Dynamical and kinematical effects of self-gravity in planet forming discs

Giuseppe Lodato - MIAPP Workshop - 11 October 2021

Self-gravitating discs

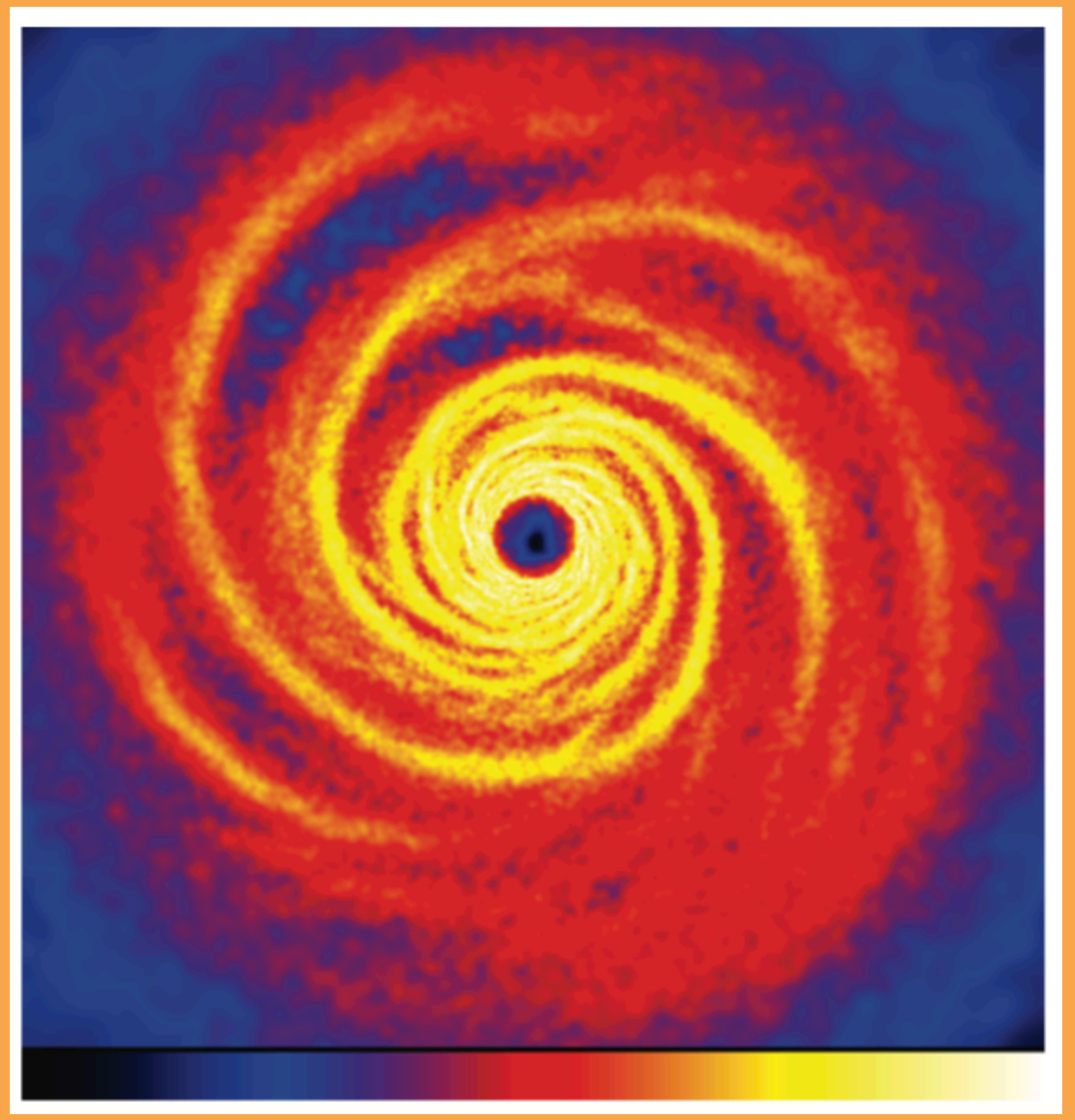
- Three main effects

Dynamical

- Ang. mom. transport
- Fragmentation
- Dust concentration

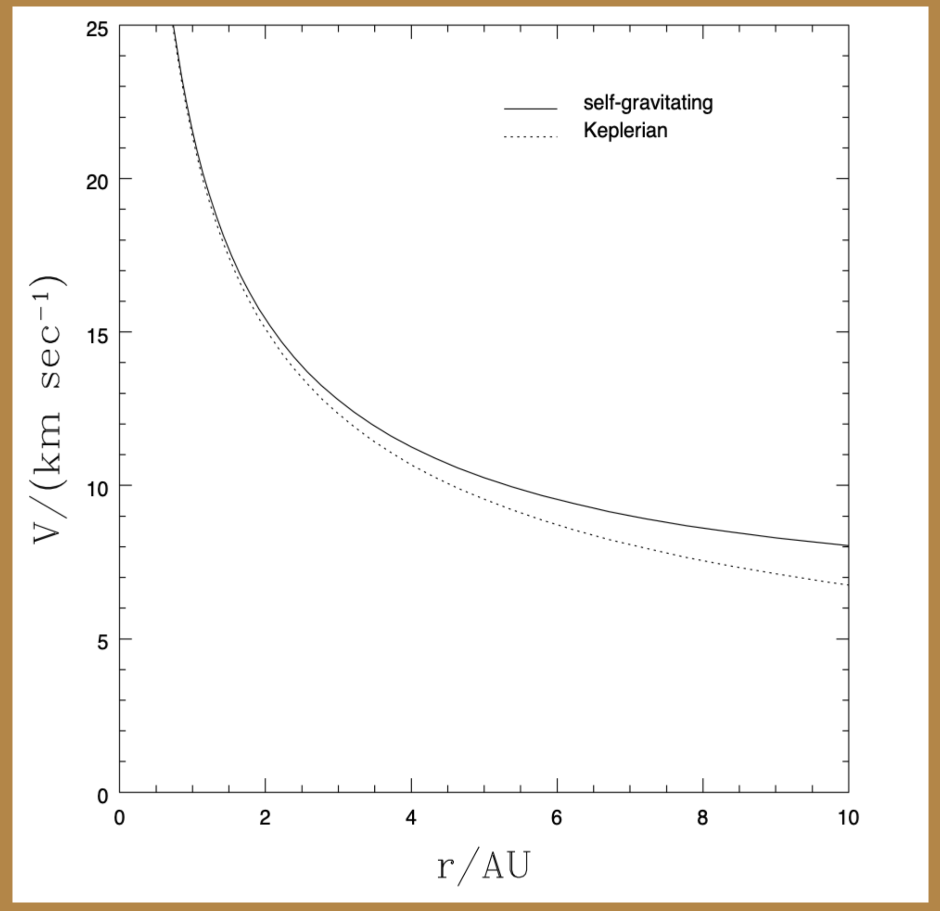
Morphological

- Spiral structure



Kinematical

- Rotation curve
- GI "wiggle"



Main parameters defining self-gravitating discs

Linear stability	Non-linear saturation	Global behaviour
$Q = \dfrac{c_s \kappa}{\pi G \Sigma}$	$\beta = t_{\rm cool}\Omega$	$\dfrac{M_{\rm disc}}{M_\star}$

- $Q \gg 1$ → stable
- $Q \lesssim 1$ → unstable
- $\beta > \beta_{\rm crit}$ → non-fragmenting
- $\beta < \beta_{\rm crit}$ → fragmenting
- $M_{\rm disc}/M_\star \ll 1$ → local
- $M_{\rm disc}/M_\star \lesssim 1$ → global

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$\beta >$ … non-fr…

$\lesssim 1$

Note that for a Keplerian disc

$$Q = \frac{c_{\rm s}\kappa}{\pi G \Sigma} \simeq \frac{c_{\rm s}\Omega}{\pi G \Sigma}$$

