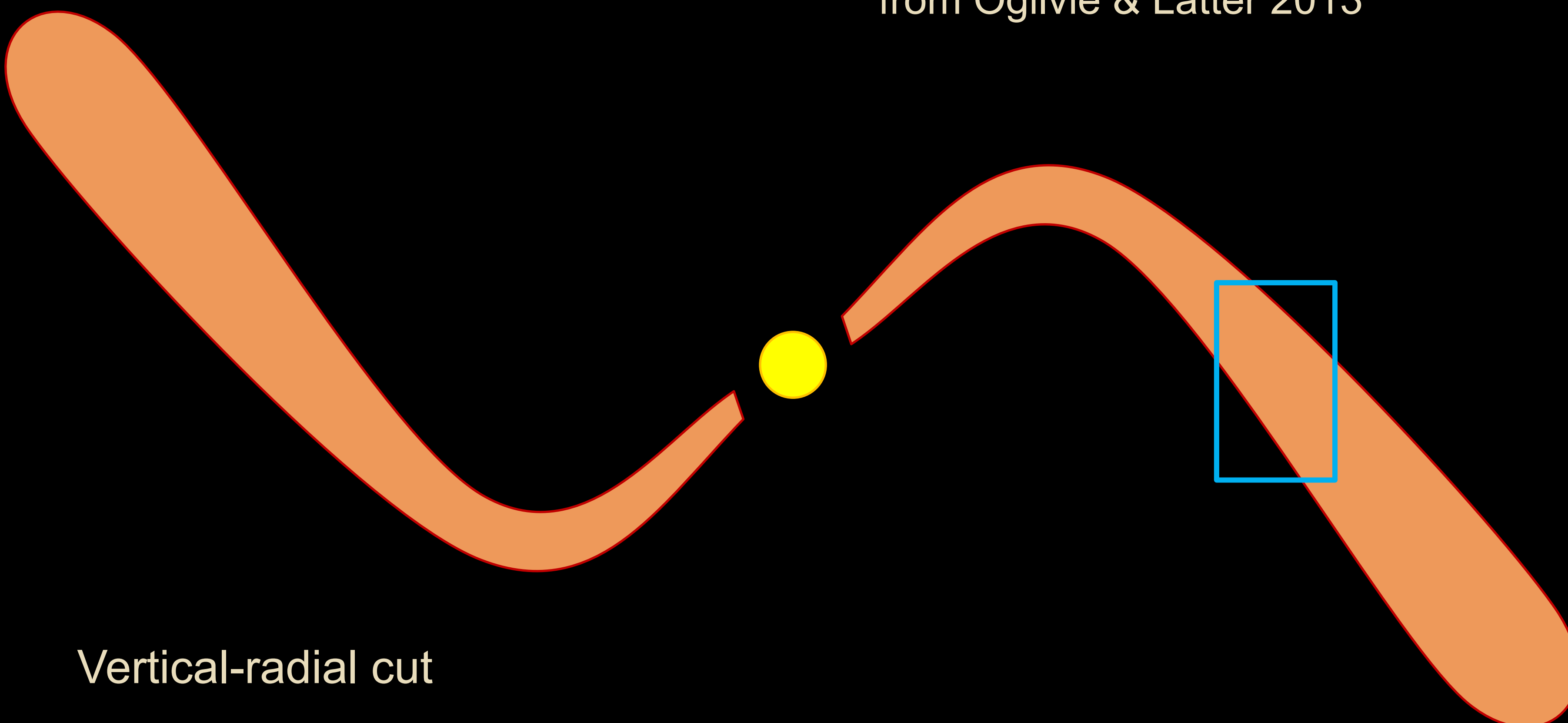
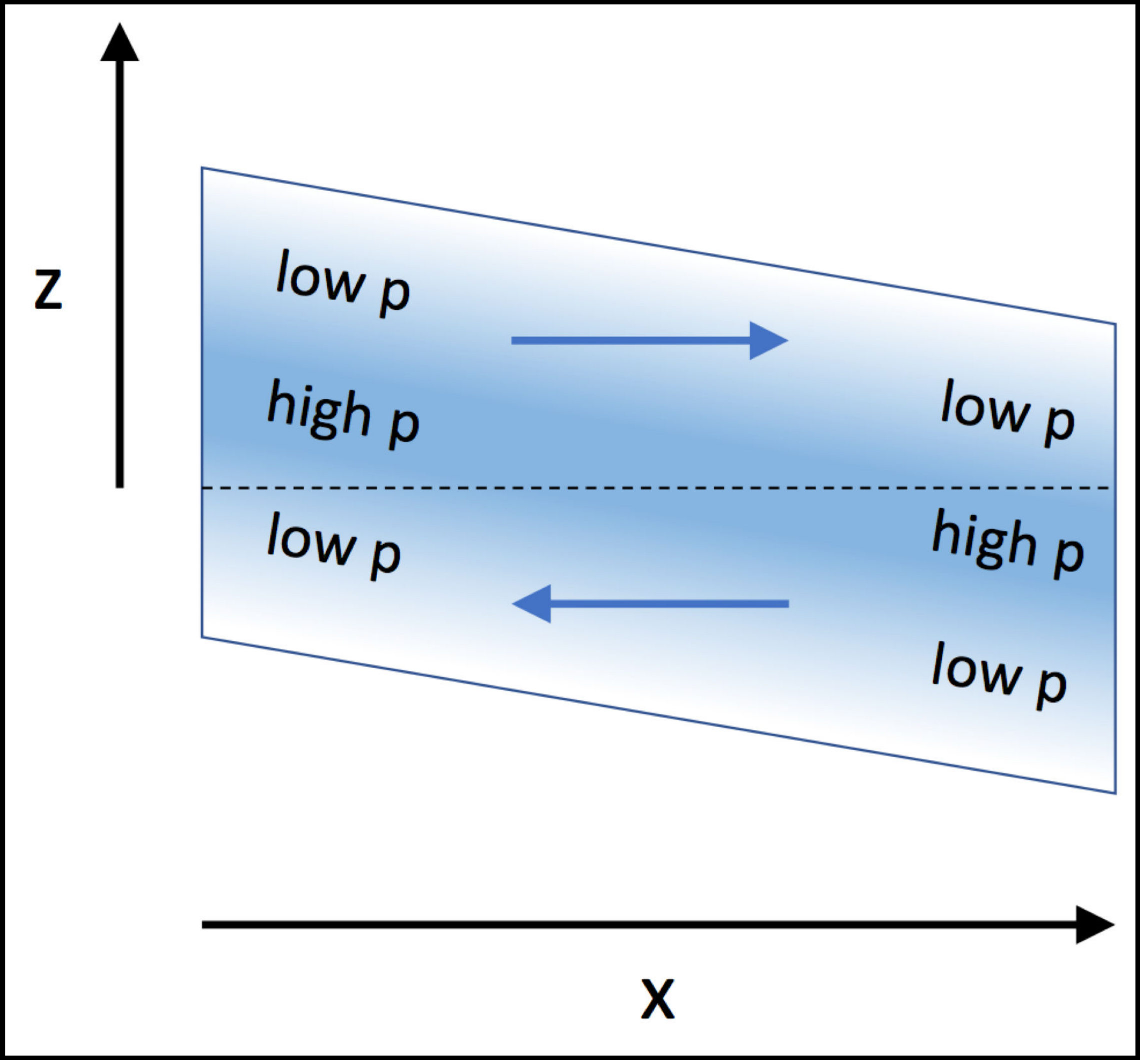


Vertical-radial cut

Sloshing motion



After an illustration in Ogilvie & Latter 2013
Principle was discovered by Papaloizou & Pringle 1983

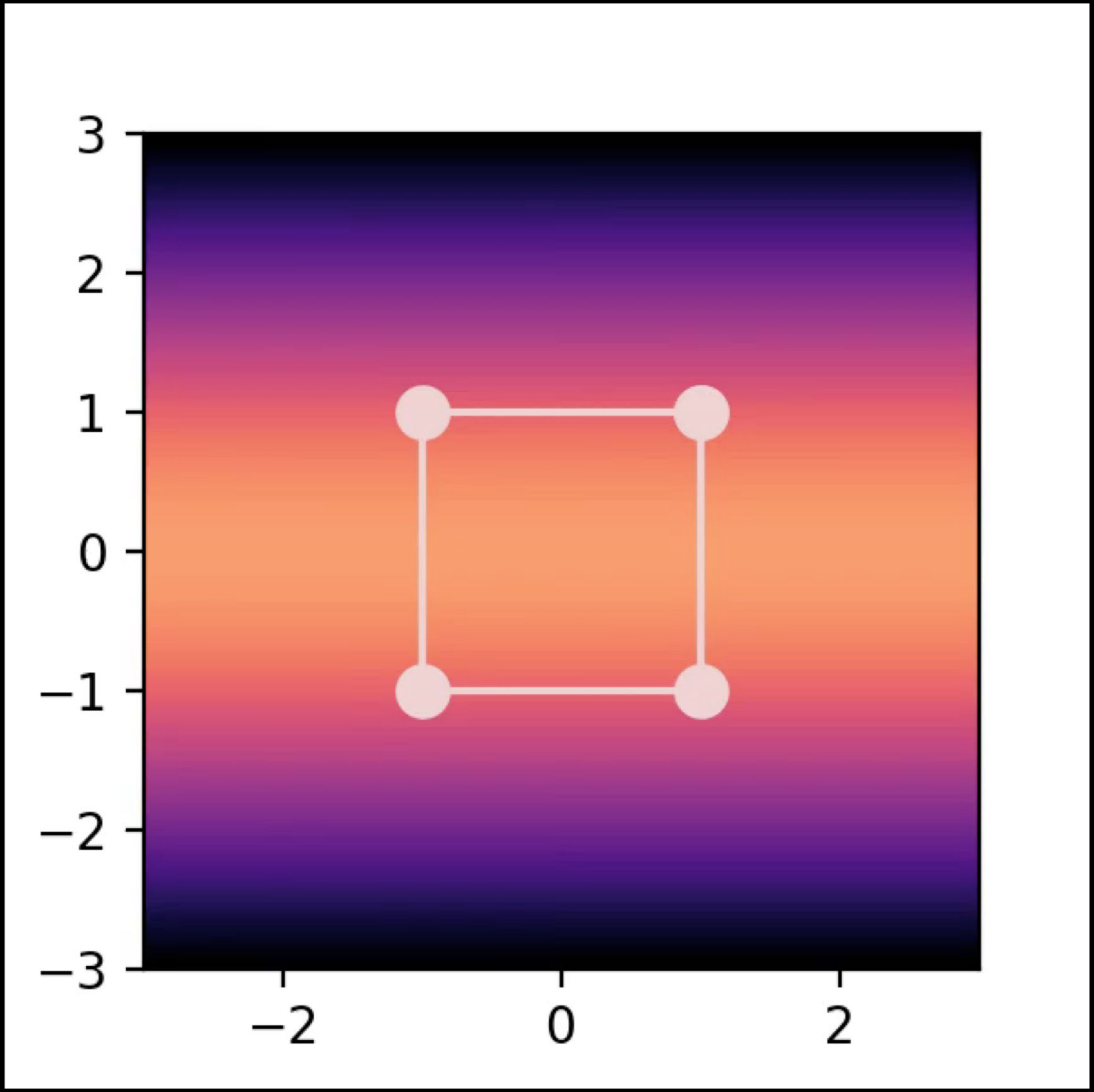
Rocking → Sloshing motion

(Papaloizou & Pringle 1983)

Sloshing creates an internal torque.

(Papaloizou & Pringle 1983)

How this works is a bit complex. It will be the topic of the second part of the tutorial.

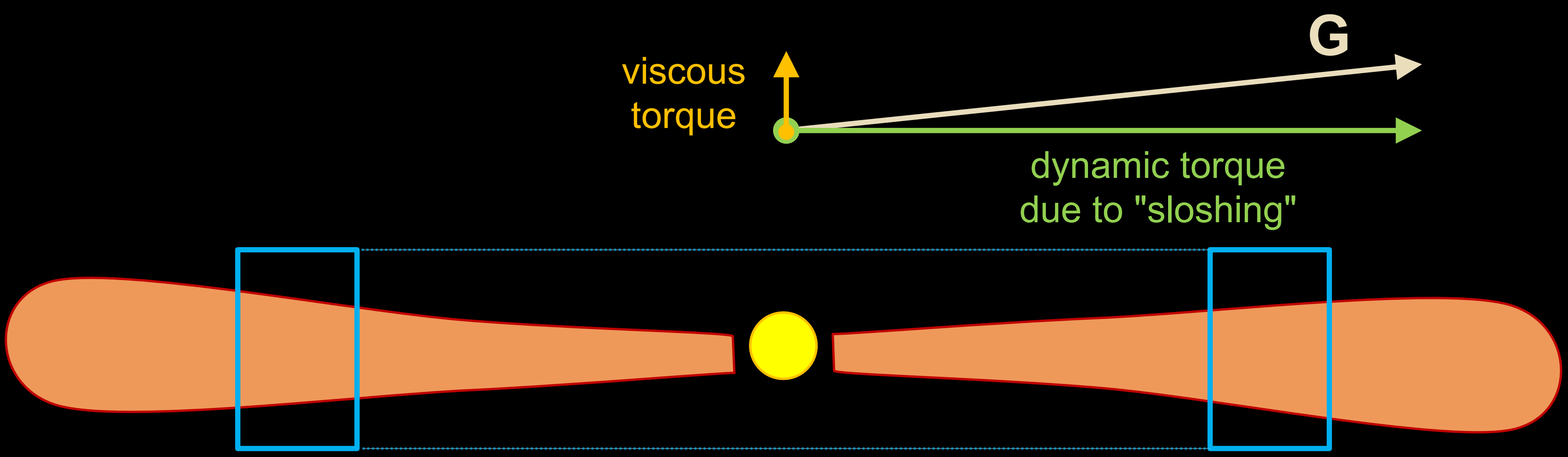


Simulation from: Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

The internal torque vector **G**

Regard G as sum of a viscous part $G^{(v)}$ and a dynamic "sloshing" part $G^{(s)}$.

$$\mathbf{G} = \mathbf{G}^{(v)} + \mathbf{G}^{(s)}$$



Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

The internal torque vector **G**

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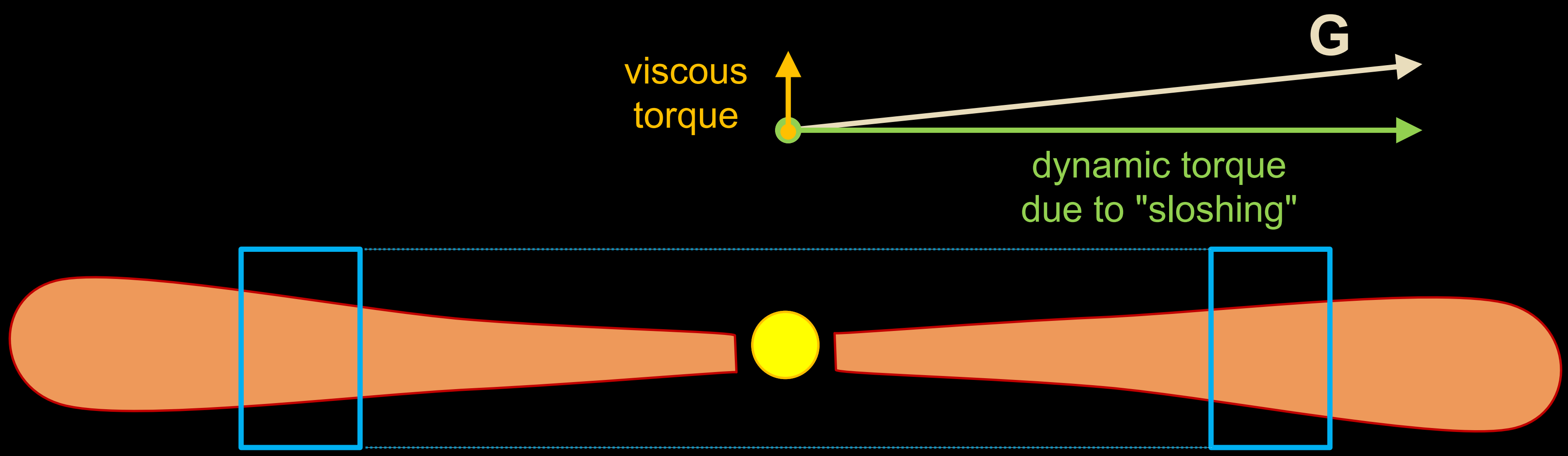


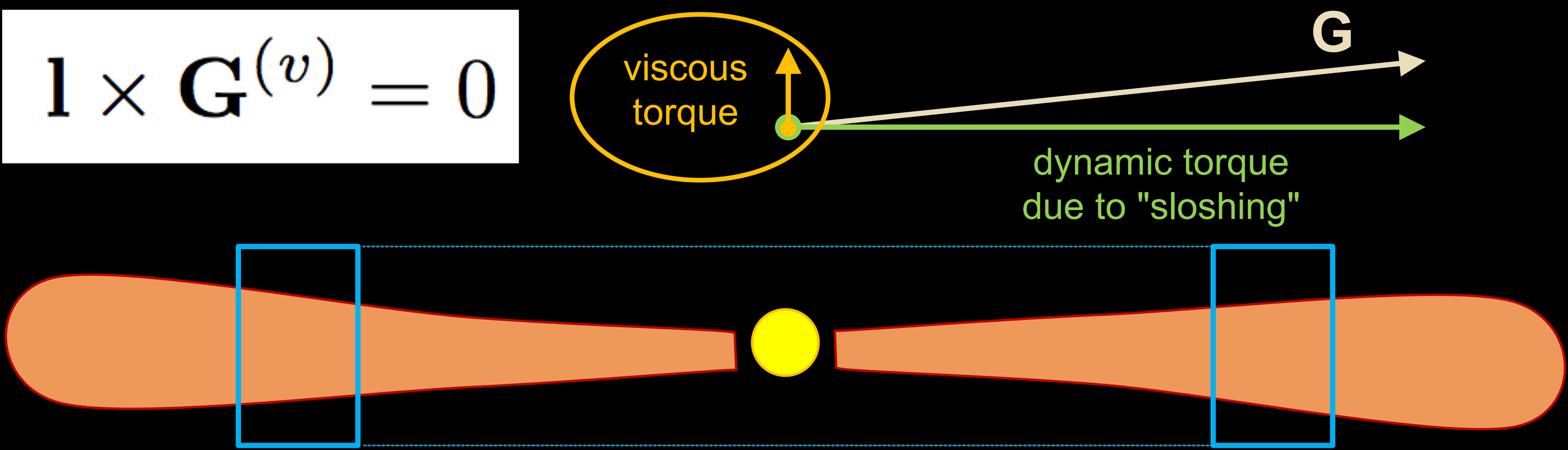
Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

The internal torque vector **G**

$$\mathbf{G} = \mathbf{G}^{(v)} + \mathbf{G}^{(s)}$$

The viscous torque drives the viscous evolution of $\Sigma(r,t)$.
It stands perpendicular to the disk.

$$\mathbf{l} \times \mathbf{G}^{(v)} = 0$$



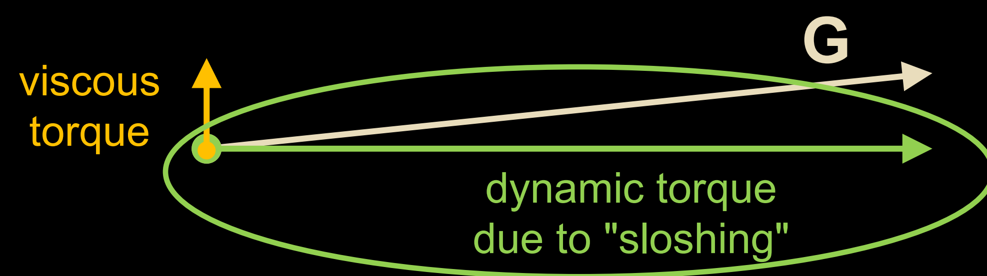
Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

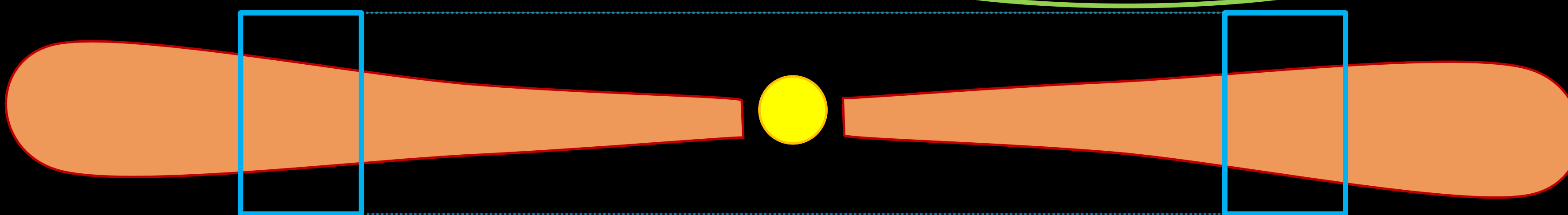
The internal torque vector **G**

$$\mathbf{G} = \mathbf{G}^{(v)} + \mathbf{G}^{(s)}$$

The sloshing torque drives the warp evolution **l**(r,t).
It lies inside the disk plane.

$$\mathbf{l} \cdot \mathbf{G}^{(s)} = 0$$



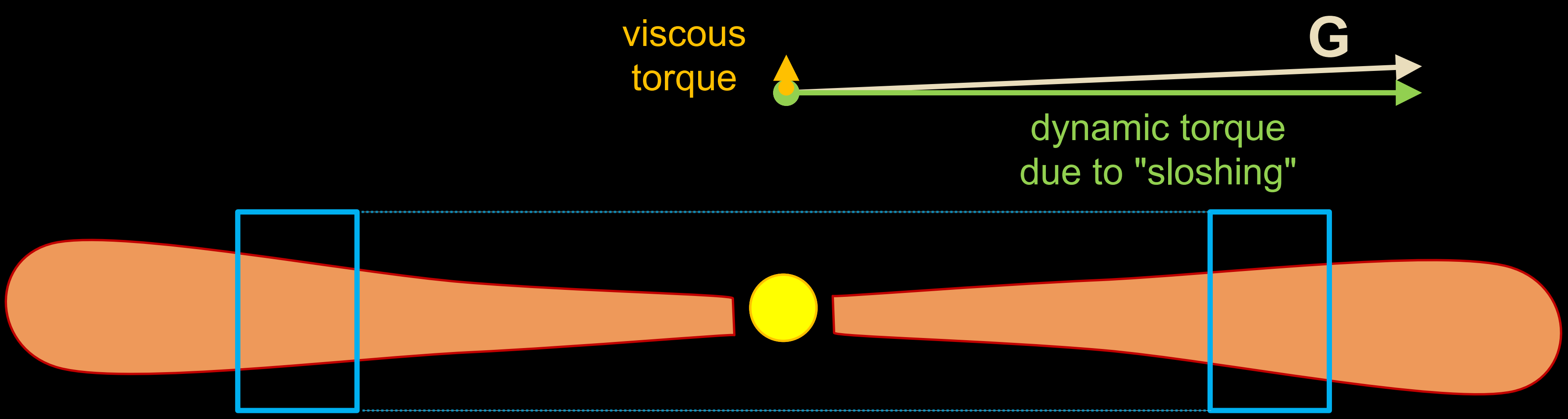


Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

The internal torque vector **G**

$$\mathbf{G} = \mathbf{G}^{(v)} + \mathbf{G}^{(s)}$$

For the usual small α the viscous torque is *tiny*, much smaller than the sloshing torque.

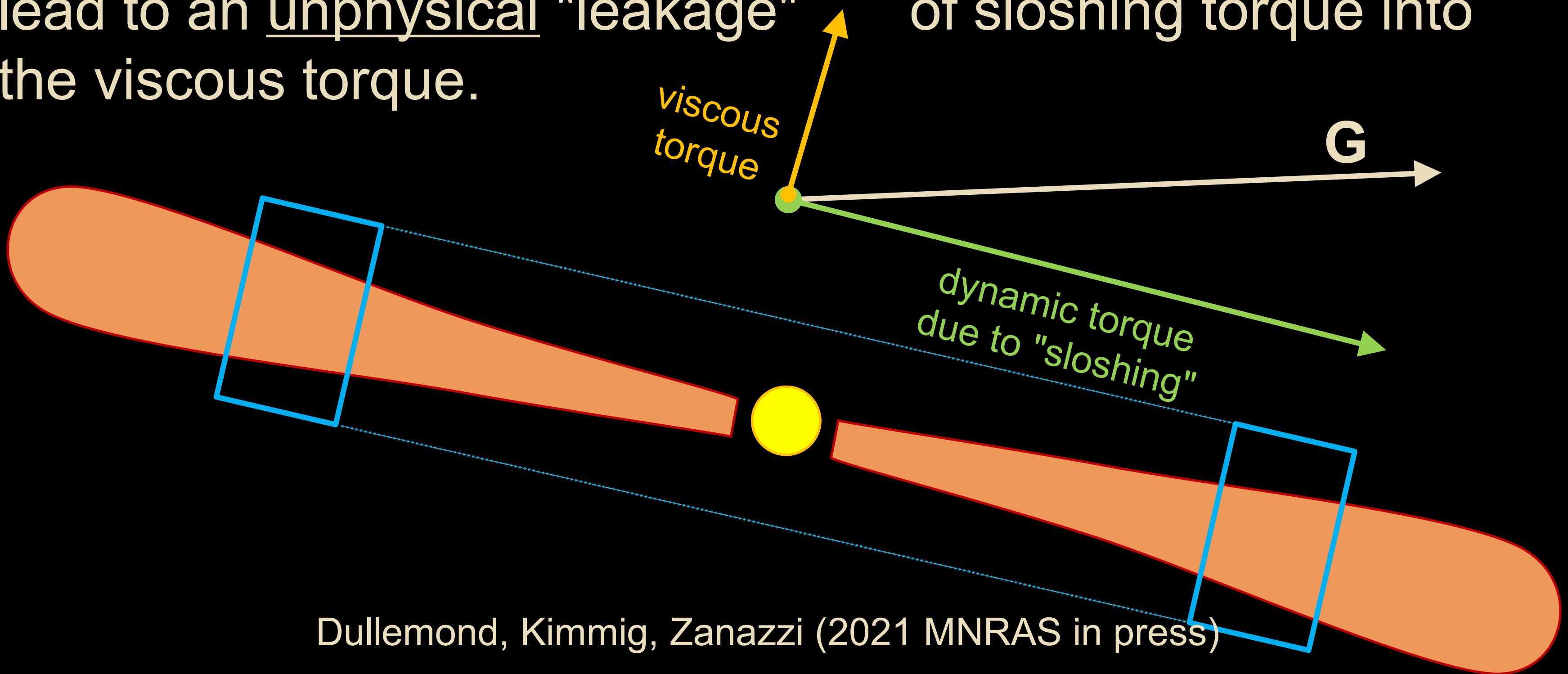


Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

The internal torque vector **G**

$$\mathbf{G} = \mathbf{G}^{(v)} + \mathbf{G}^{(s)}$$

As a result of the torque, the disk may tilt, yielding a new disk plane (a new vector **l**(r,t)). If left untreated, this may lead to an unphysical "leakage" of sloshing torque into the viscous torque.

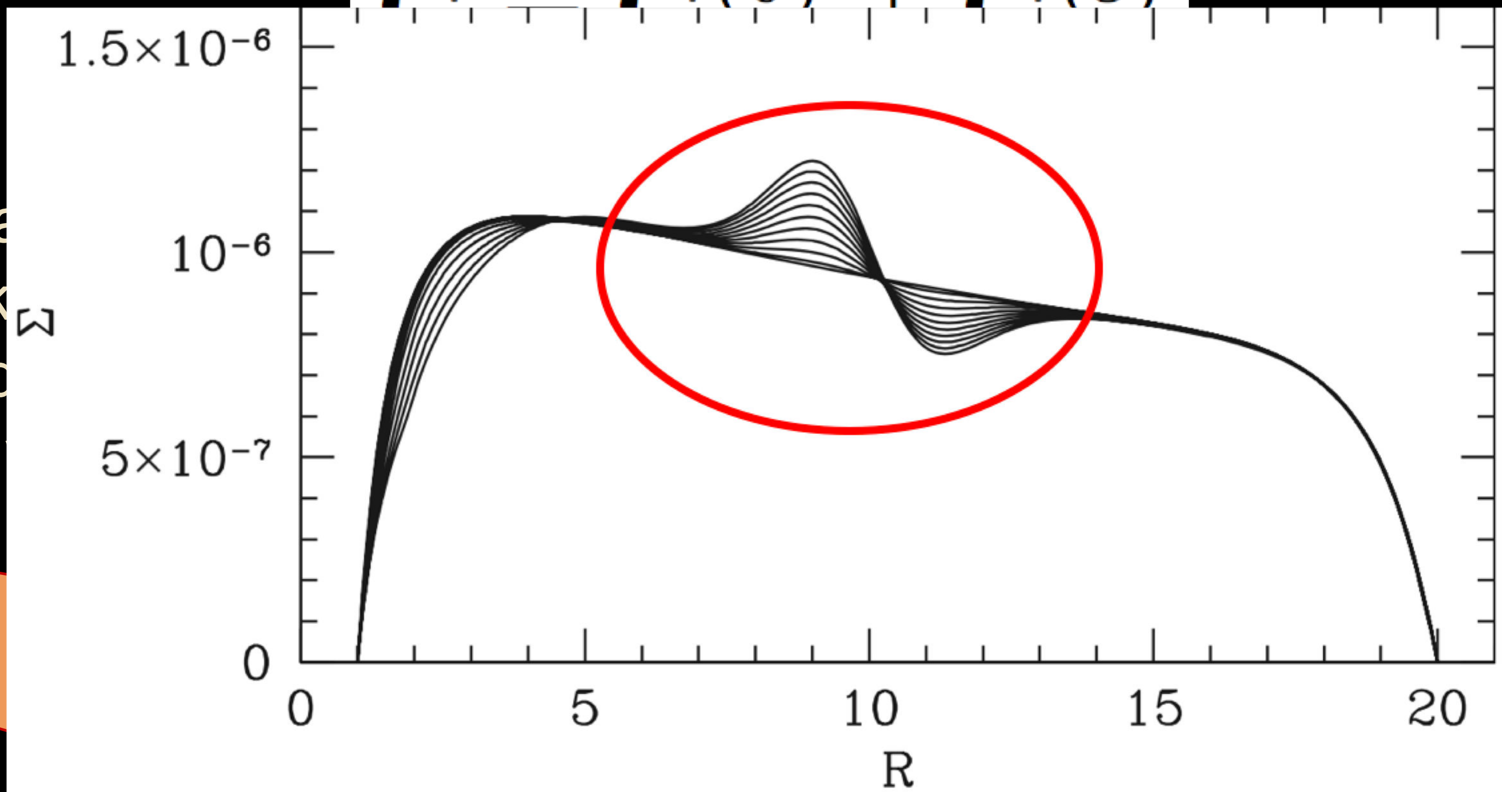


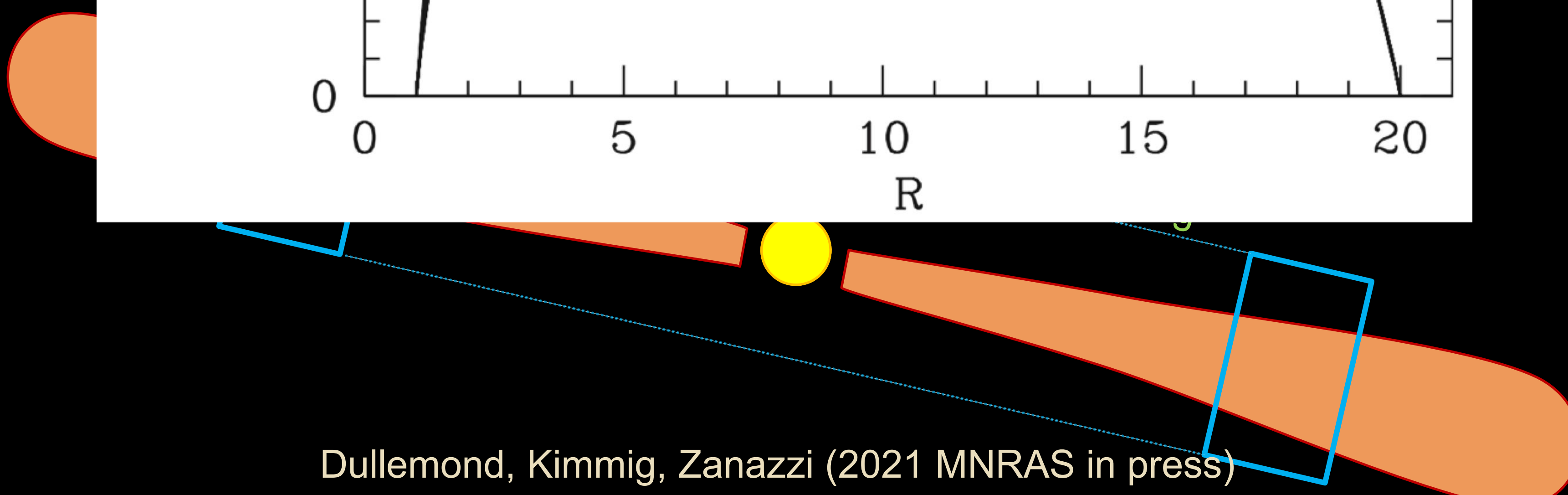
Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

The internal torque vector **G**

$$\mathbf{G} = \mathbf{G}^{(v)} + \mathbf{G}^{(s)}$$

As a
disk
lead o
the



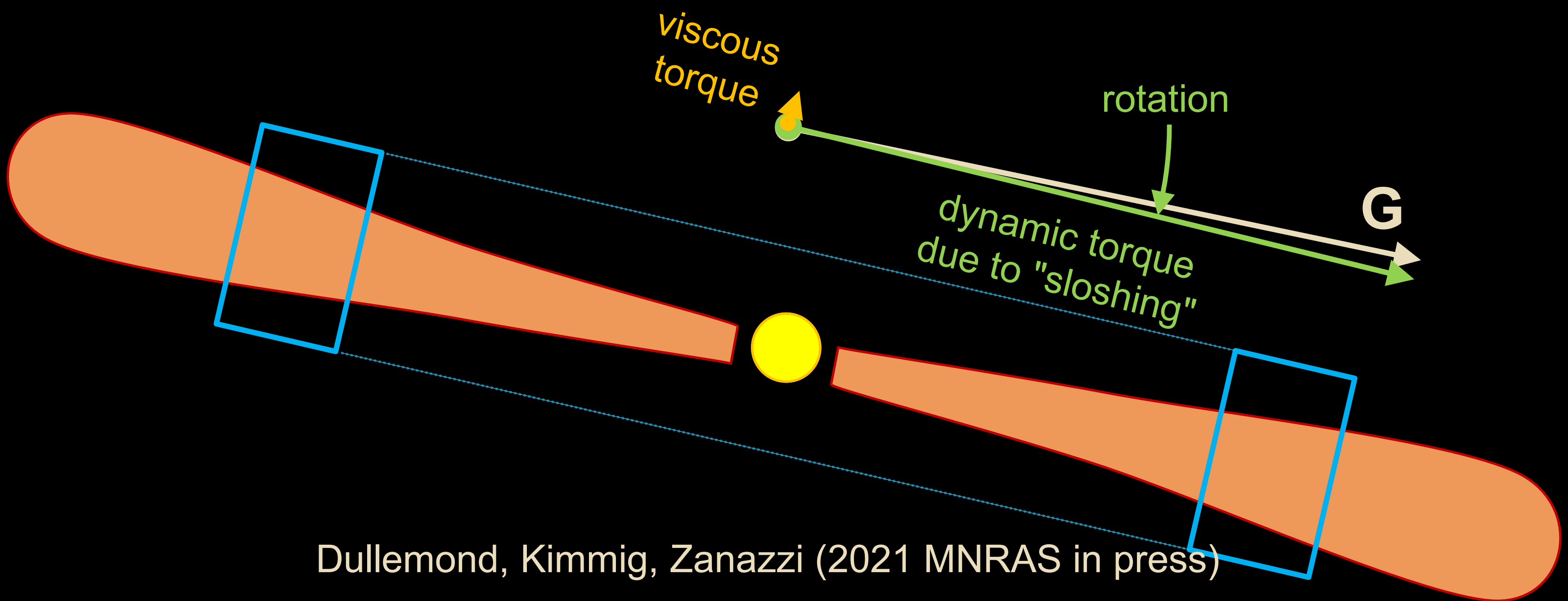


Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

The internal torque vector **G**

$$\frac{\partial \mathbf{G}^{(s)}}{\partial t} + \alpha\Omega\mathbf{G}^{(s)} = -\frac{g_0}{4}\frac{d\mathbf{l}}{d\ln r} + \left(\mathbf{l}\times\frac{d\mathbf{l}}{dt}\right)\times\mathbf{G}^{(s)}$$

In our new formalism we rotate the sloshing torque with the plane of the disk.



Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

Summary: Our new equations

$$\mathbf{G} = \mathbf{G}^{(v)} + \mathbf{G}^{(s)}$$

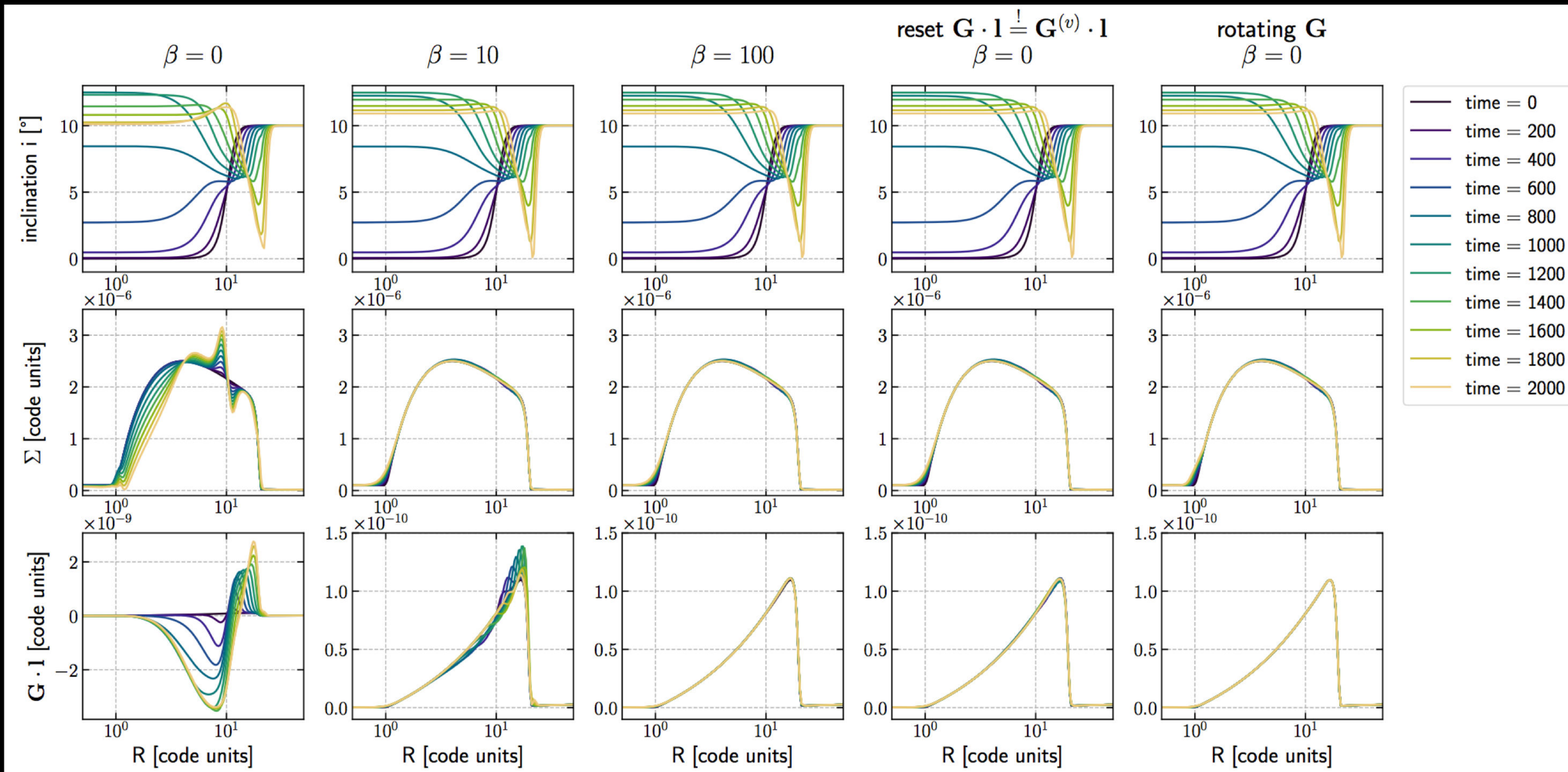
$$\mathbf{G}^{(v)} = \frac{3\,g_0}{2\alpha\Omega}\alpha^2 \mathbf{l}$$

Static viscous torque

$$\frac{\partial \mathbf{G}^{(s)}}{\partial t} + \alpha\Omega\mathbf{G}^{(s)} = -\frac{g_0}{4}\frac{d\mathbf{l}}{d\ln r} + \left(\mathbf{l} \times \frac{d\mathbf{l}}{dt}\right) \times \mathbf{G}^{(s)}$$

Dynamic sloshing torque

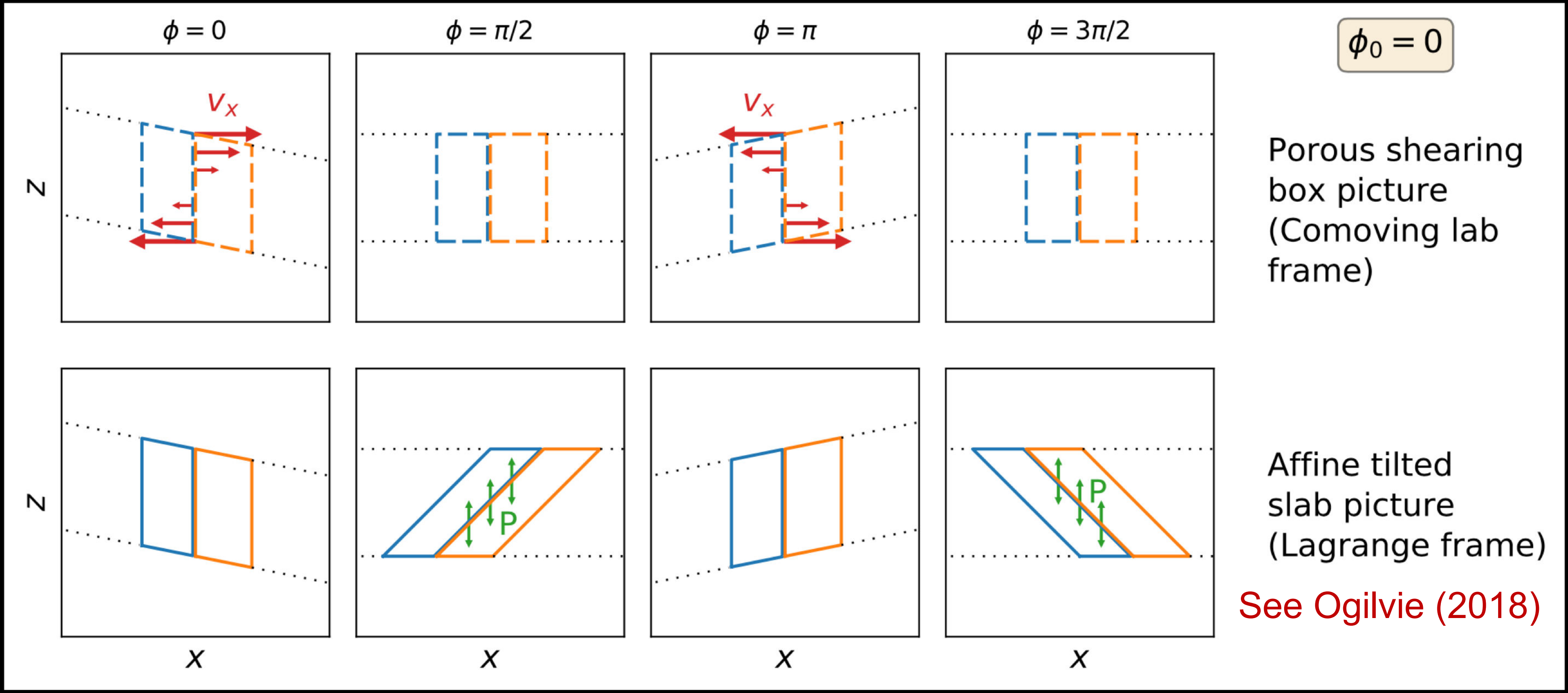
Numerical test of the equations



Dullemond, Kimmig, Zanazzi (2021 MNRAS in press)

So... Why does the "sloshing motion" create a torque??

Two perspectives: Lab vs. Lagrange

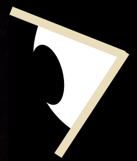


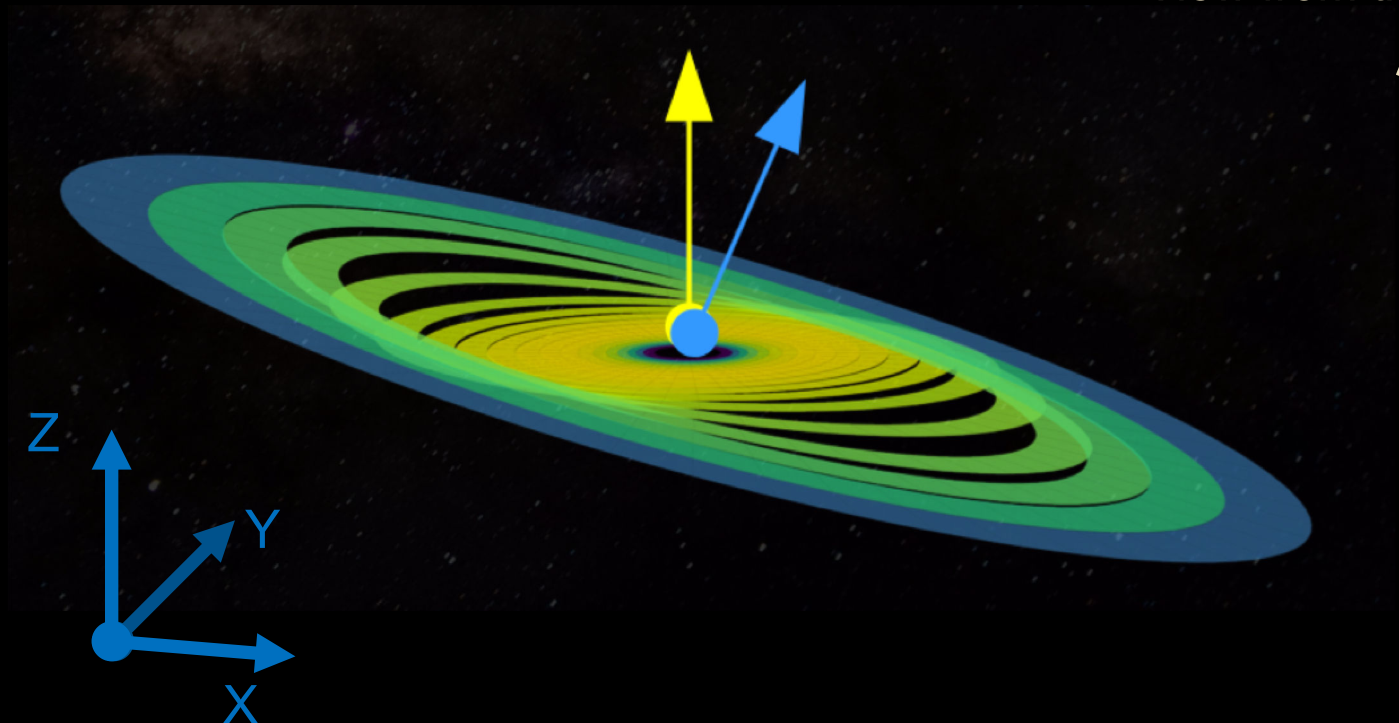
See Ogilvie (2018)

Here: Case of perfectly Keplerian disk, i.e. no apsidal precession. That implies: $\Omega_{\text{epicycle}} = \Omega_{\text{Kepler}}$ and thus ϕ_0=0 → pure warp damping, no warp twisting.

Two perspectives: Lab vs. Lagrange

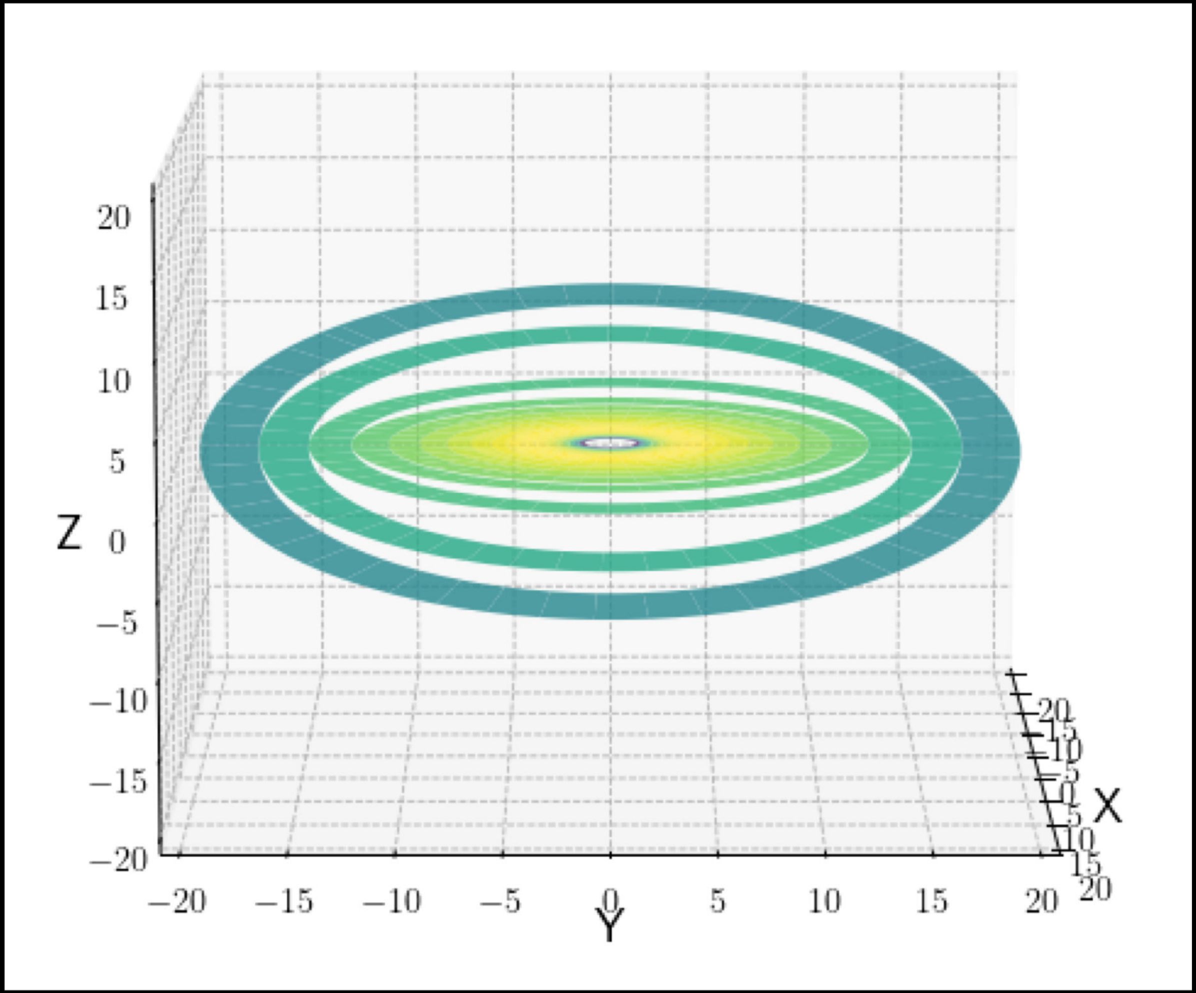
View from this perspective:





<u>Here:</u> Case of perfectly Keplerian disk, i.e. no apsidal precession. That implies: $\Omega_{epicycle} = \Omega_{Kepler}$ and thus $\phi_0=0$ → <u>pure warp damping</u>, no warp twisting.