For a marginally stable disc with Q~1

$$\frac{M_{\rm disc}}{M_\star} \simeq \frac{H}{R}$$

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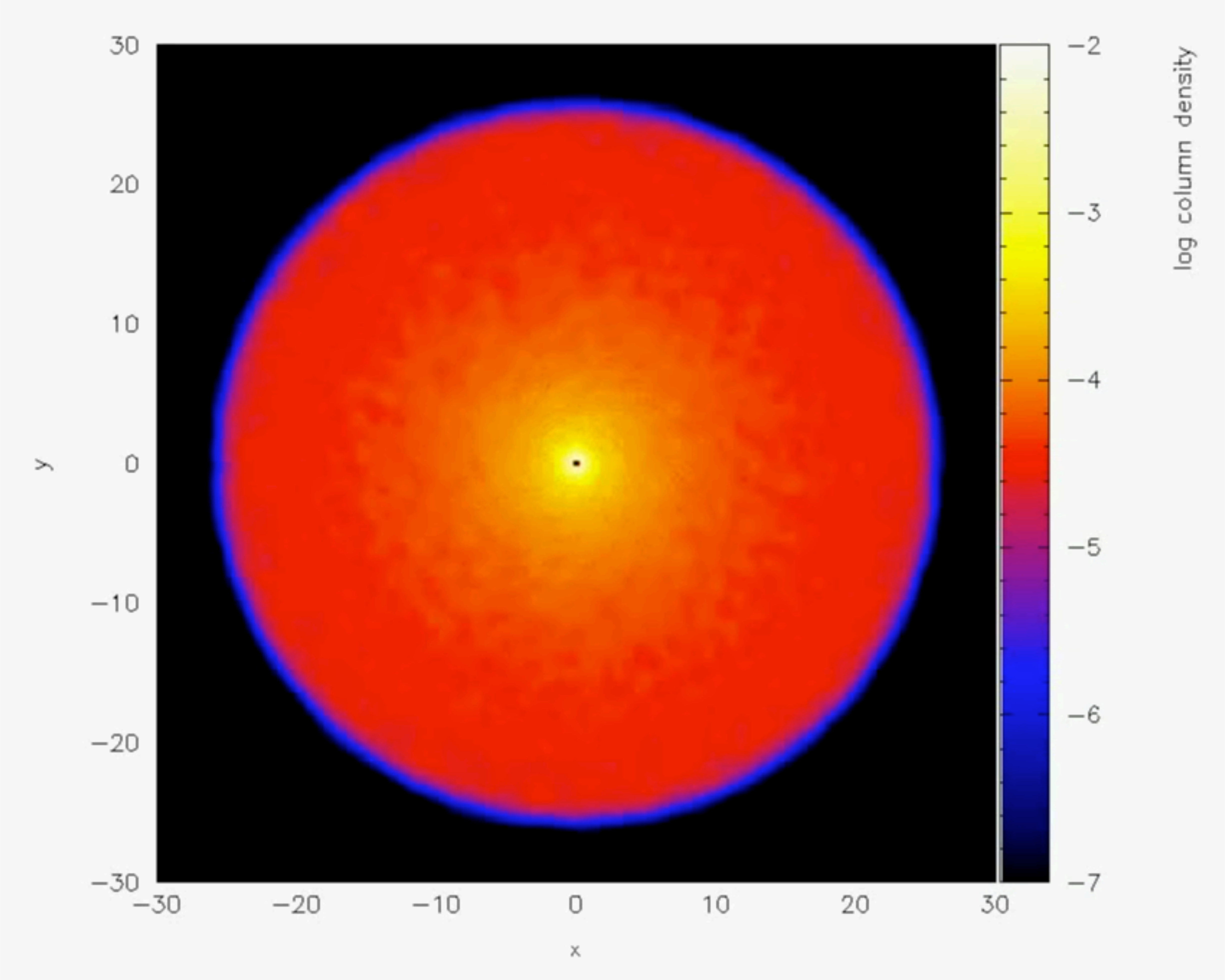
stable | global

What is the critical cooling for fragmentation?