Two perspectives: Lab vs. Lagrange



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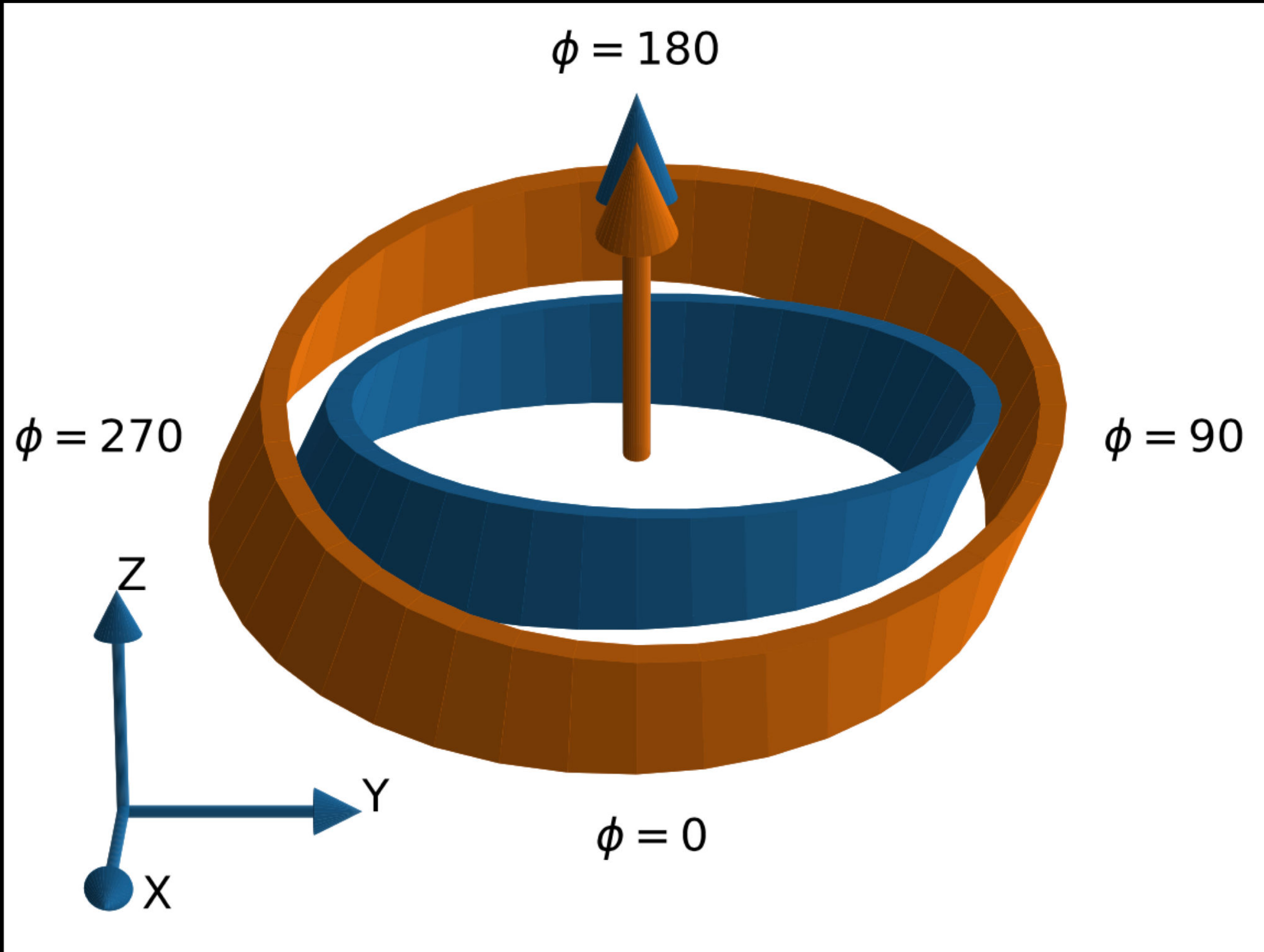
Two perspectives: Lab vs. Lagrange



Here: Case of perfectly Keplerian disk, i.e. no apsidal precession. That implies: $\Omega_{\text{epicycle}} = \Omega_{\text{Kepler}}$ and thus ϕ_0=0 → pure warp damping, no warp twisting.

Conclusion Part 1

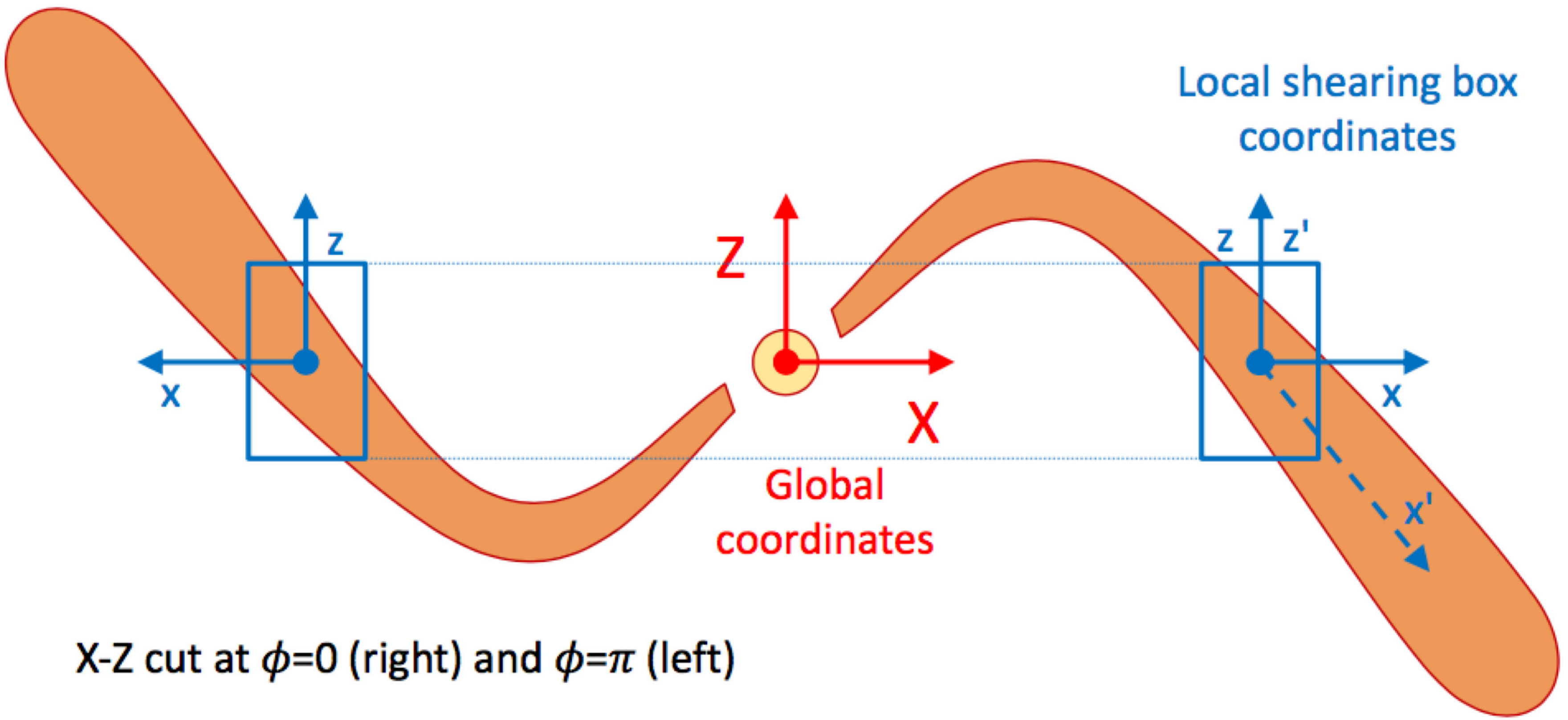
- The generalized warped disk equations of Martin et al. 2019 are correct
- But they require an ad-hoc β-damping to avoid unphysical "leakage" of sloshing torque into viscous torque.
- This β-damping makes the equations "stiff", and the value of β is ad-hoc.
- Our new generalized warped disk equations are:
  - Derived from first principles (using shearing box eqs)
  - Solve the "leaking" by rotation
  - Useful for cases where disks evolve over long times.

Part 2

The math of sloshing in a local shearing box

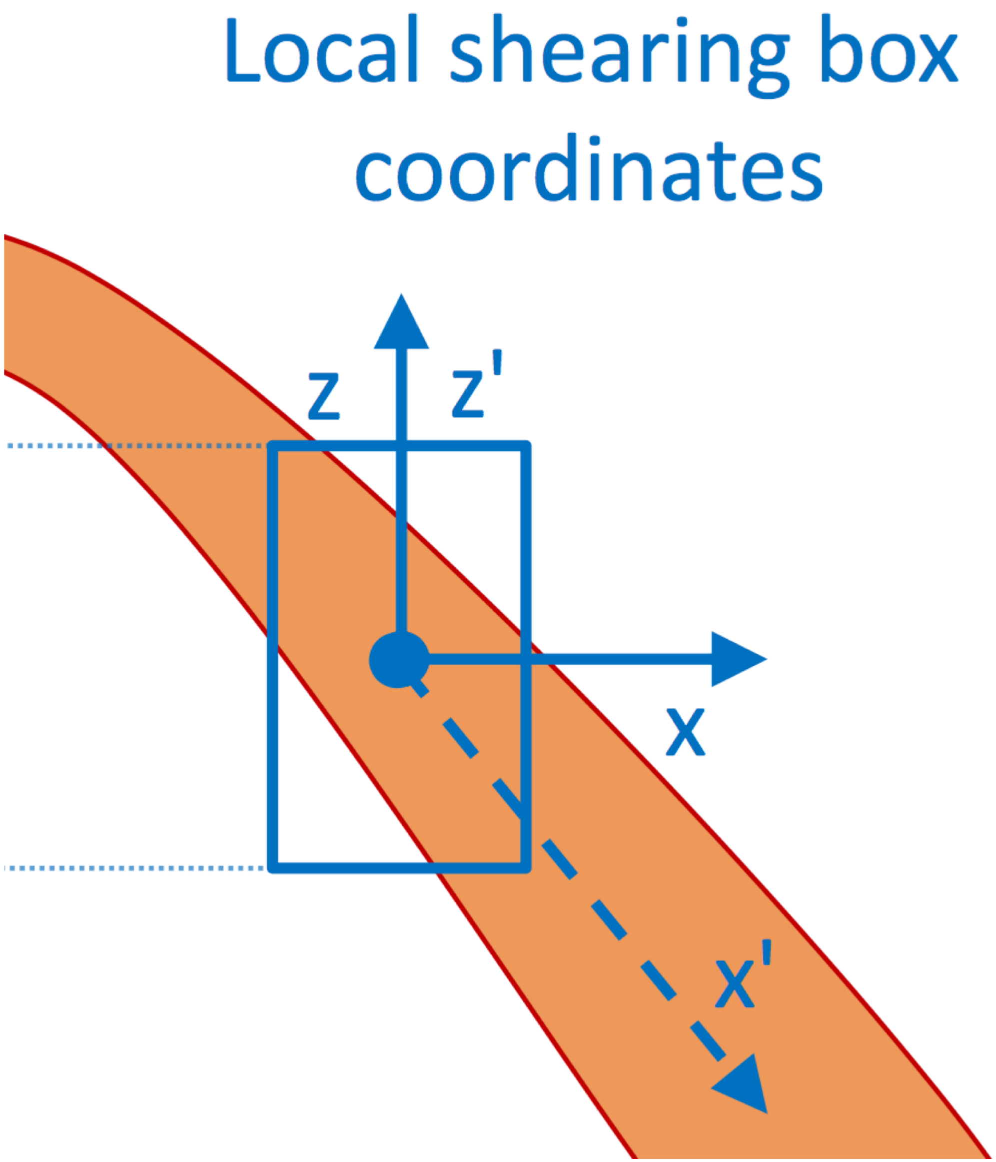
Ogilvie & Latter (2013a)
Dullemond, Kimmig & Zanazzi (2021)

Shearing box analysis

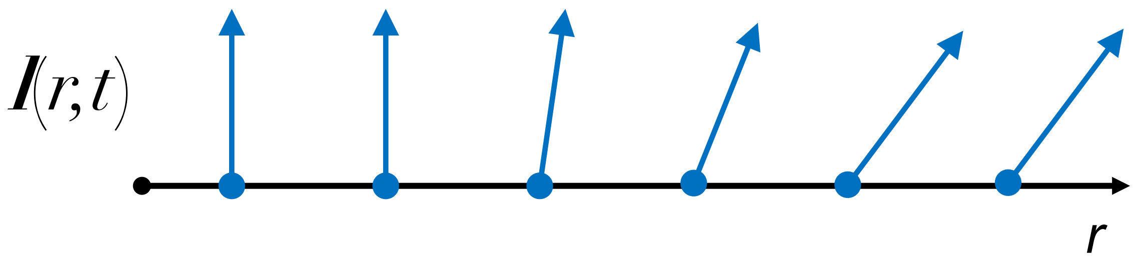


X-Z cut at ϕ=0 (right) and ϕ=π (left)

Shearing box analysis



Shearing box analysis



Warp amplitude vector:

$$\boldsymbol{\psi}(r) = \frac{\mathrm{d}\boldsymbol{l}(r)}{\mathrm{d}\ln r},$$

Warp amplitude:

$$\psi(r) = |\boldsymbol{\psi}(r)|\,.$$

Shearing box analysis

Deviations from Kepler from pressure gradients:

$$v_{\phi,g} - v_K = \frac{1}{2}\frac{c_s^2}{v_K}\left(\frac{d\ln p}{d\ln r}\right)$$

Deviations from Kepler for our analysis:

$$q = -\frac{\mathrm{d}\ln\Omega}{\mathrm{d}\ln r},$$

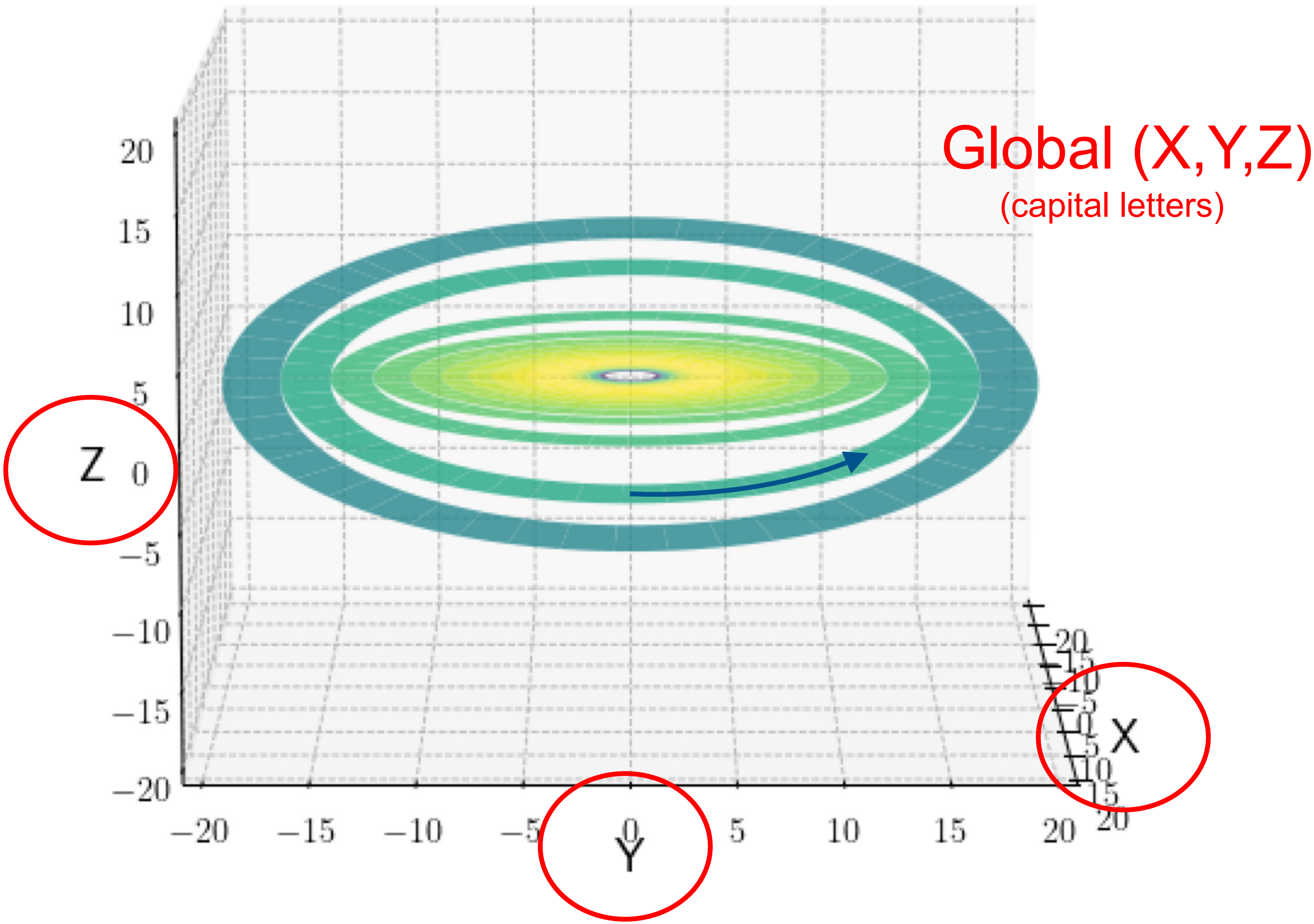
Kepler: q=3/2

$$\Omega_e = \sqrt{2(2-q)}\,\Omega$$

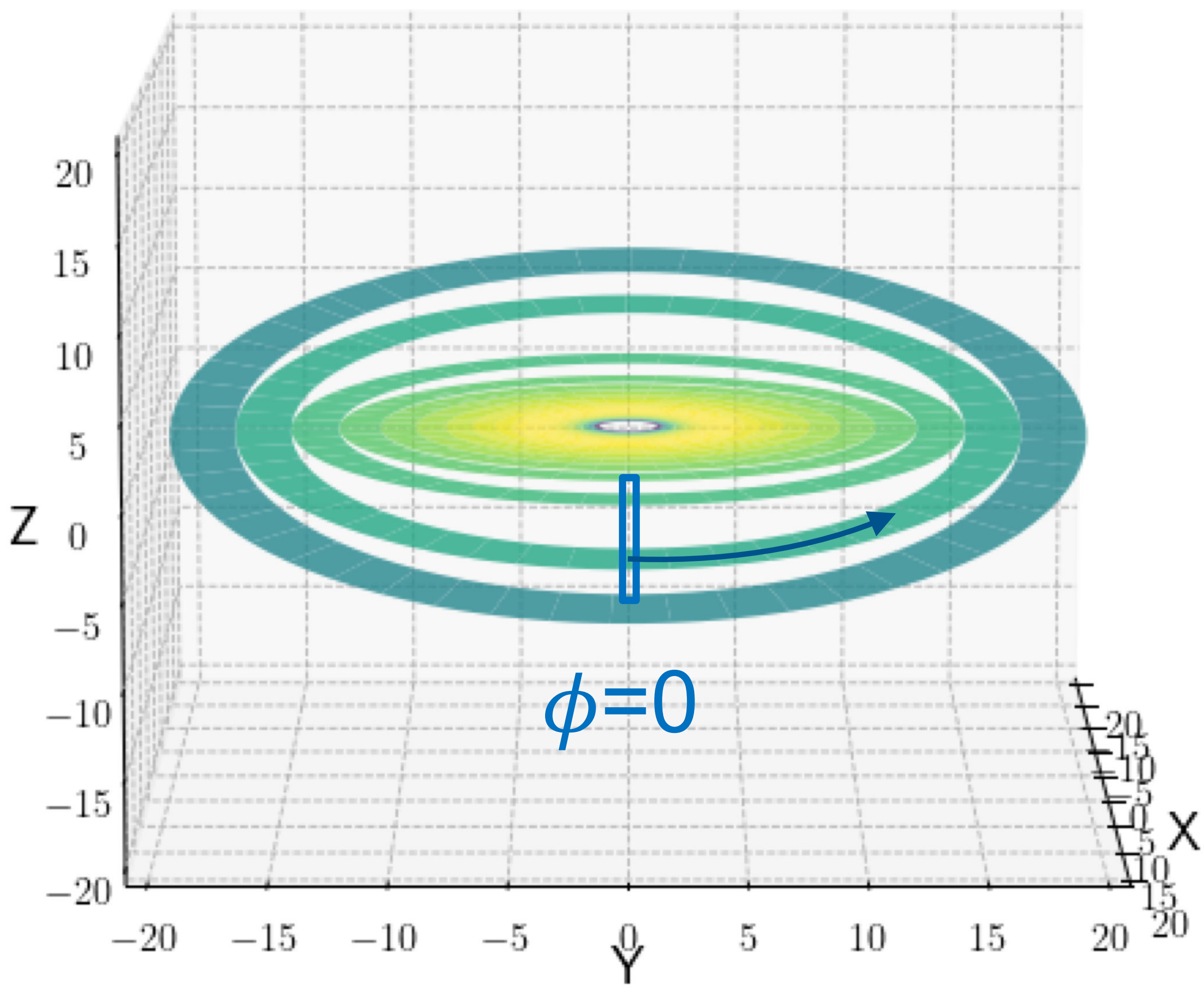
Epicyclic frequency

Kepler: $\Omega_e = \Omega$

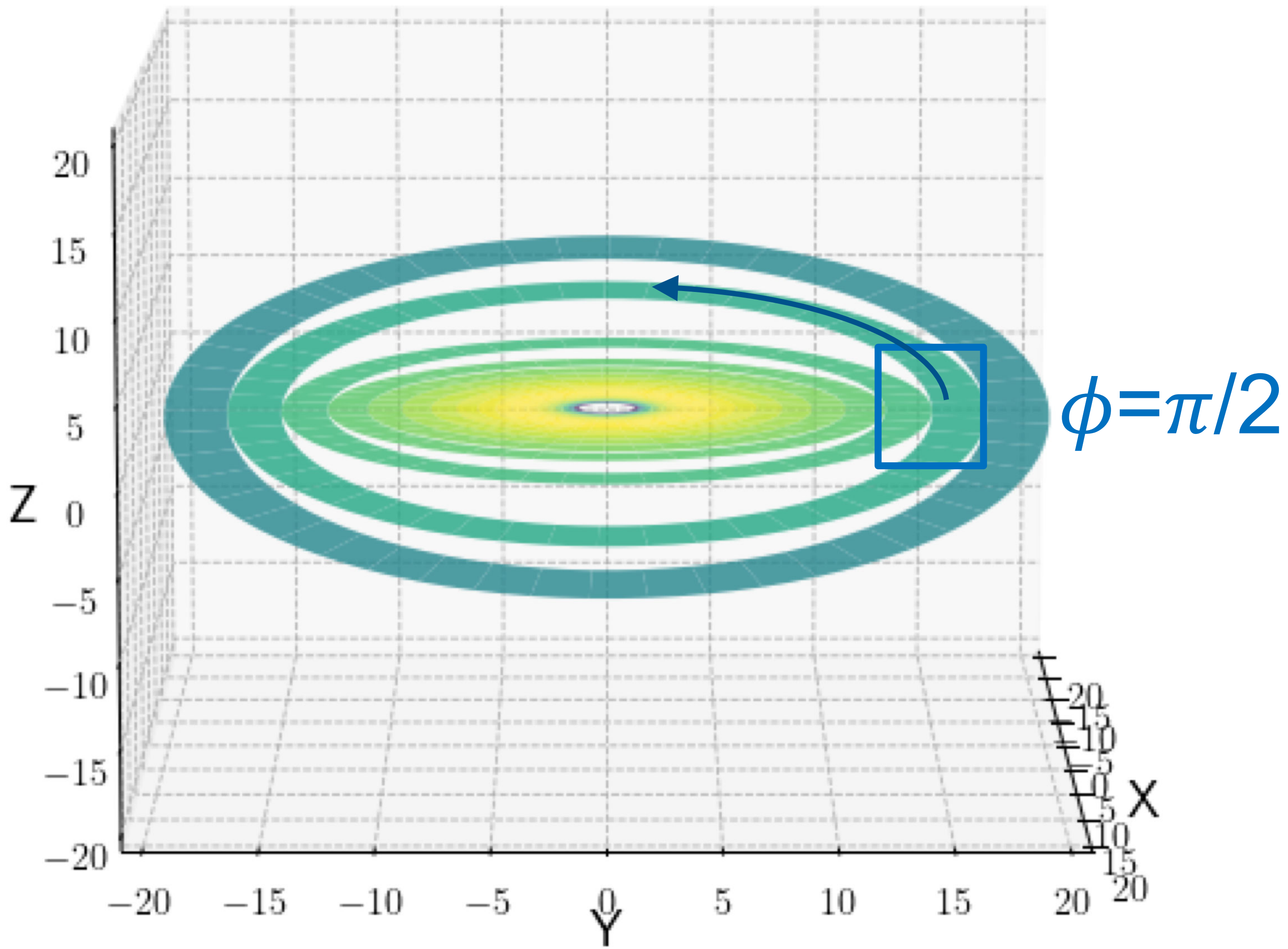
Shearing box analysis



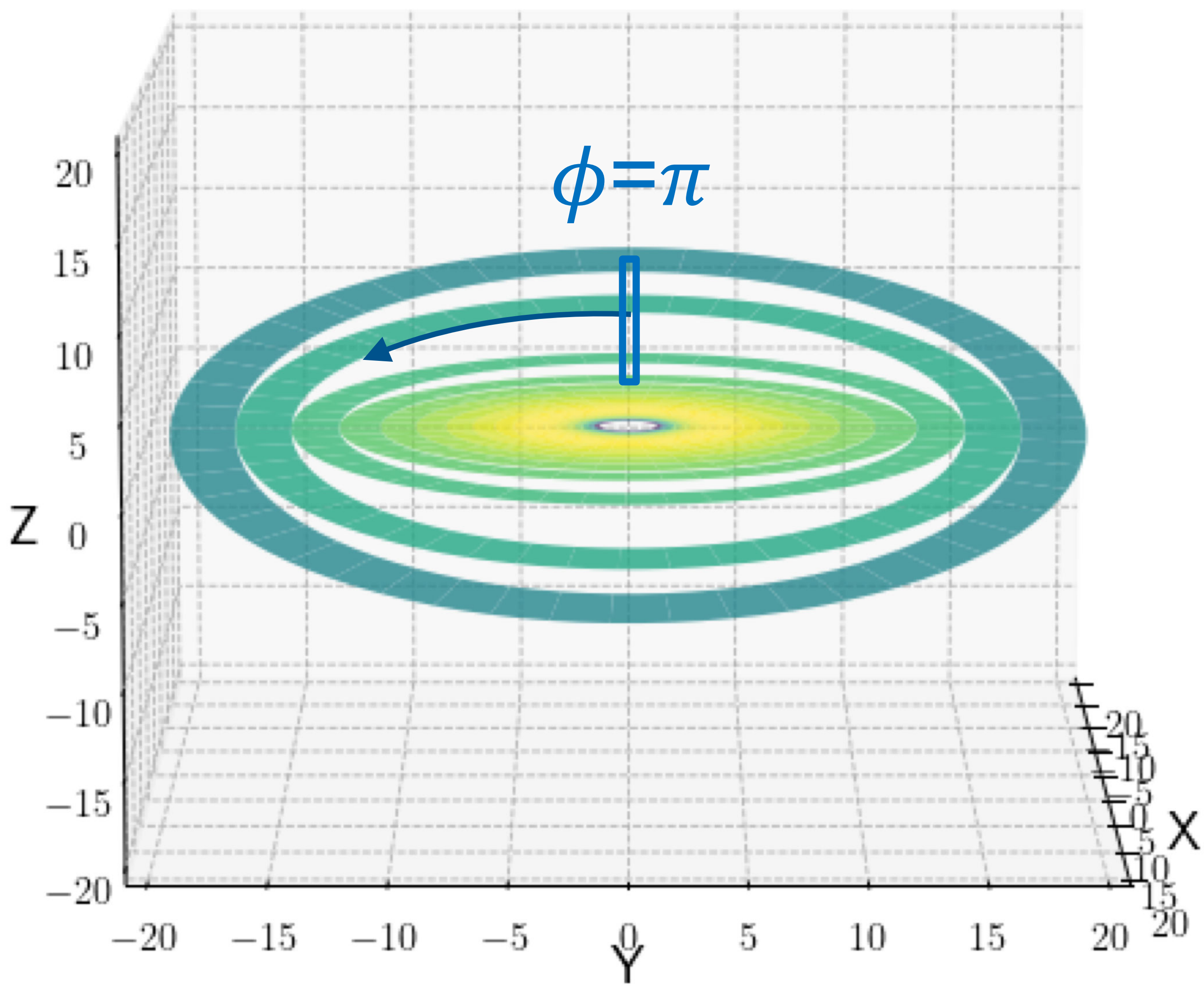
Shearing box analysis



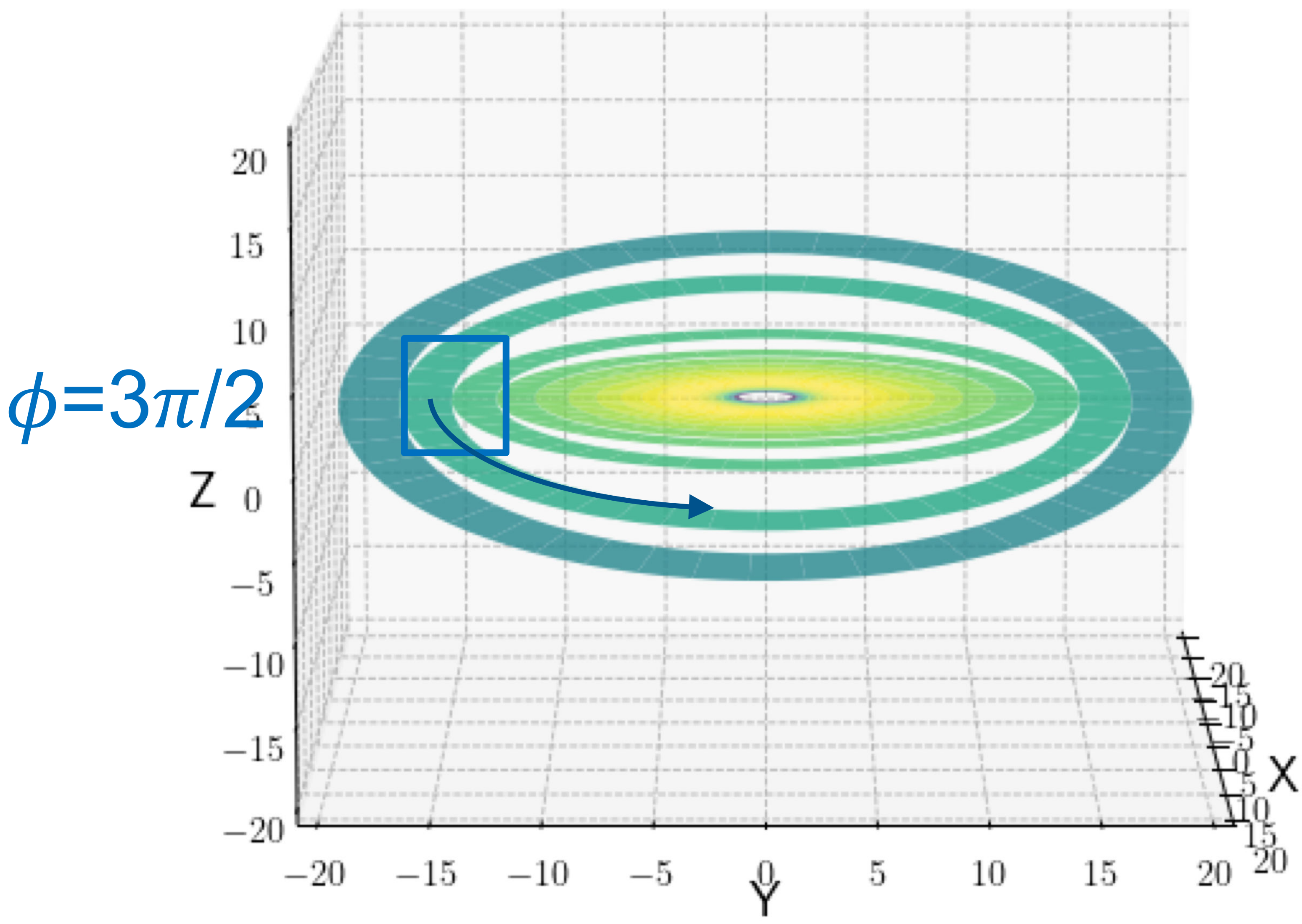
Shearing box analysis



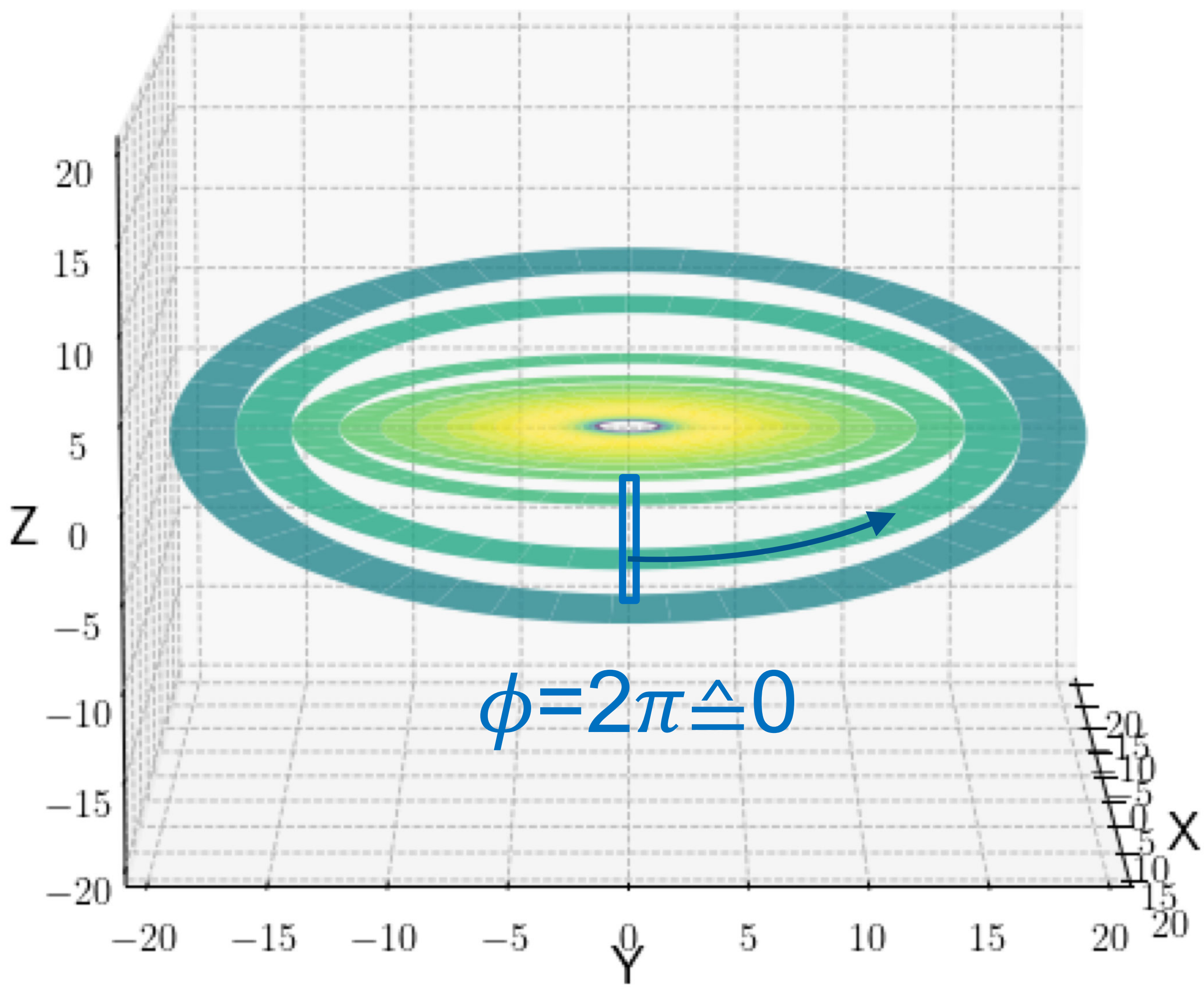
Shearing box analysis



Shearing box analysis

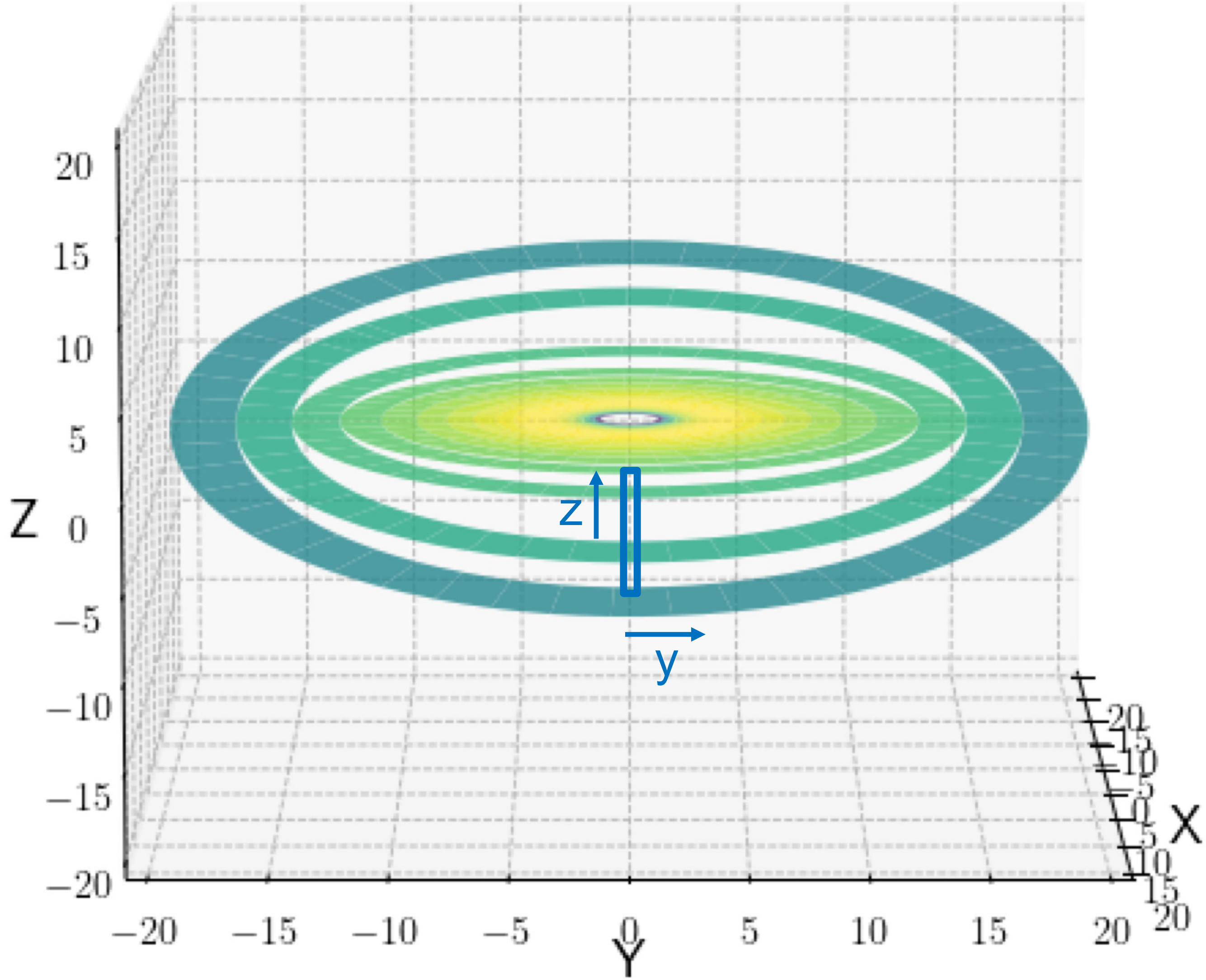


Shearing box analysis



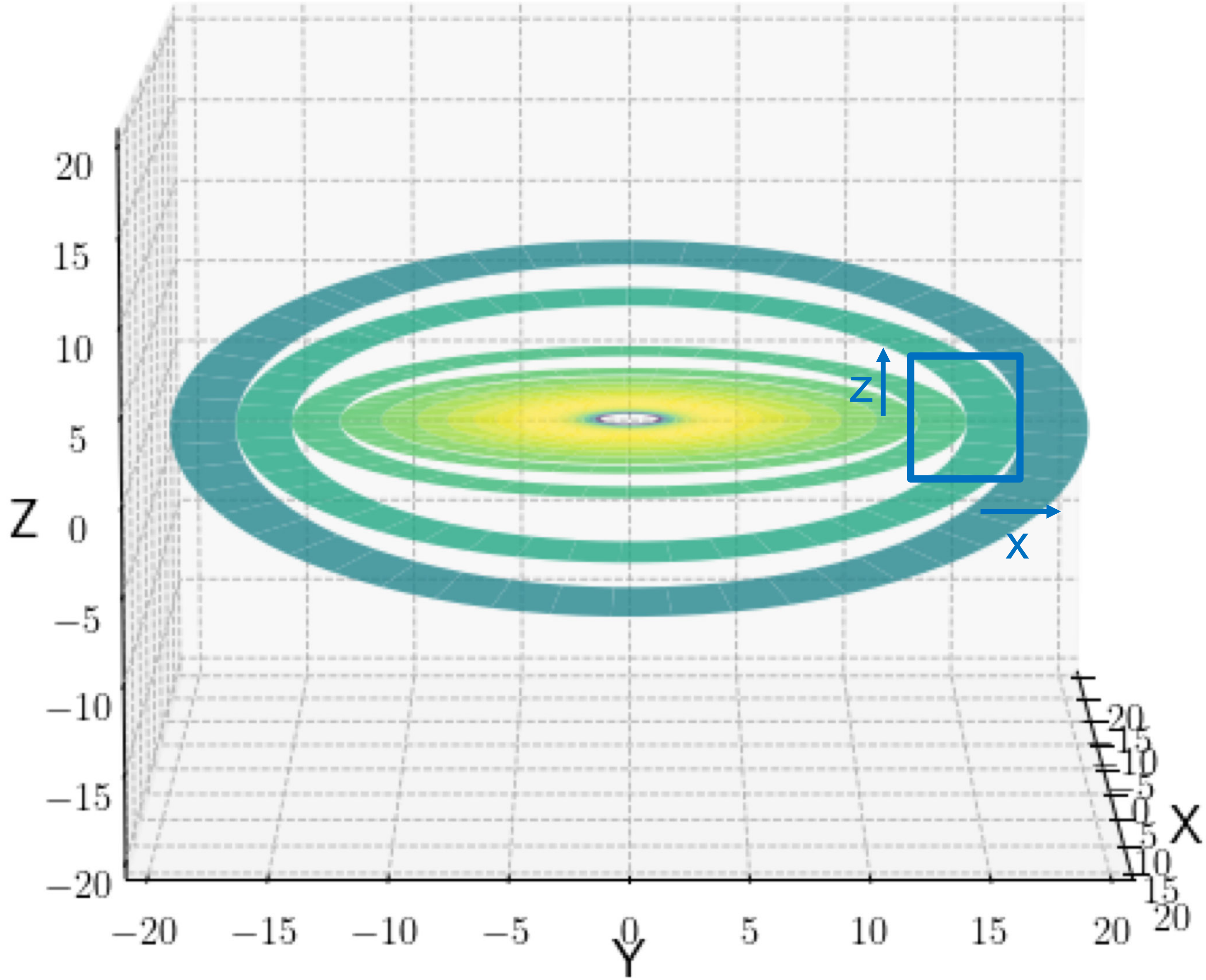
Shearing box analysis

Unwarped local (x,y,z) coordinates



Shearing box analysis

Unwarped local (x,y,z) coordinates



Shearing box analysis

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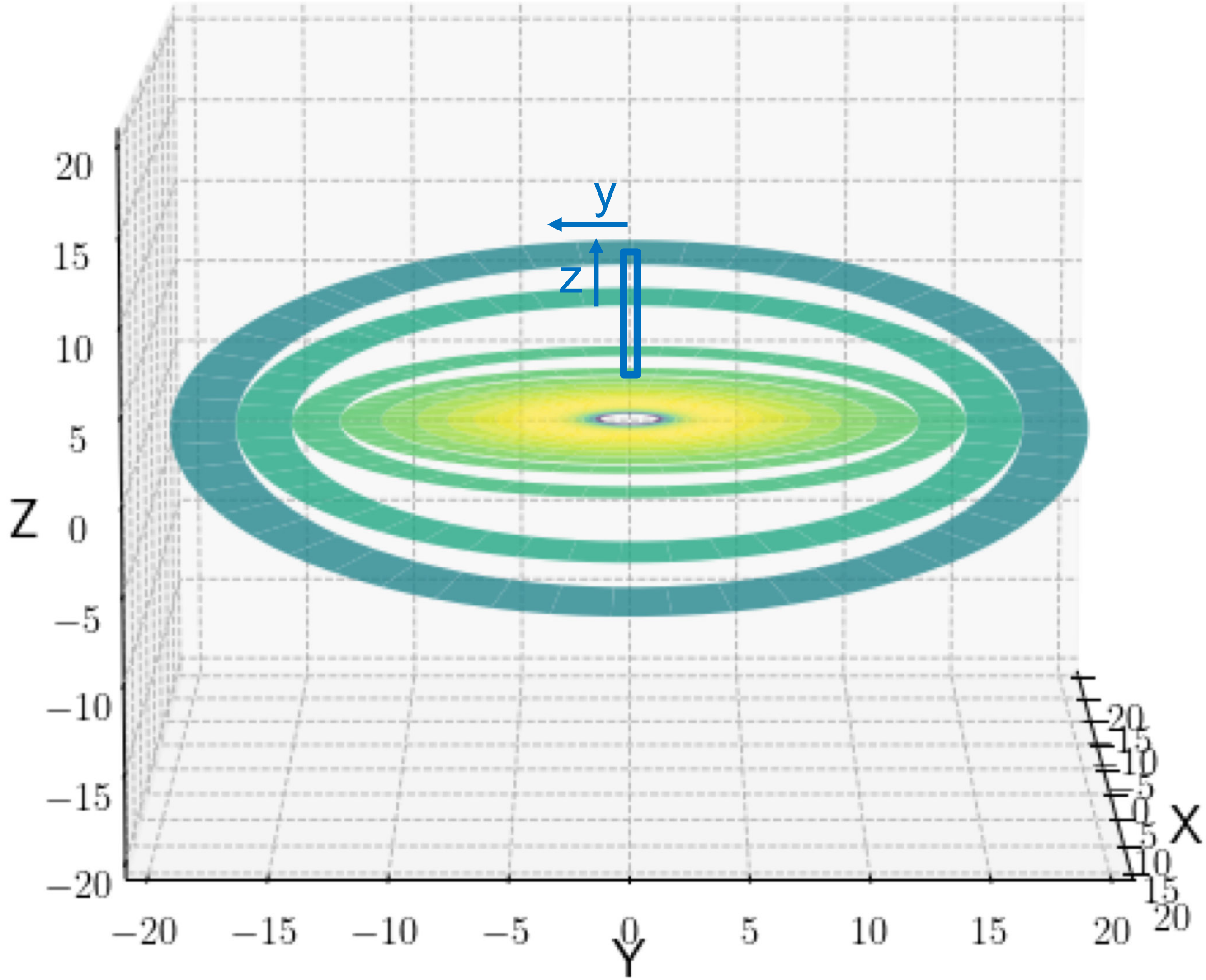
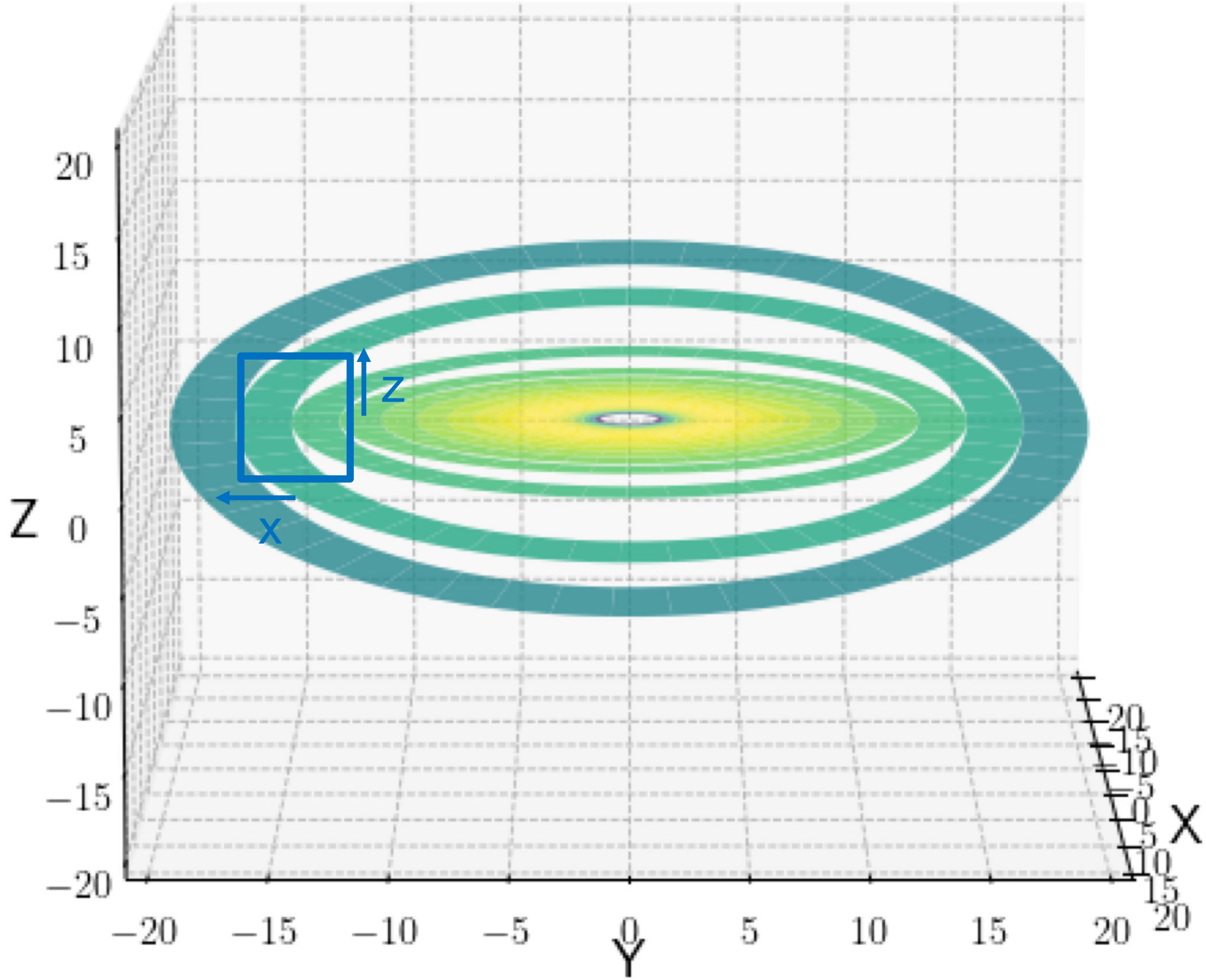


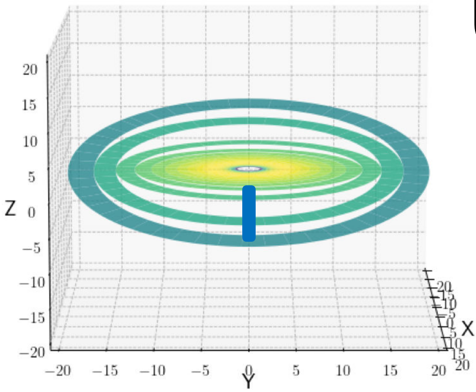
Shearing box analysis

Unwarped local (x,y,z) coordinates

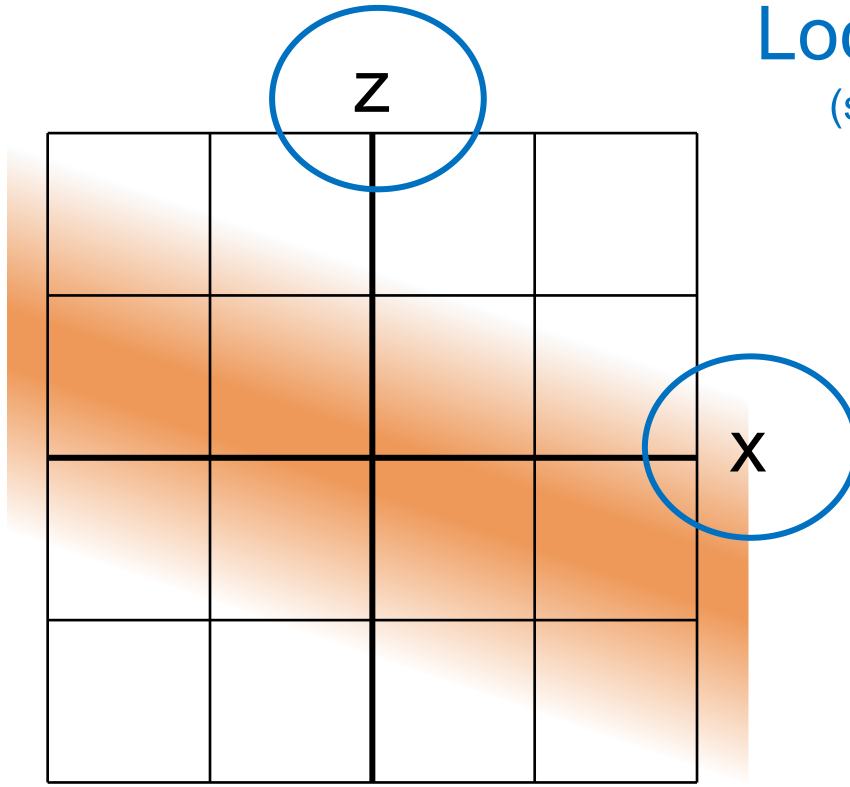


Shearing box analysis

Unwarped local (x,y,z) coordinates



ϕ=0

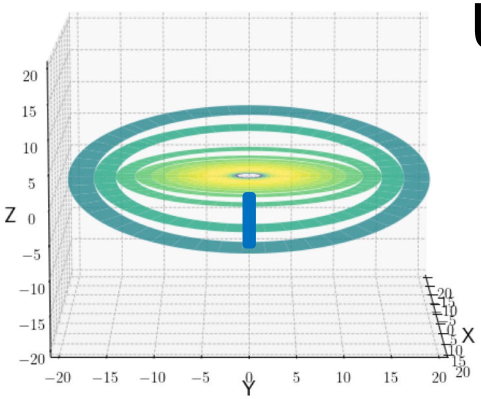


Local (x,y,z)

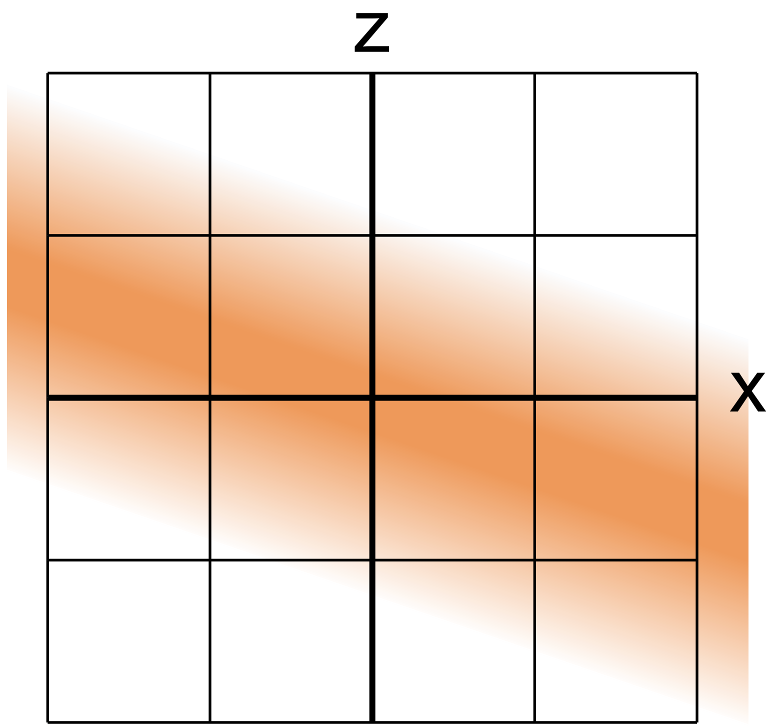
(small letters)

Shearing box analysis

Unwarped local (x,y,z) coordinates

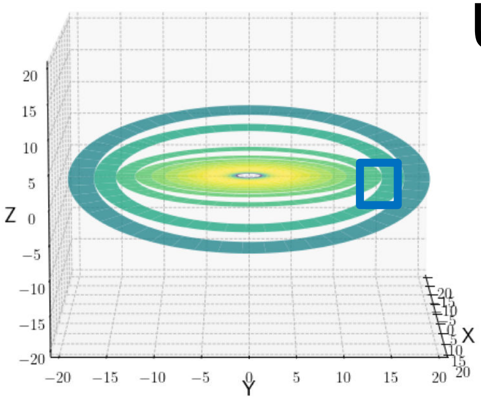


ϕ=0

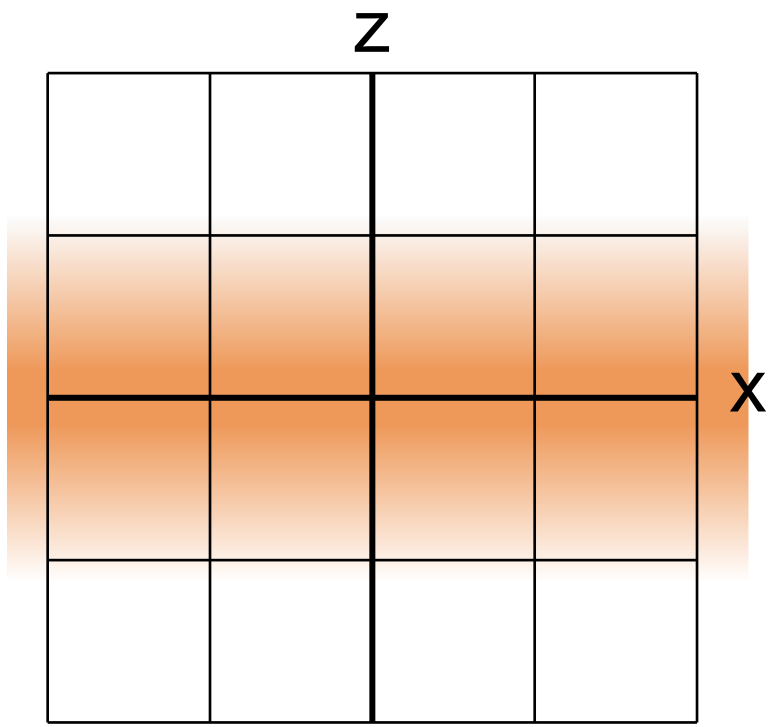


Shearing box analysis

Unwarped local (x,y,z) coordinates

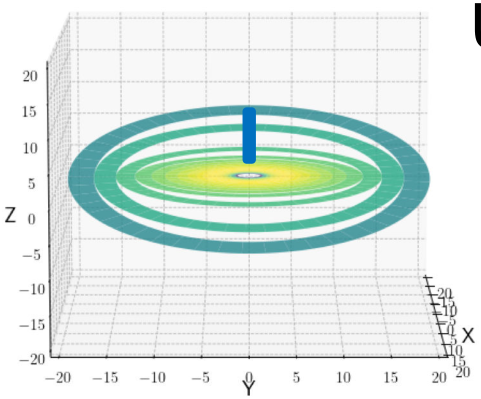


$\phi = \pi/2$

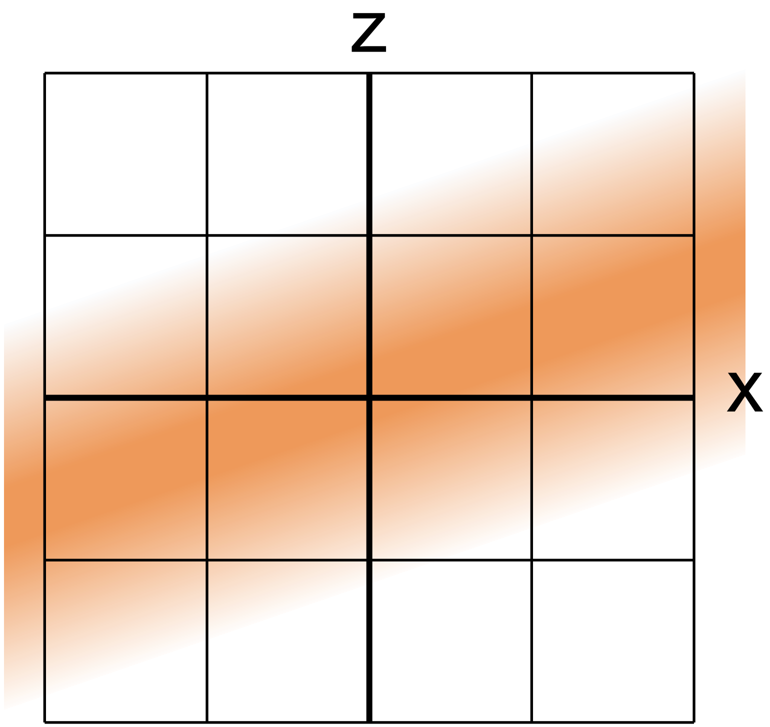


Shearing box analysis

Unwarped local (x,y,z) coordinates

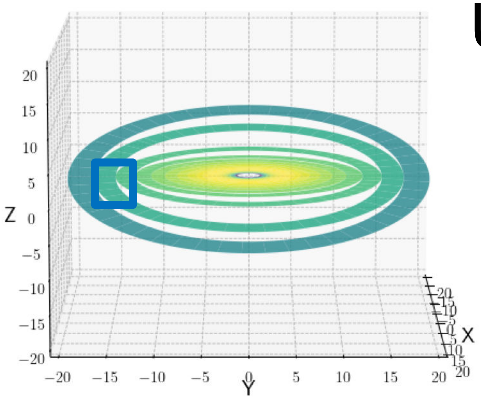


$\phi = \pi$

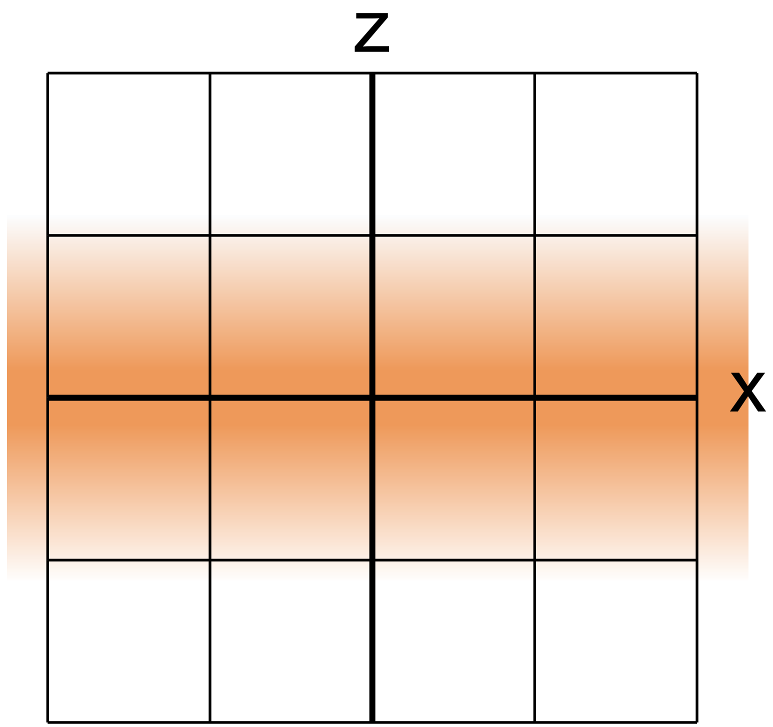


Shearing box analysis

Unwarped local (x,y,z) coordinates

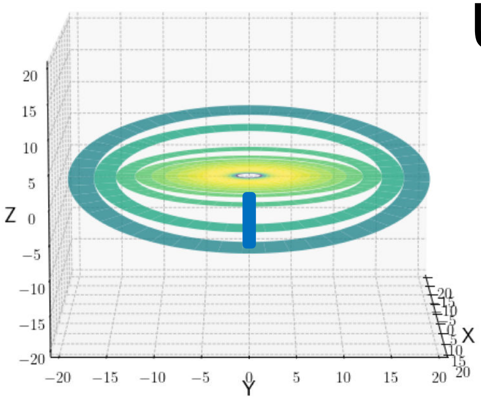


$\phi=3\pi/2$

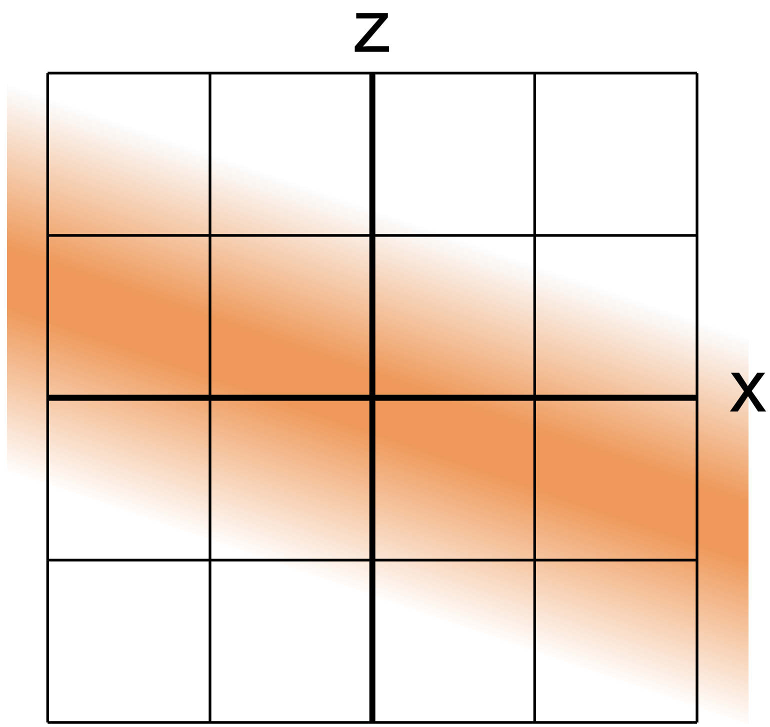


Shearing box analysis

Unwarped local (x,y,z) coordinates



$\phi = 2\pi \triangleq 0$



Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

$$
\begin{aligned}
\mathrm{D}_t x &= u_x, \\
\mathrm{D}_t y &= u_y, \\
\mathrm{D}_t z &= u_z, \\
\mathrm{D}_t u_x - 2\Omega_0 u_y &= f_x + 2q\Omega_0^2 x, \\
\mathrm{D}_t u_y + 2\Omega_0 u_x &= f_y, \\
\mathrm{D}_t u_z &= f_z - \Omega_0^2 z,
\end{aligned}
$$

Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

Comoving time derivative

$$
\begin{aligned}
\mathrm{D}_t x &= u_x\,, \\
\mathrm{D}_t y &= u_y\,, \\
\mathrm{D}_t z &= u_z\,, \\
\mathrm{D}_t u_x - 2\Omega_0 u_y &= f_x + 2q\Omega_0^2 x, \\
\mathrm{D}_t u_y + 2\Omega_0 u_x &= f_y\,, \\
\mathrm{D}_t u_z &= f_z - \Omega_0^2 z,
\end{aligned}
$$

Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

$$
\begin{aligned}
\mathrm{D}_t x &= u_x\,,\\
\mathrm{D}_t y &= u_y\,,\\
\mathrm{D}_t z &= u_z\,,
\end{aligned}
$$

dx/dt = u

$$
\begin{aligned}
\mathrm{D}_t u_x - 2\Omega_0 u_y &= f_x + 2q\Omega_0^2 x,\\
\mathrm{D}_t u_y + 2\Omega_0 u_x &= f_y\,,\\
\mathrm{D}_t u_z &= f_z - \Omega_0^2 z,
\end{aligned}
$$

Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

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\mathrm{D}_t z &= u_z\,, \\
\mathrm{D}_t u_x - 2\Omega_0 u_y &= f_x + 2q\Omega_0^2 x, \\
\mathrm{D}_t u_y + 2\Omega_0 u_x &= f_y\,, \\
\mathrm{D}_t u_z &= f_z - \Omega_0^2 z,
\end{aligned}
$$

d**u**/dt = **f**

Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

$$
\begin{aligned}
\mathrm{D}_t x &= u_x\,, \\
\mathrm{D}_t y &= u_y\,, \\
\mathrm{D}_t z &= u_z\,, \\
\mathrm{D}_t u_x - 2\Omega_0 u_y &= f_x + 2q\Omega_0^2 x, \\
\mathrm{D}_t u_y + 2\Omega_0 u_x &= f_y\,, \\
\mathrm{D}_t u_z &= f_z - \Omega_0^2 z,
\end{aligned}
$$

Coriolis force

Coriolis force leads to epicyclic oscillations (circular motion in the x-y plane)

This is what later will become the "sloshing motion"

Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

$$
\begin{aligned}
\mathrm{D}_t x &= u_x\,, \\
\mathrm{D}_t y &= u_y\,, \\
\mathrm{D}_t z &= u_z\,, \\
\mathrm{D}_t u_x - 2\Omega_0 u_y &= f_x + 2q\Omega_0^2 x, \\
\mathrm{D}_t u_y + 2\Omega_0 u_x &= f_y\,, \\
\mathrm{D}_t u_z &= f_z - \Omega_0^2 z,
\end{aligned}
$$

Centrifugal force

Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

$$\begin{aligned}
\mathrm{D}_t x &= u_x\,, \\
\mathrm{D}_t y &= u_y\,, \\
\mathrm{D}_t z &= u_z\,, \\
\mathrm{D}_t u_x - 2\Omega_0 u_y &= f_x + 2q\Omega_0^2 x, \\
\mathrm{D}_t u_y + 2\Omega_0 u_x &= f_y\,, \\
\mathrm{D}_t u_z &= f_z \boxed{- \Omega_0^2 z,}
\end{aligned}$$

Vertical gravity force

Shearing box analysis

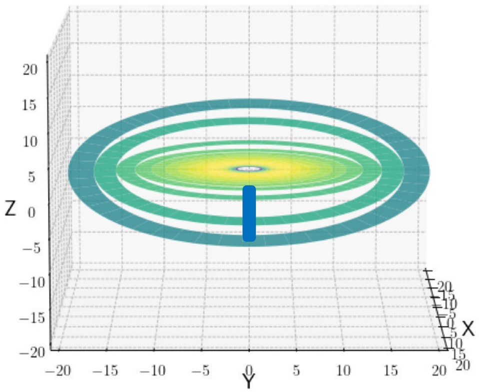
"Simple" Newtonian dynamics of a gas parcel

$$
\begin{aligned}
\mathrm{D}_t x &= u_x\,, \\
\mathrm{D}_t y &= u_y\,, \\
\mathrm{D}_t z &= u_z\,, \\
\mathrm{D}_t u_x - 2\Omega_0 u_y &= f_x + 2q\Omega_0^2 x, \\
\mathrm{D}_t u_y + 2\Omega_0 u_x &= f_y\,, \\
\mathrm{D}_t u_z &= f_z - \Omega_0^2 z,
\end{aligned}
$$

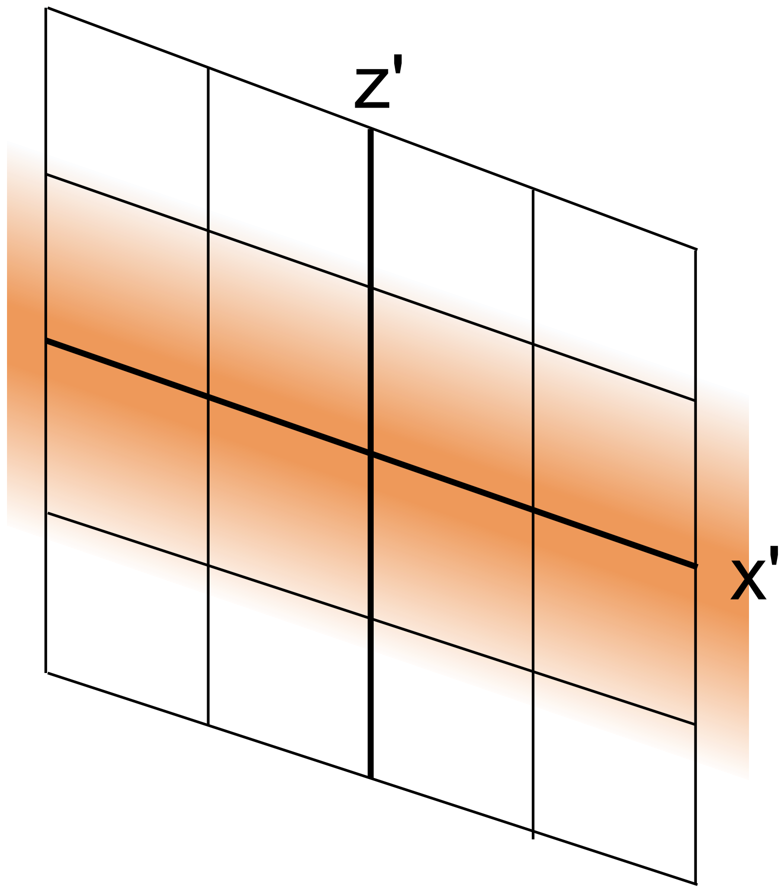
Pressure forces + viscous forces

Shearing box analysis

Warped (x',y',z') coordinates



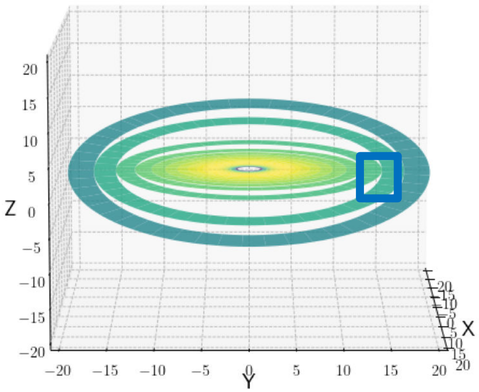
ϕ=0



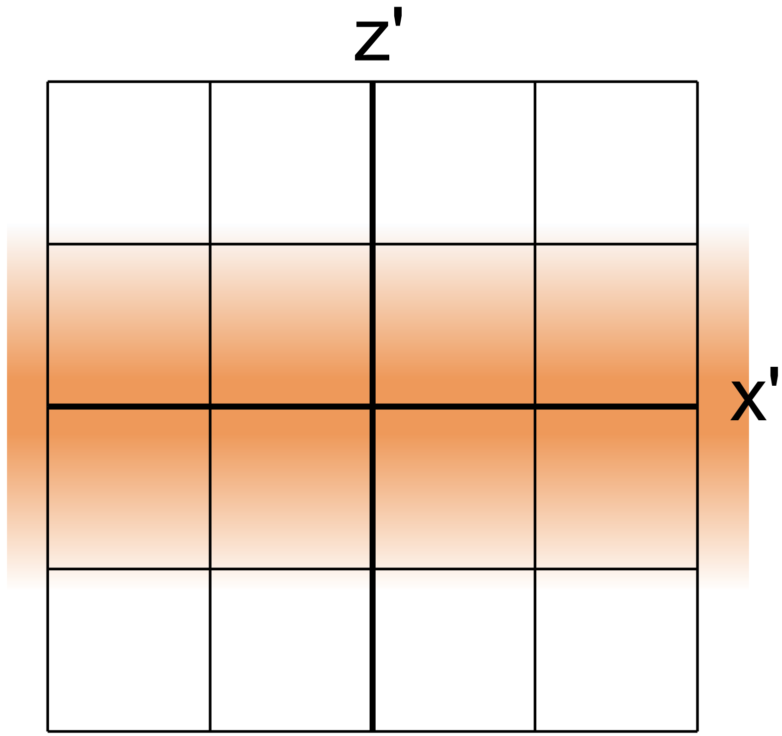
See Ogilvie & Latter (2013a)

Shearing box analysis

Warped (x',y',z') coordinates



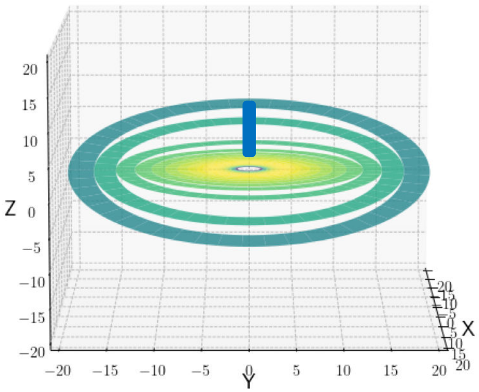
$\phi = \pi/2$

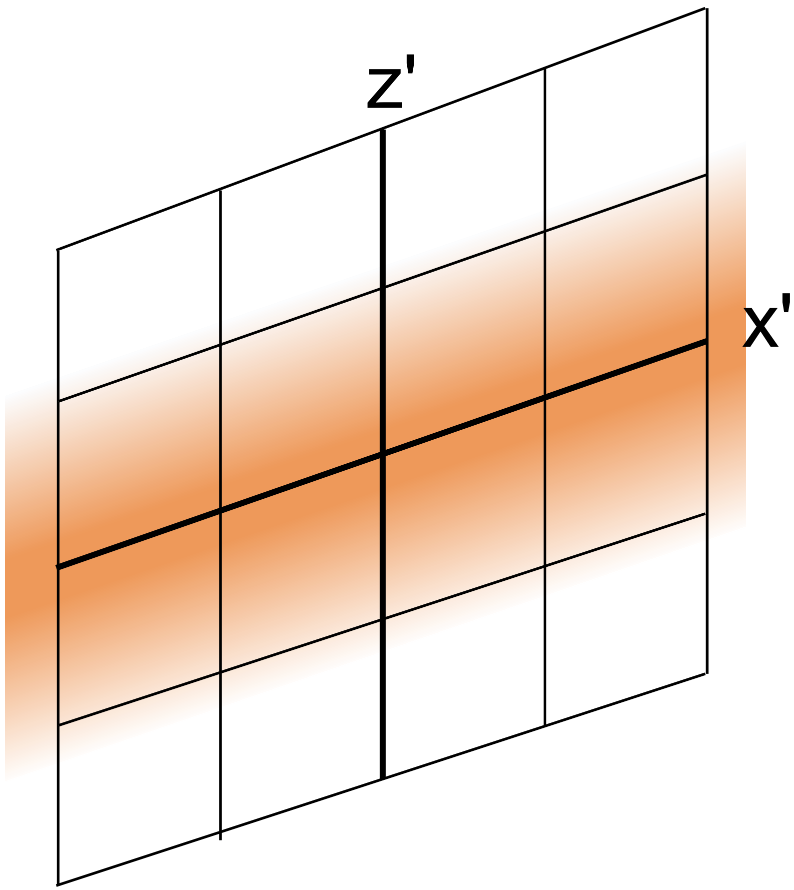


See Ogilvie & Latter (2013a)

Shearing box analysis

Warped (x',y',z') coordinates



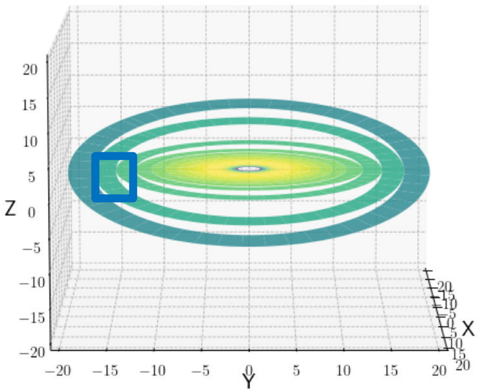


$\phi = \pi$

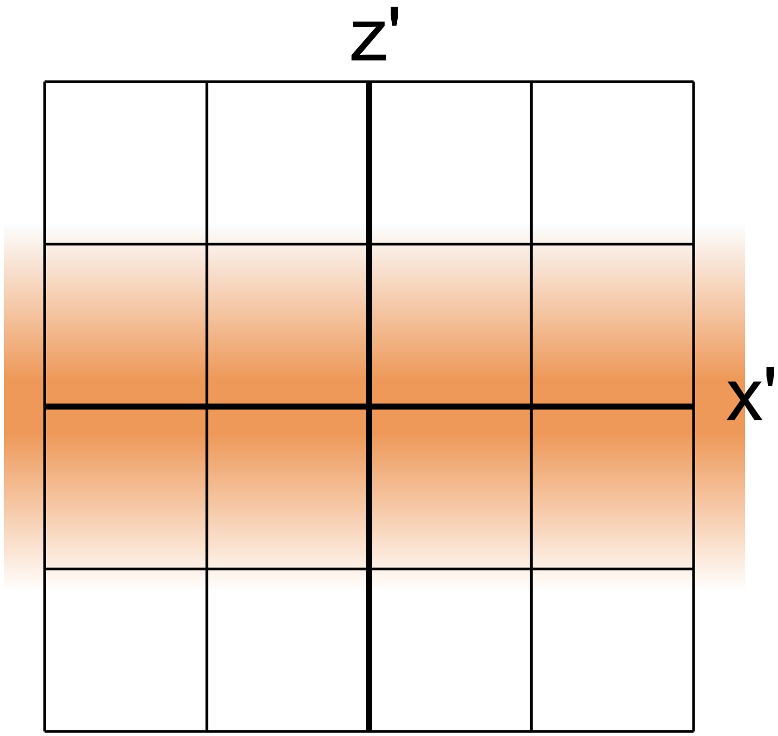
See Ogilvie & Latter (2013a)

Shearing box analysis

Warped (x',y',z') coordinates



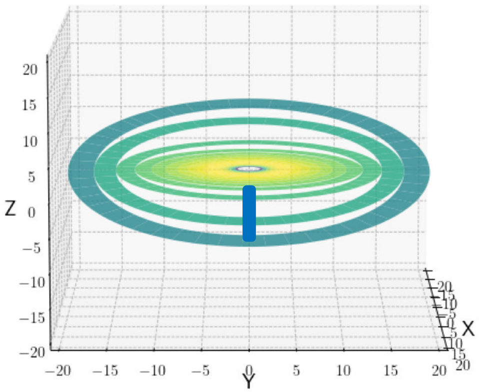
$\phi = 3\pi/2$



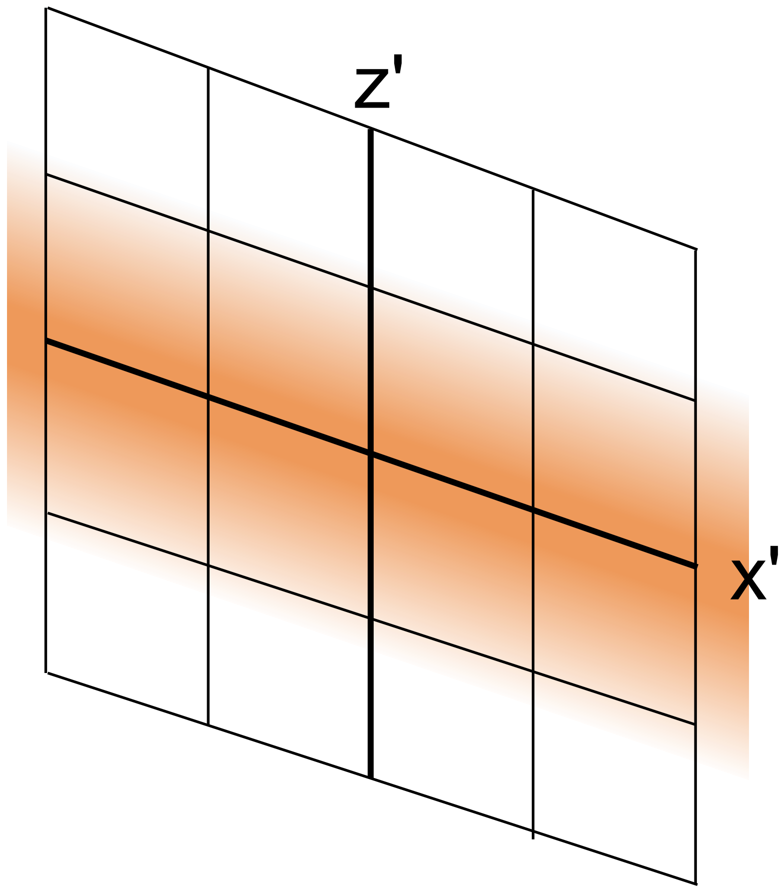
See Ogilvie & Latter (2013a)

Shearing box analysis

Warped (x',y',z') coordinates



ϕ=0



See Ogilvie & Latter (2013a)

Shearing box analysis

Warped (x',y',z') coordinates and (v_x',v_y',v_z') velocities

$$x = x',$$
$$y = y',$$
$$z = z' - \psi x' \cos(\phi).$$

green = new variables

$$u_x = v'_x,$$
$$u_y = v'_y - q\Omega_0 x',$$
$$u_z = v'_z + \psi\Omega_0 x' \sin(\phi),$$

Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

Original (x,y,z)

$$
\begin{aligned}
\mathrm{D}_t x &= u_x\,, \\
\mathrm{D}_t y &= u_y\,, \\
\mathrm{D}_t z &= u_z\,, \\
\mathrm{D}_t u_x - 2\Omega_0 u_y &= f_x + 2q\Omega_0^2 x, \\
\mathrm{D}_t u_y + 2\Omega_0 u_x &= f_y\,, \\
\mathrm{D}_t u_z &= f_z - \Omega_0^2 z,
\end{aligned}
$$

Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

New (x',y',z')

$$
\begin{aligned}
\mathrm{D}_t x' &= v'_x, \\
\mathrm{D}_t y' &= v'_y - q\Omega_0 x', \\
\mathrm{D}_t z' &= v'_z + \psi v'_x \cos(\phi), \\
\mathrm{D}_t v'_x - 2\Omega_0 v'_y &= f_x, \\
\mathrm{D}_t v'_y + (2-q)\Omega_0 v'_x &= f_y, \\
\mathrm{D}_t v'_z + \psi\Omega_0 \sin(\phi) v'_x &= f_z - \Omega_0^2 z'.
\end{aligned}
$$

Shearing box analysis

"Simple" Newtonian dynamics of a gas parcel

New (x',y',z')

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\mathrm{D}_t v'_z + \psi\Omega_0 \sin(\phi) v'_x &= f_z - \Omega_0^2 z'\,.
\end{aligned}
$$

Let's focus only on the horizontal motions

Shearing box analysis

$$
\begin{aligned}
\mathrm{D}_t v'_x - 2\Omega_0 v'_y &= f_x, \\
\mathrm{D}_t v'_y + (2-q)\Omega_0 v'_x &= f_y,
\end{aligned}
$$

Forces are: Pressure gradient and viscous forces:

$$f_i = f_i^p + f_i^v$$

Driven damped harmonic oscillator!

Let's focus only on the horizontal motions

Shearing box analysis

Let's put rhs to 0 for a moment

$$\begin{aligned} \mathrm{D}_t v_x' - 2\Omega_0 v_y' &= 0 \\ \mathrm{D}_t v_y' + (2-q)\Omega_0 v_x' &= 0 \end{aligned}$$

Homogeneous solution is:

$$v_x'(t) = v_{x0}' \, e^{i\Omega_{\mathrm{hom}} t}$$

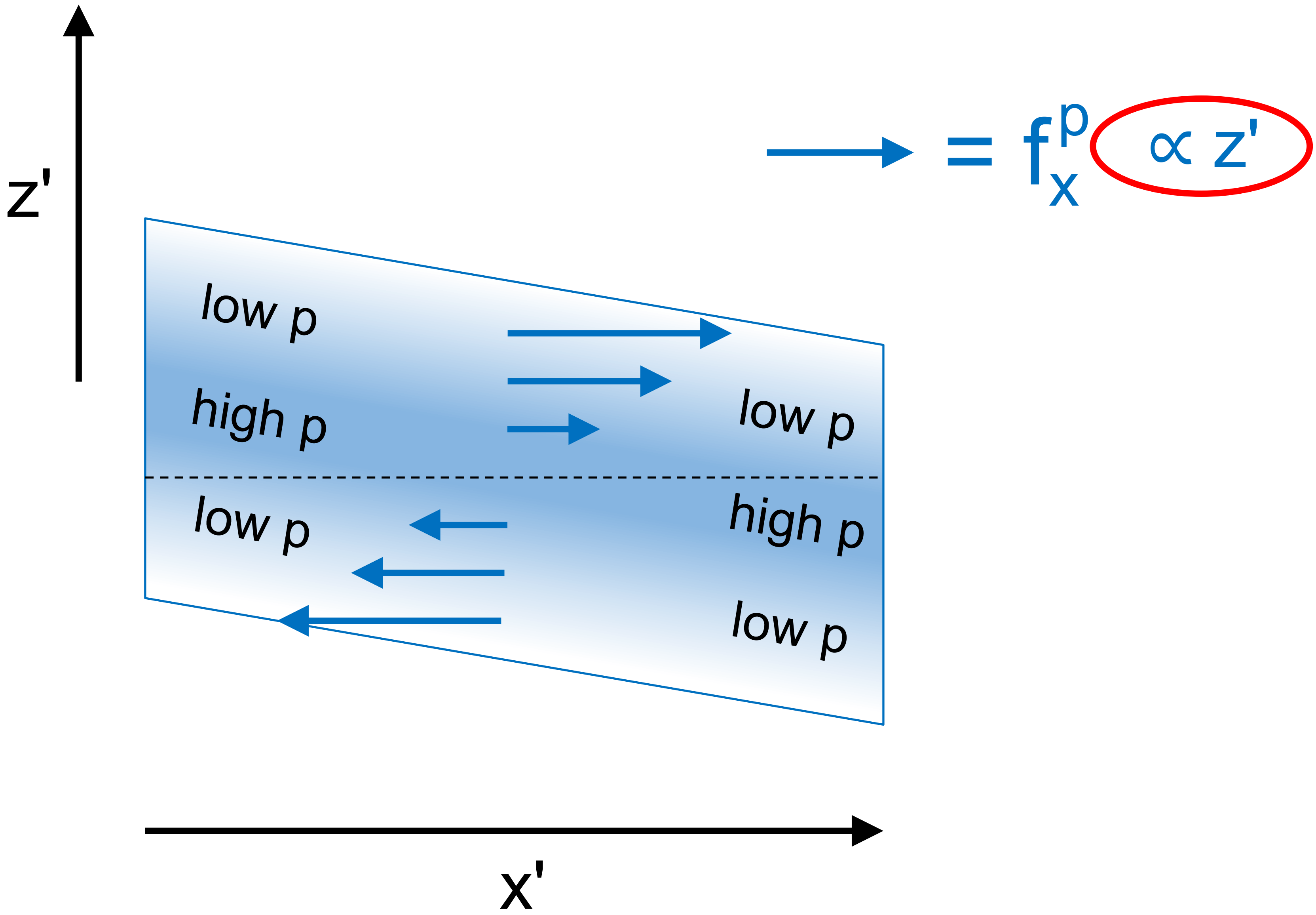
with

$$\Omega_{\mathrm{hom}}^2 = 2(2-q)\Omega_0^2 \equiv \Omega_e^2$$

which is the epicyclic oscillation frequency

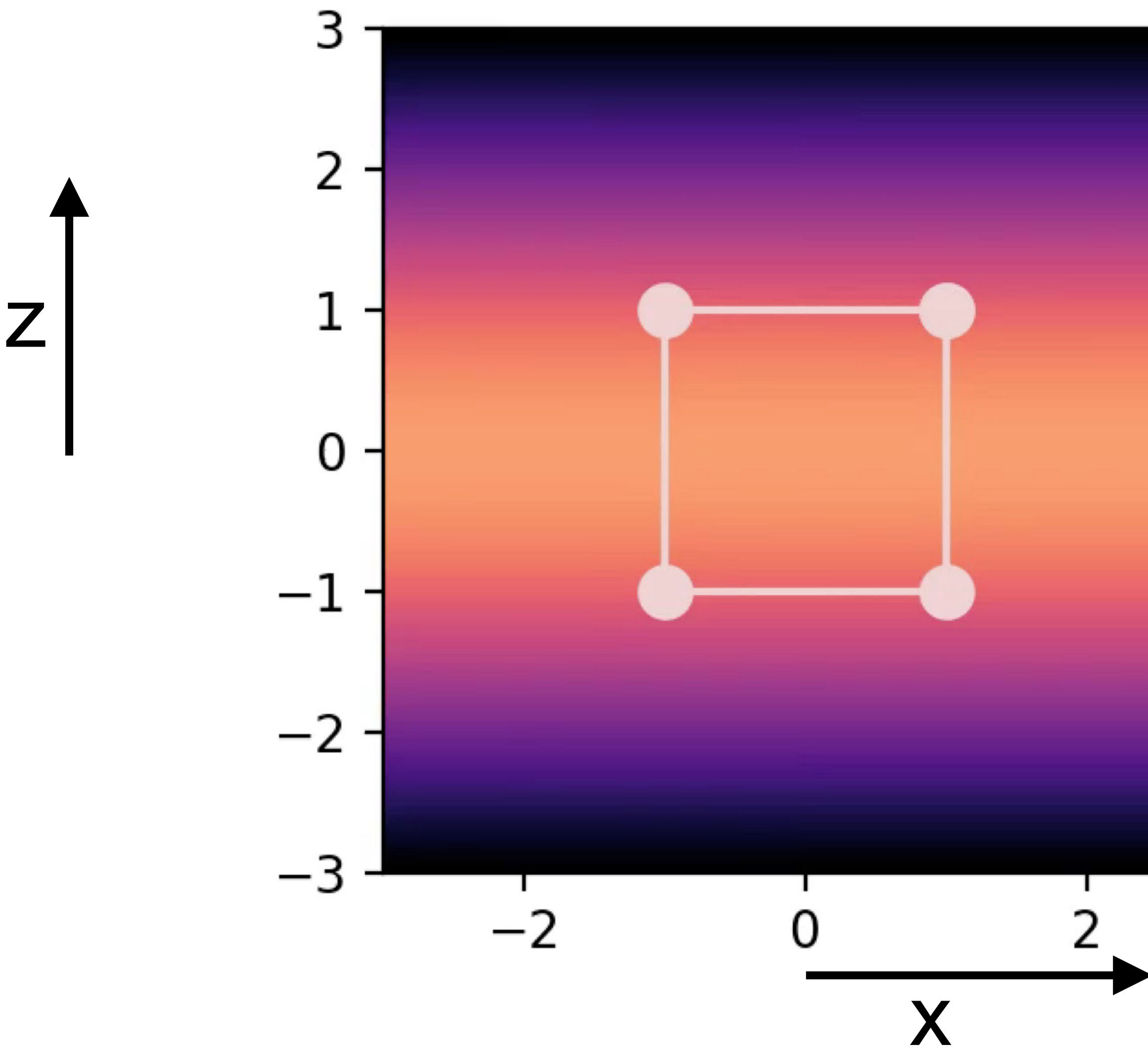
Shearing box analysis

Driving force is horizontal pressure gradient



Shearing box analysis

Driving force is horizontal pressure gradient



Shearing box analysis

Forcing ∝ z'

$$f_x^p \propto z'$$

Everything ∝ z':

Now find solution for these

$$\begin{aligned} v'_x(z',\tau) &= V_x(\tau)\Omega_0 z', \\ v'_y(z',\tau) &= V_y(\tau)\Omega_0 z', \\ v'_z(z',\tau) &= V_z(\tau)\Omega_0 z'. \end{aligned}$$

with $\tau = \Omega_0 t$

Shearing box analysis

Original

$$
\begin{aligned}
\mathrm{D}_t v'_x - 2\Omega_0 v'_y &= f_x, \\
\mathrm{D}_t v'_y + (2-q)\Omega_0 v'_x &= f_y,
\end{aligned}
$$

Shearing box analysis

Dimensionless time τ

$$
\begin{aligned}
\mathrm{D}_\tau v_x' - 2v_y' &= \Omega_0^{-1} f_x , \\
\mathrm{D}_\tau v_y' + (2-q) v_x' &= \Omega_0^{-1} f_y ,
\end{aligned}
$$

$$\tau = \Omega_0 t$$

Shearing box analysis

Now in terms of V_x and V_y

$$\bar{\mathrm{D}}_\tau V_x - 2V_y = (\Omega_0^2 z')^{-1} f_x,$$
$$\bar{\mathrm{D}}_\tau V_y + (2-q)V_x = (\Omega_0^2 z')^{-1} f_y,$$

$$v'_x(z',\tau) = V_x(\tau)\Omega_0 z',$$
$$v'_y(z',\tau) = V_y(\tau)\Omega_0 z',$$
$$v'_z(z',\tau) = V_z(\tau)\Omega_0 z'.$$

Shearing box analysis

And define F to divide out z'

$$\begin{aligned} \bar{\mathrm{D}}_\tau V_x - 2V_y &= F_x \\ \bar{\mathrm{D}}_\tau V_y + (2-q)V_x &= F_y \end{aligned}$$

$$F_i \equiv (\Omega_0^2 z')^{-1} f_i$$

Shearing box analysis

Write F as pressure driving + viscous damping

$$\partial_\tau V_x - 2V_y = \boxed{\psi \cos(\phi)} + \boxed{F_x^{\mathrm{v}}},$$

$$\partial_\tau V_y + (2-q)V_x = \boxed{F_y^{\mathrm{v}}},$$

Shearing box analysis

Write F as pressure driving + viscous damping

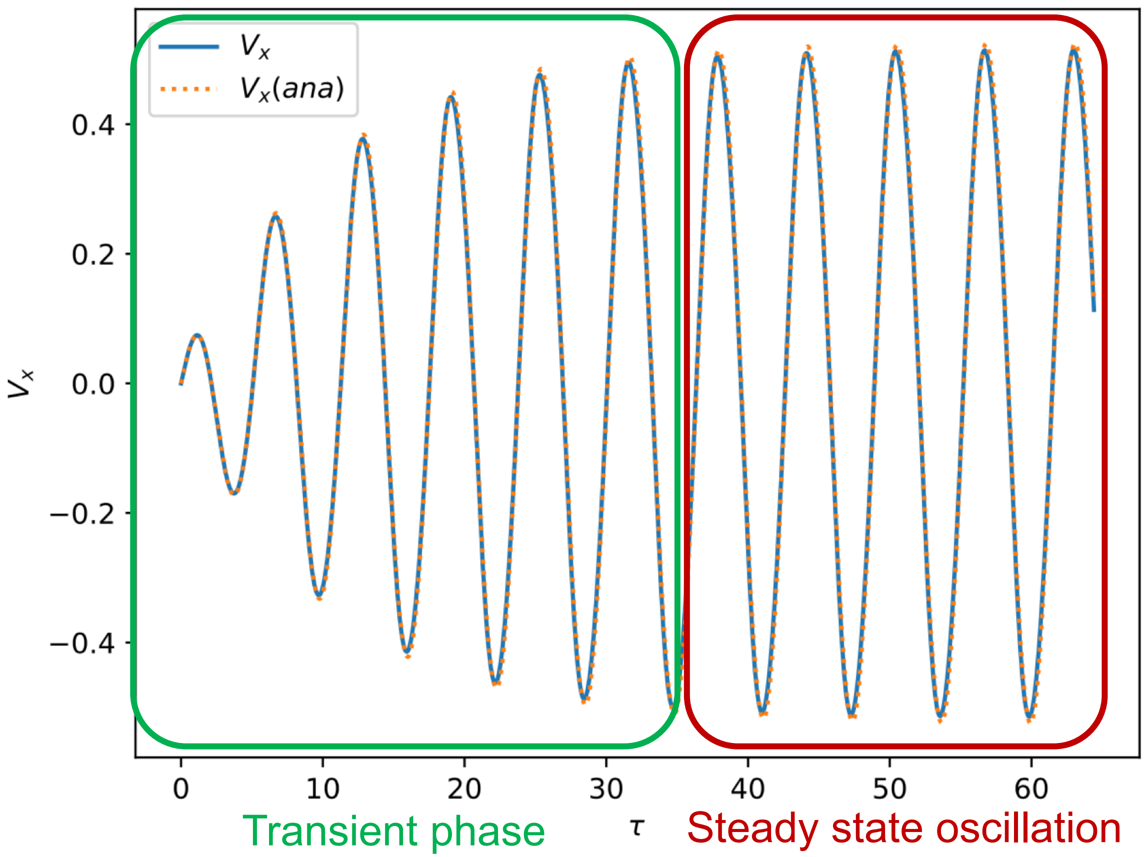
$$\partial_\tau V_x - 2V_y = \psi \cos(\phi) + F_x^{\mathrm{v}} \,,$$

$$\partial_\tau V_y + (2-q)V_x = F_y^{\mathrm{v}} \,,$$

Solution for q=3/2, ψ=0.1, α=0.1

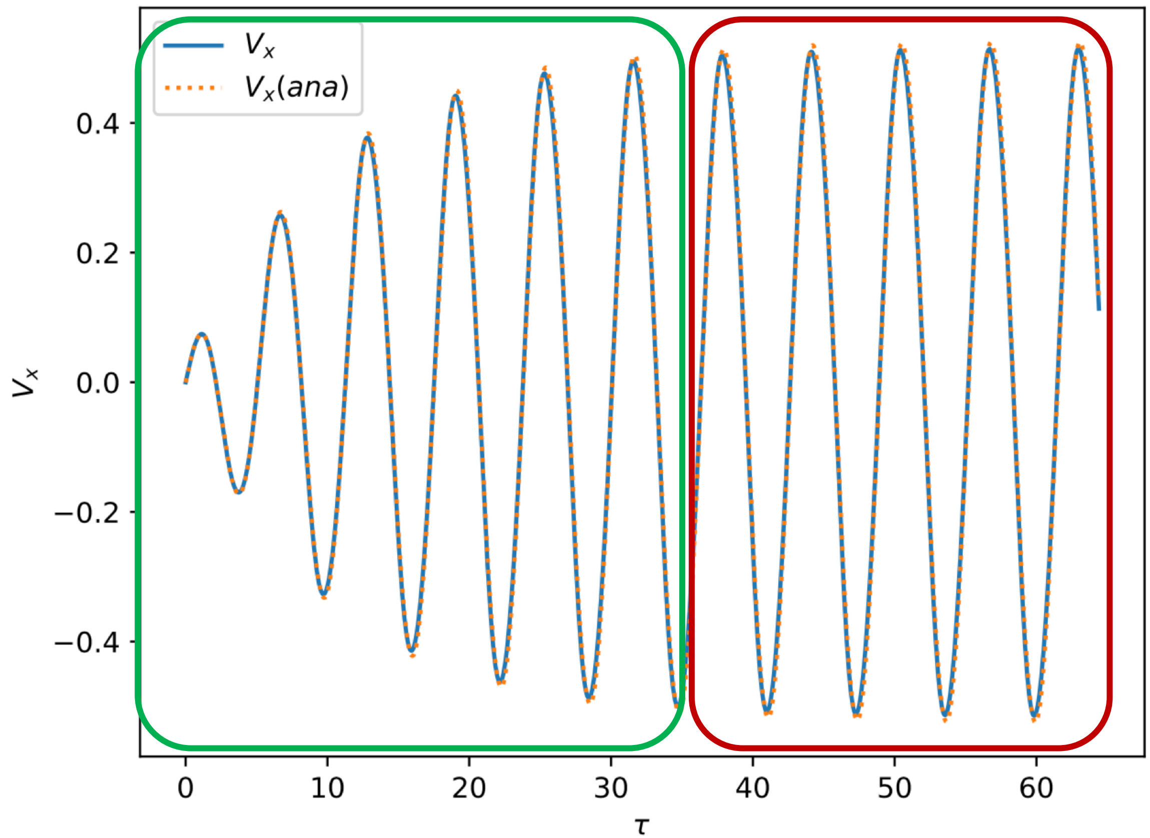
This is the sloshing motion!

This will lead to the torque ***G***.



Shearing box analysis

Solution for q=3/2, ψ=0.1, α=0.1

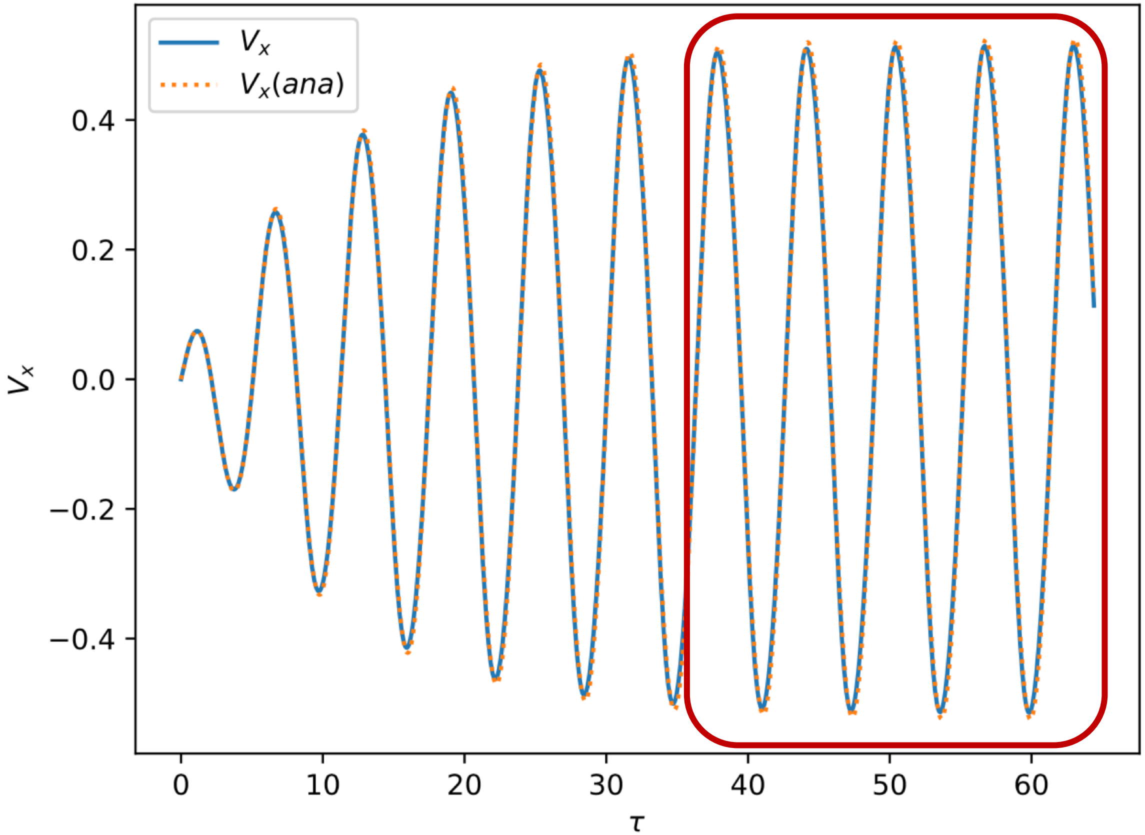


All solutions are sum of particular (=steady state oscillation) solution plus the homogeneous (=transient) solution:

$$V_x(\tau) = V_{xp}(\tau) + V_{xh}(\tau),$$

Shearing box analysis
Unification of diffusive and wavelike regime

Solution for q=3/2, ψ=0.1, α=0.1

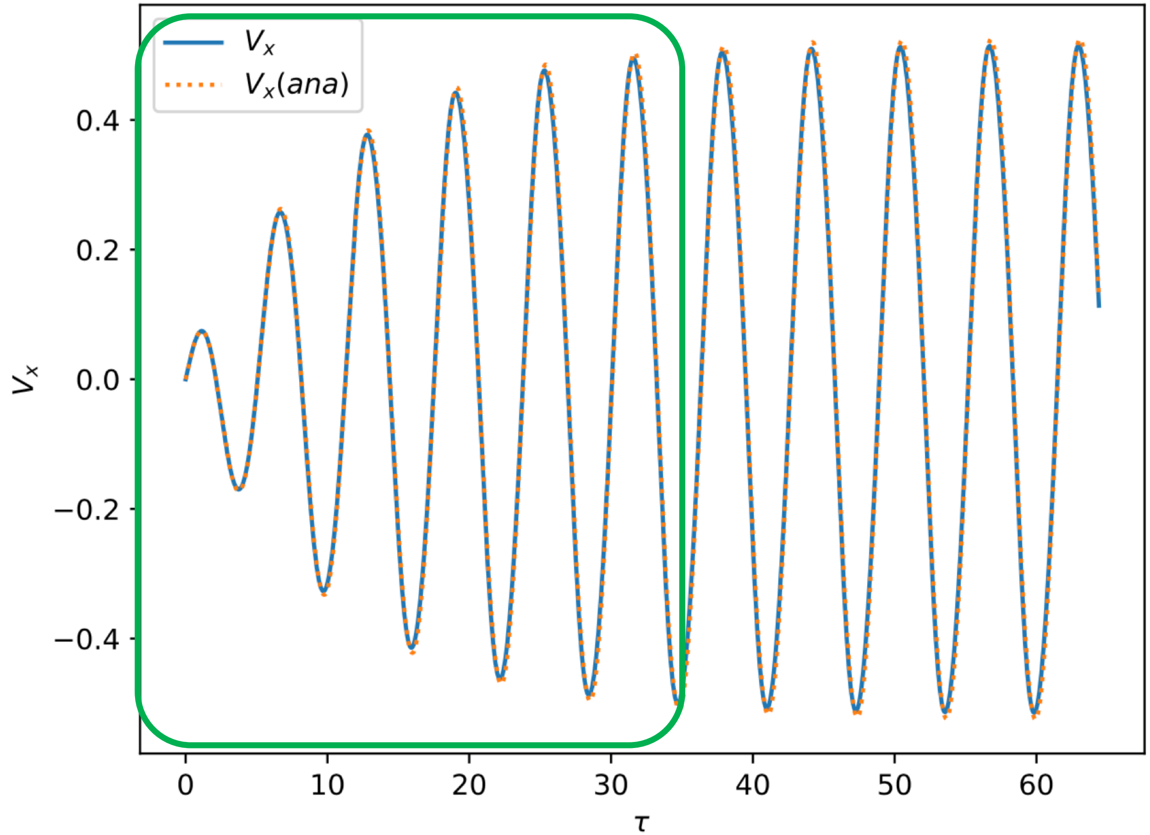


In the **diffusive** regime, the sloshing is always near **steady state**. This is the regime studied by Ogilvie & Latter (2013a)

Shearing box analysis

Unification of diffusive and wavelike regime

Solution for q=3/2, ψ=0.1, α=0.1

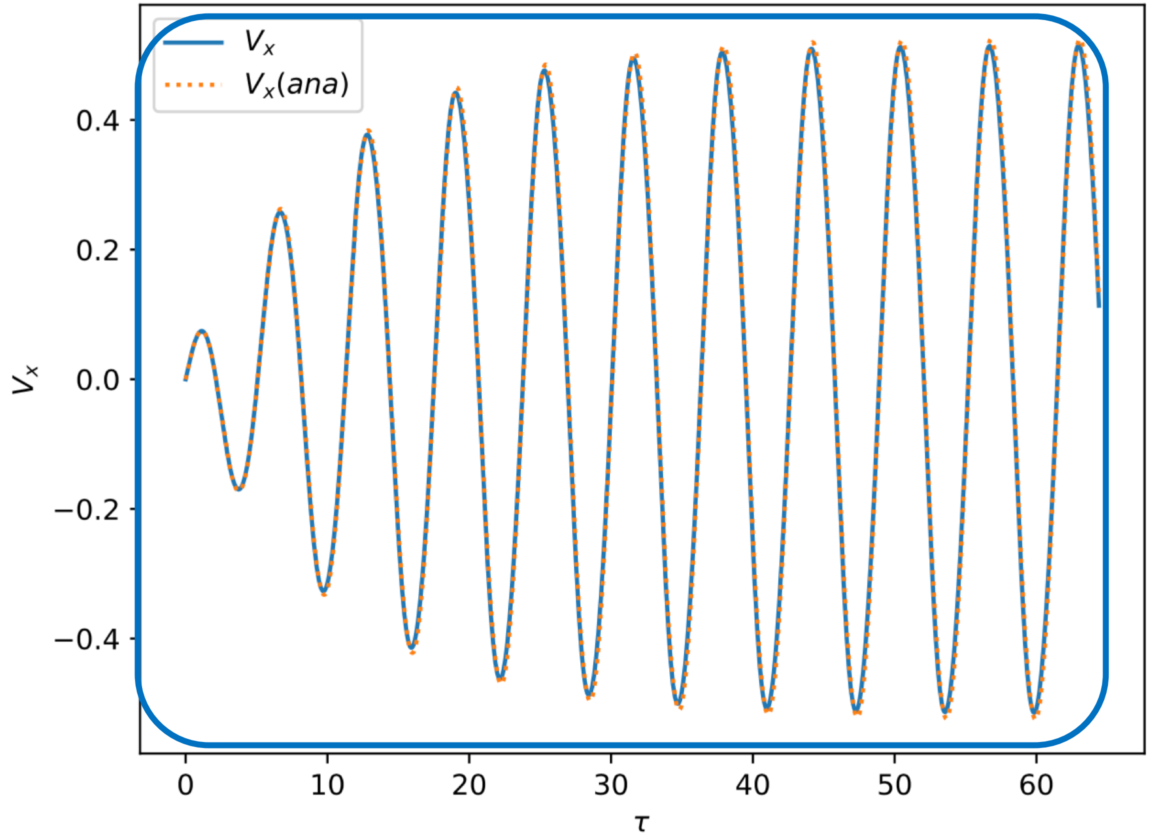


In the **wavelike** regime, the sloshing is always in the **transient regime**, never reaching the steady state oscillation. This is the regime studied by Lubow & Ogilvie (2000), though not in the shearing box picture.

Shearing box analysis

Unification of diffusive and wavelike regime

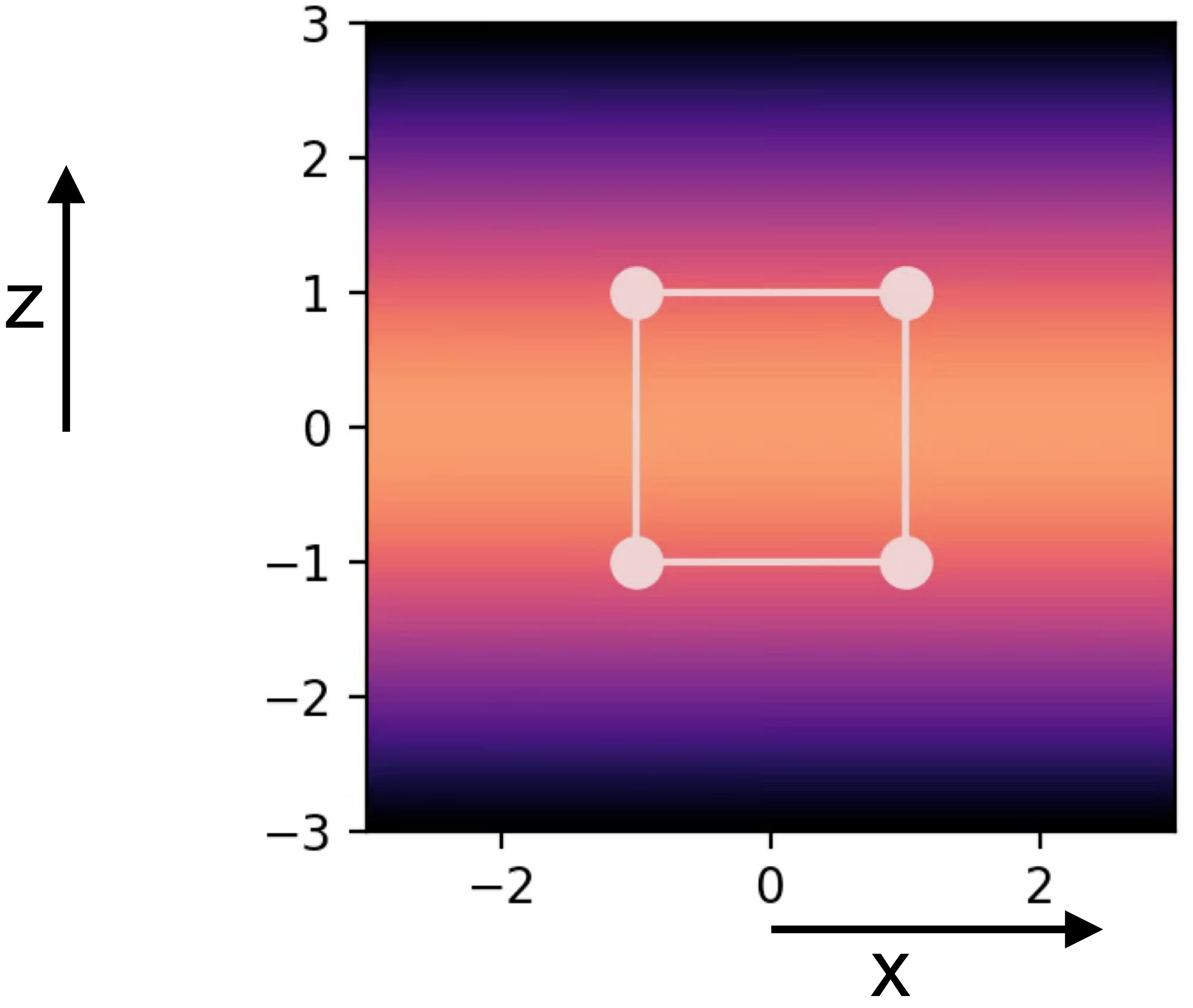
Solution for q=3/2, ψ=0.1, α=0.1



The **unification** of the two regimes, in the shearing box picture, is to consider the full solution: Transient all the way to steady state, i.e. particular + homogeneous solution.

Shearing box analysis

From transient to steady state oscillation



Conclusion Part 2 (a)

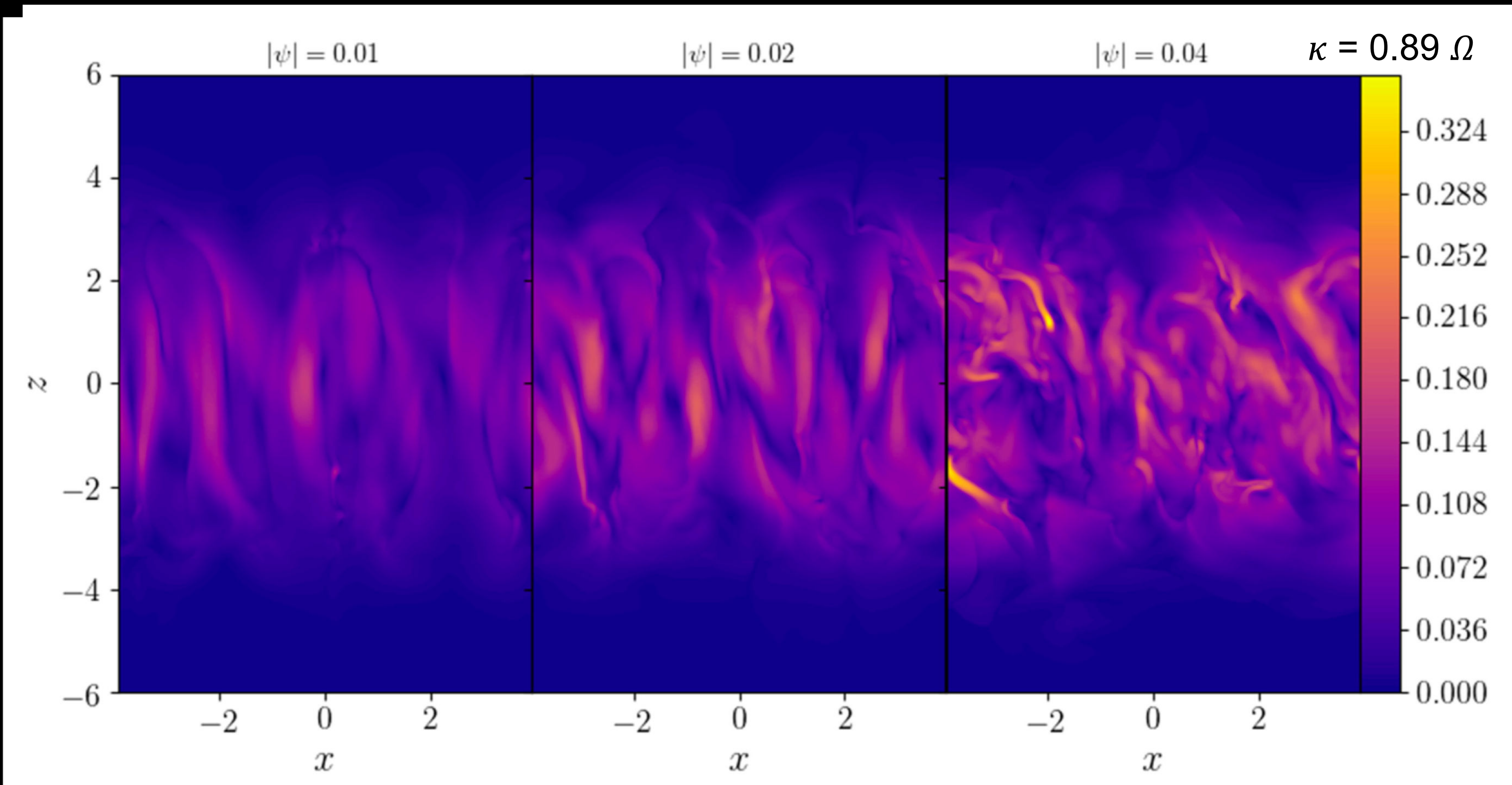
- From the warped shearing box analysis we find:
  - Sloshing motions (Papaloizou & Pringle 1983)
  - ...with a transient amplification
  - ...ending (for static warp) in a steady-state oscillation
- The transient and steady-state oscillation solutions unify the wavelike (transient) with the diffusive (steady-state) regime.
- For low viscosity: The disk shape changes long before the steady-state oscillation is reached (wavelike regime)
- For high viscosity: The disk shape changes so slowly that the oscillation is always near steady state

Conclusion Part 2 (b)

- In viscous regime(α>>h/r): Increasing viscosity → Weaker damping of the warp! Because sloshing is suppressed by the viscosity
- Reducing viscosity → stronger damping, *until* you reach α=h/r
- Further reduction → α<h/r → wavelike regime.
- If you now "fix" the warp by e.g. a companion (in the α<h/r regime):
  - Sloshing will continue to amplify until reaching the steady state oscillation.
  - Strong vertical shear → turbulence → increases α ? (see Kumar & Coleman 1993, Ogilvie & Latter 2013b, Paardekooper & Ogilvie 2019)

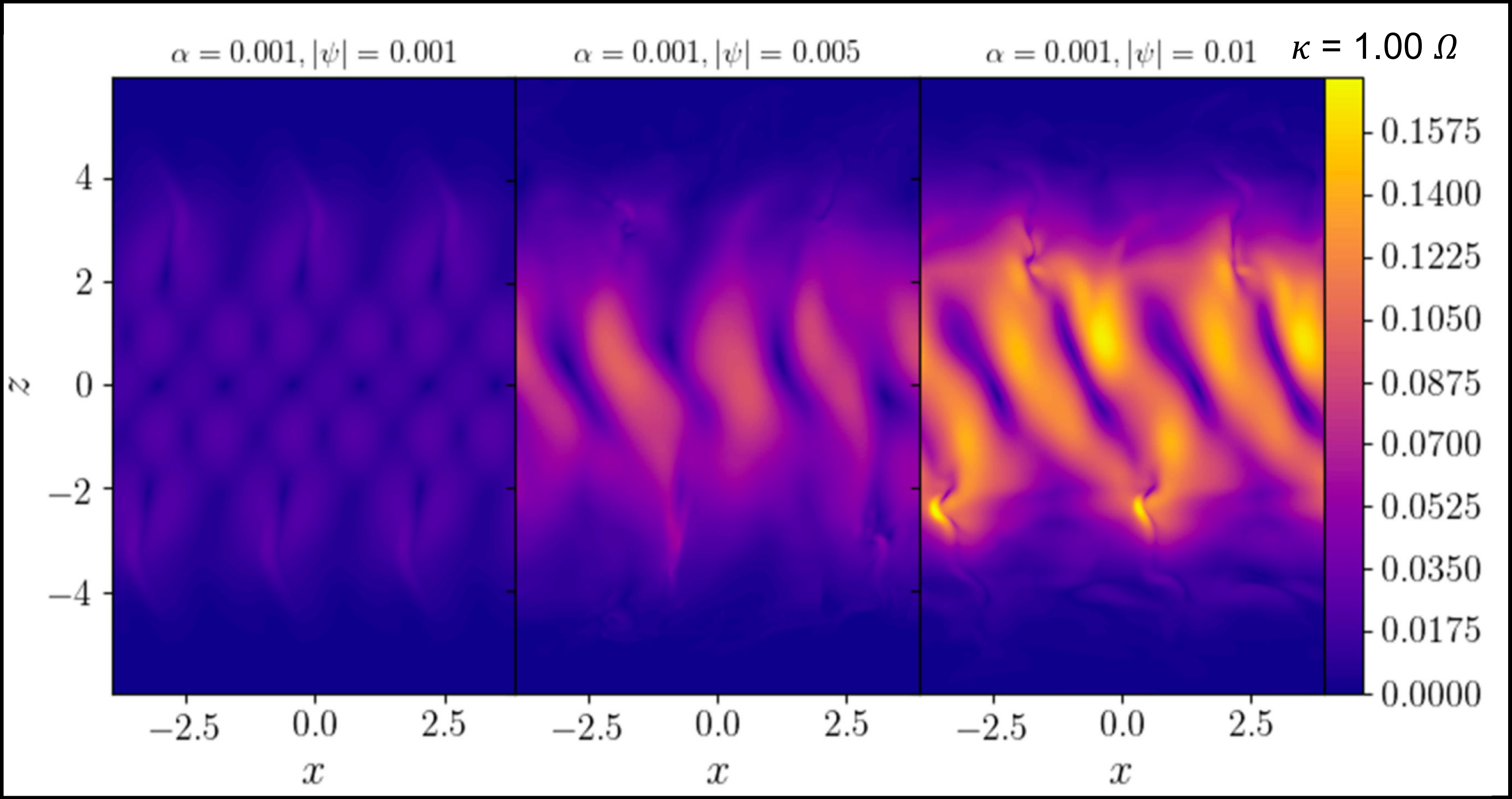
Outlook / Ideas

Outlook: effect on dust



Paardekooper & Ogilvie 2019

Outlook: effect on dust

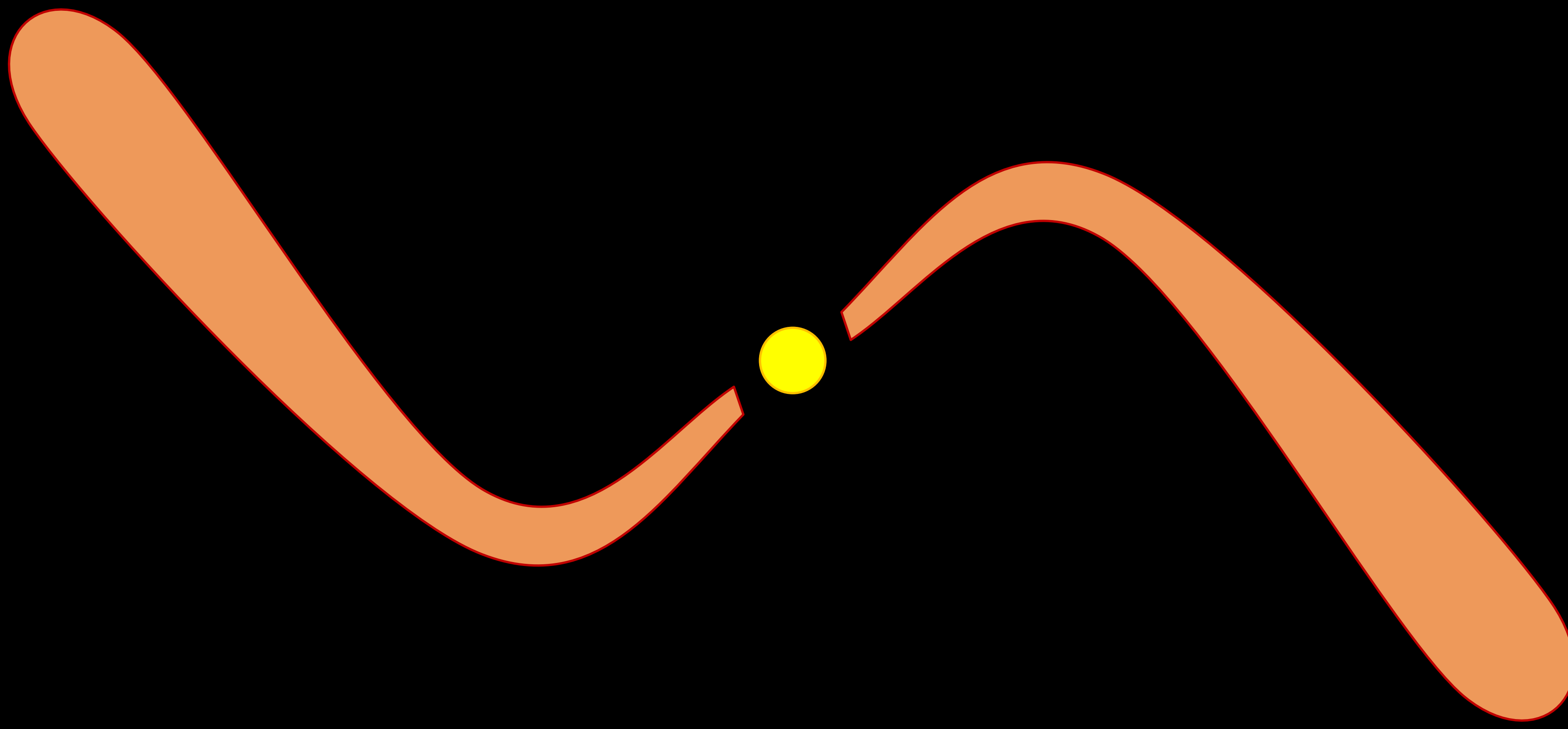


Paardekooper & Ogilvie 2019

How will these oscillations and/or turbulent eddies affect the dust growth?

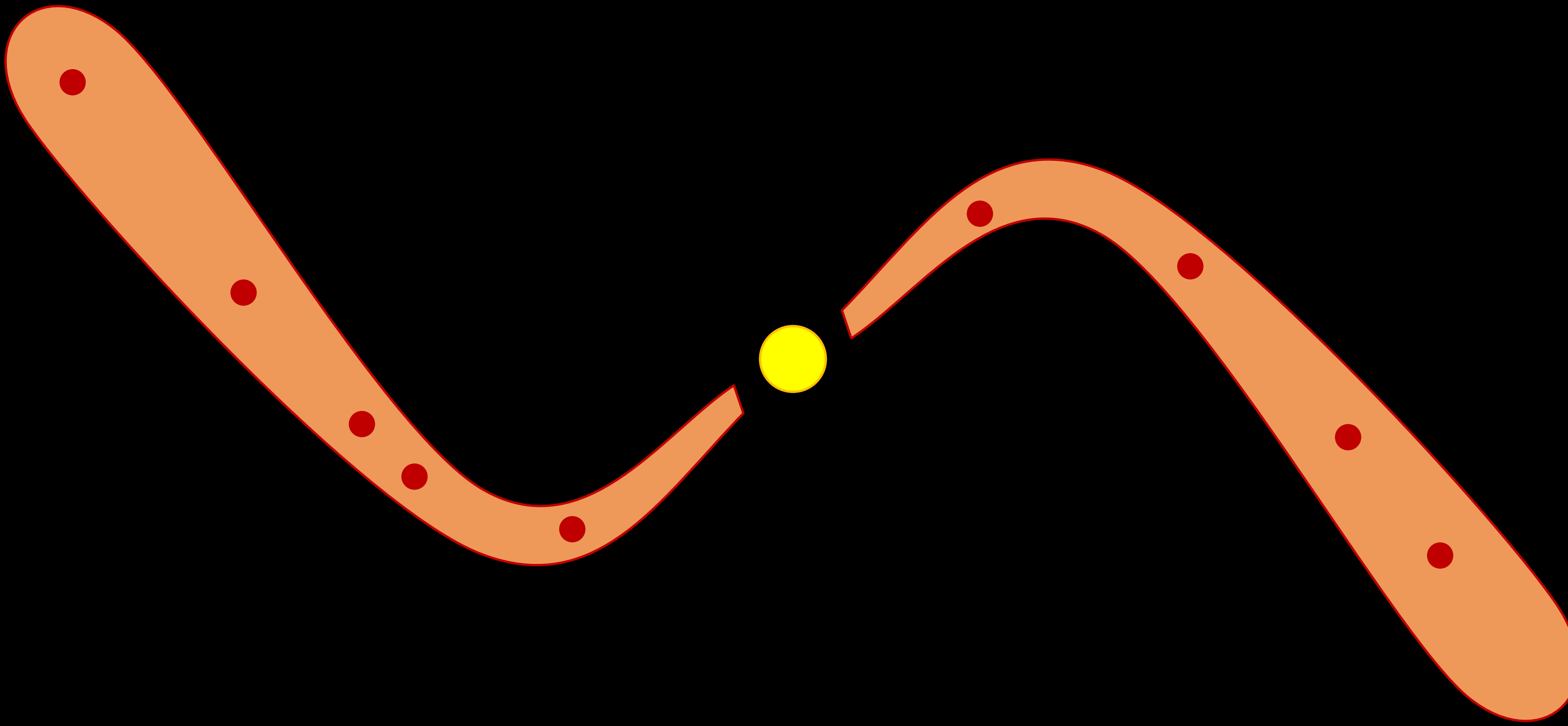
Outlook: Planetesimals in warped disks

Sprinkle a couple of 1-100 km planetesimals in the disk



Outlook: Planetesimals in warped disks

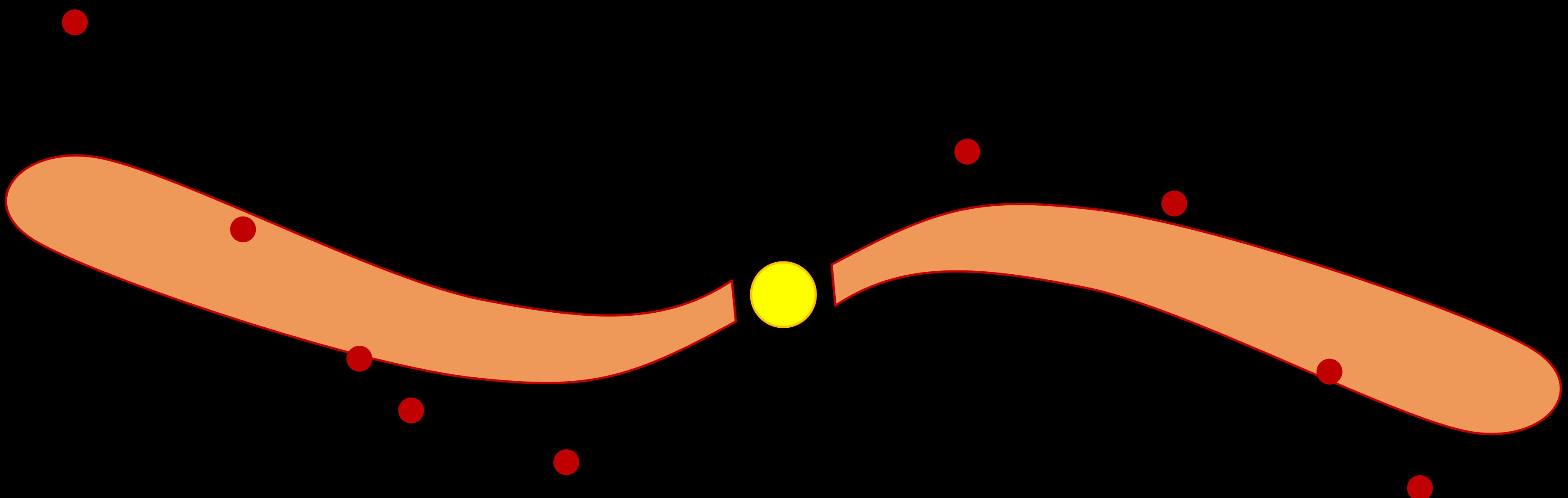
Sprinkle a couple of 1-100 km planetesimals in the disk



What if the warp precesses or changes?

(see also Hossam Aly et al. 2021)

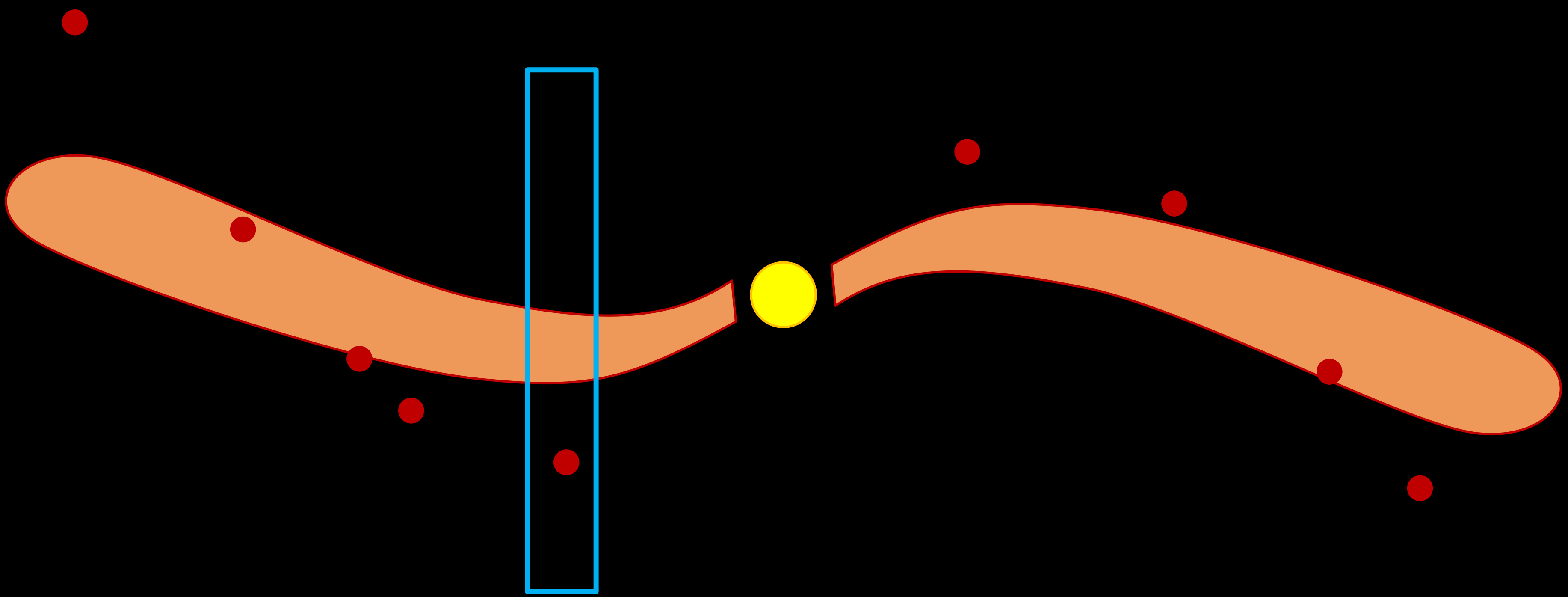
Outlook: Planetesimals in warped disks



These planetesimals will pass through the disk at supersonic speeds

(see also Hossam Aly et al. 2021)

Outlook: Planetesimals in warped disks

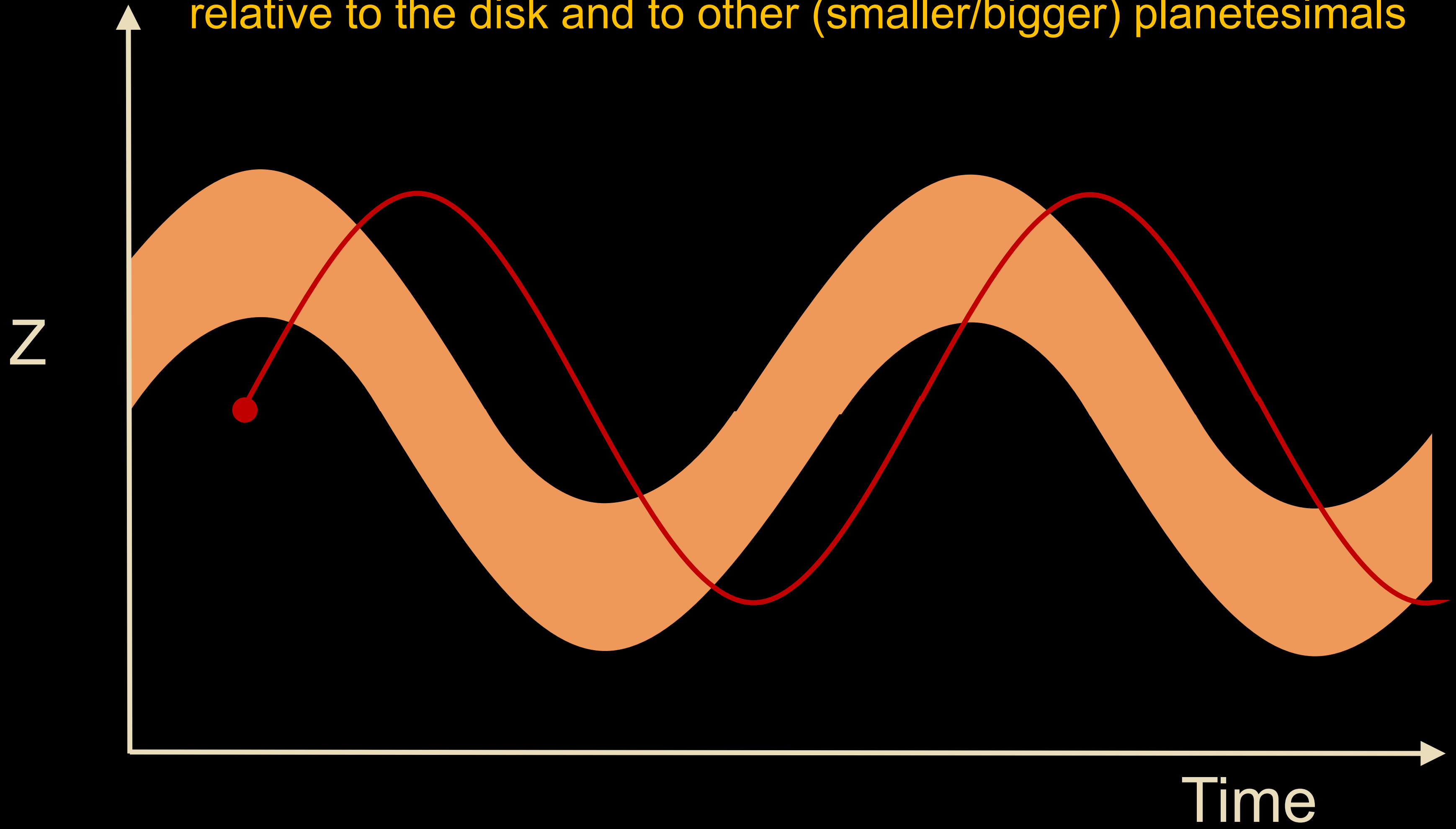


Let's look at this in a time-series for a single annulus

(see also Hossam Aly et al. 2021)

Outlook: Planetesimals in warped disks

Planetesimals may acquire huge velocities
relative to the disk and to other (smaller/bigger) planetesimals



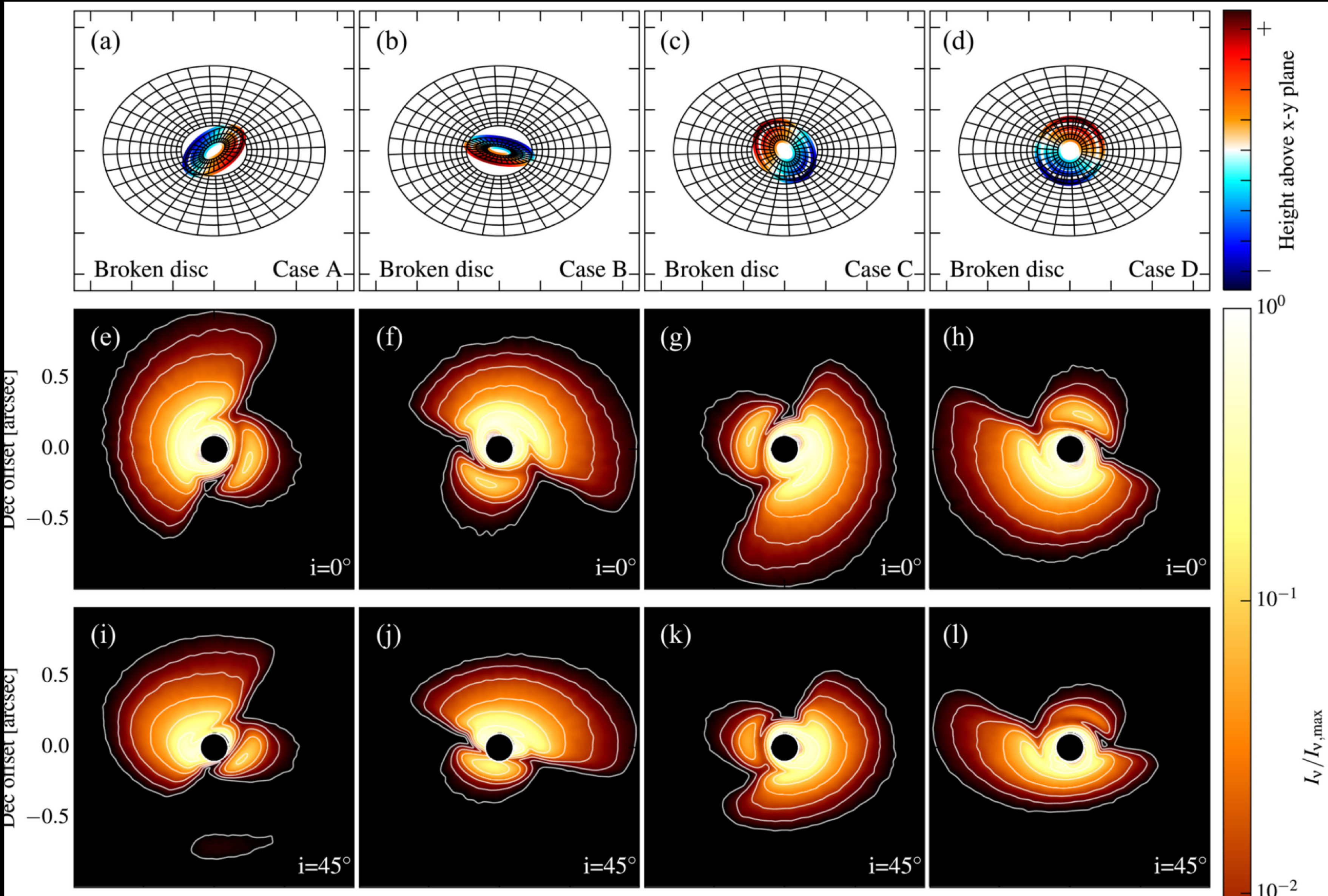
Let's look at this in a time-series for a single annulus

(see also Hossam Aly et al. 2021)

Topic 3

New RADMC-3D model setup template for warped disks

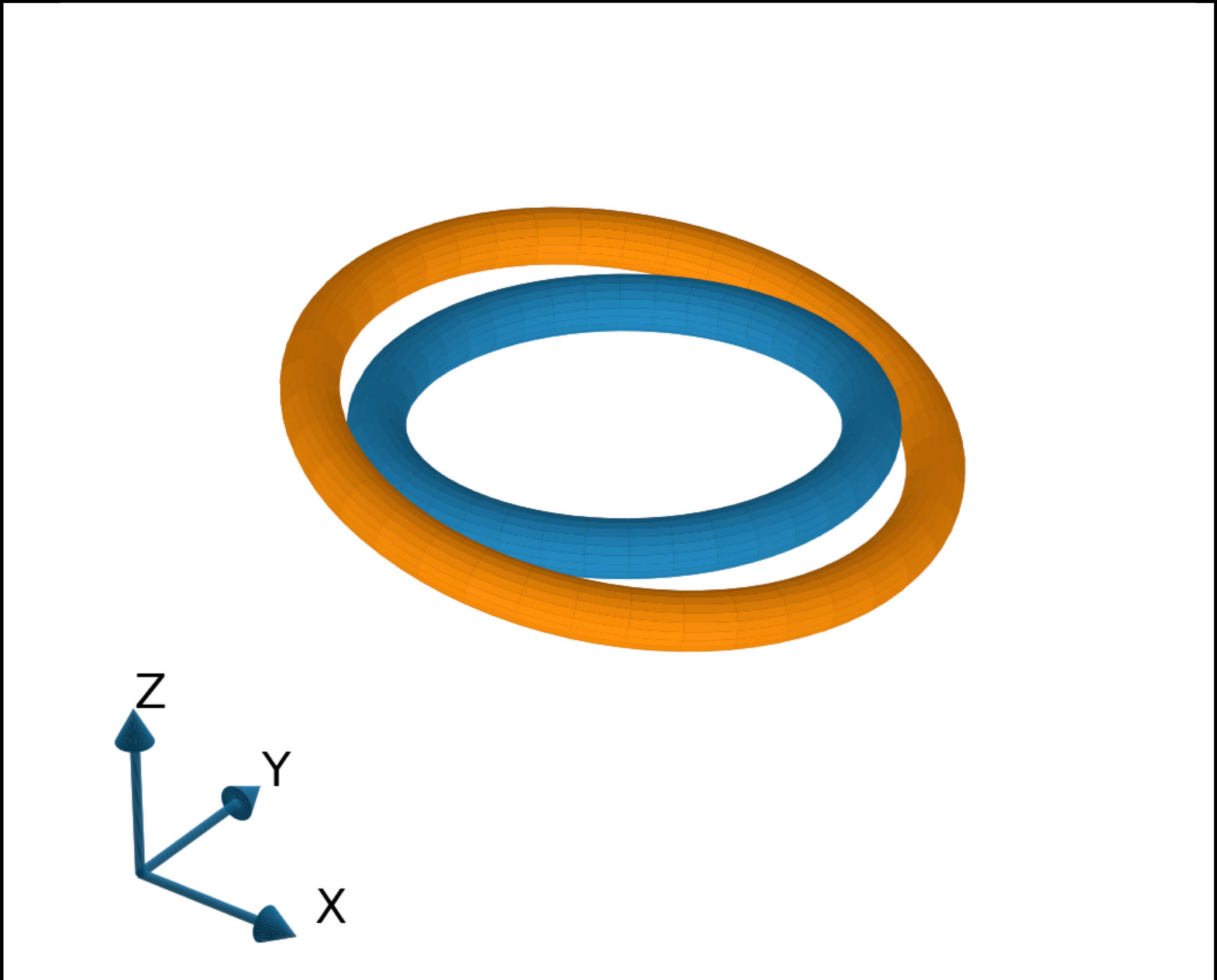
Broken disks in scattered light



Facchini, Juhasz & Lodato (2018)

Additional Slides

Two adjacent rings



Two adjacent rings