From 2D simulations (Gammie 2001)	$\beta \simeq 3$
From 3D simulations (Rice et al 2005)	$\beta \simeq 6 - 7$
Non-convergence? (Meru & Bate 2011, 2012)	$\beta \gtrsim 10 - 15$
Convergence! (Deng et al 2017)	$\beta \simeq 3$

Cossins, Lodato & Clarke 2009

Self-regulation vs fragmentation



$$\beta = 4$$

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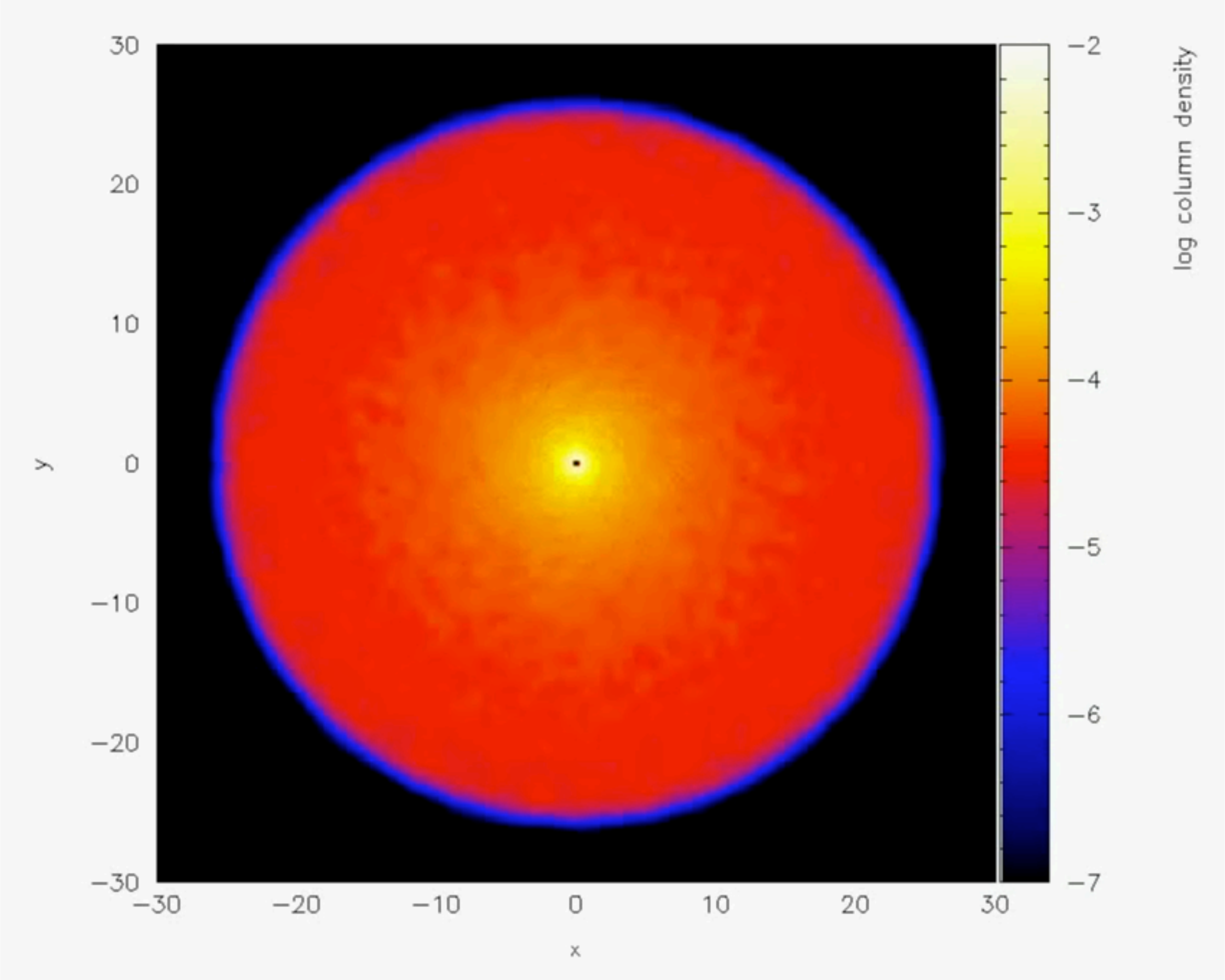

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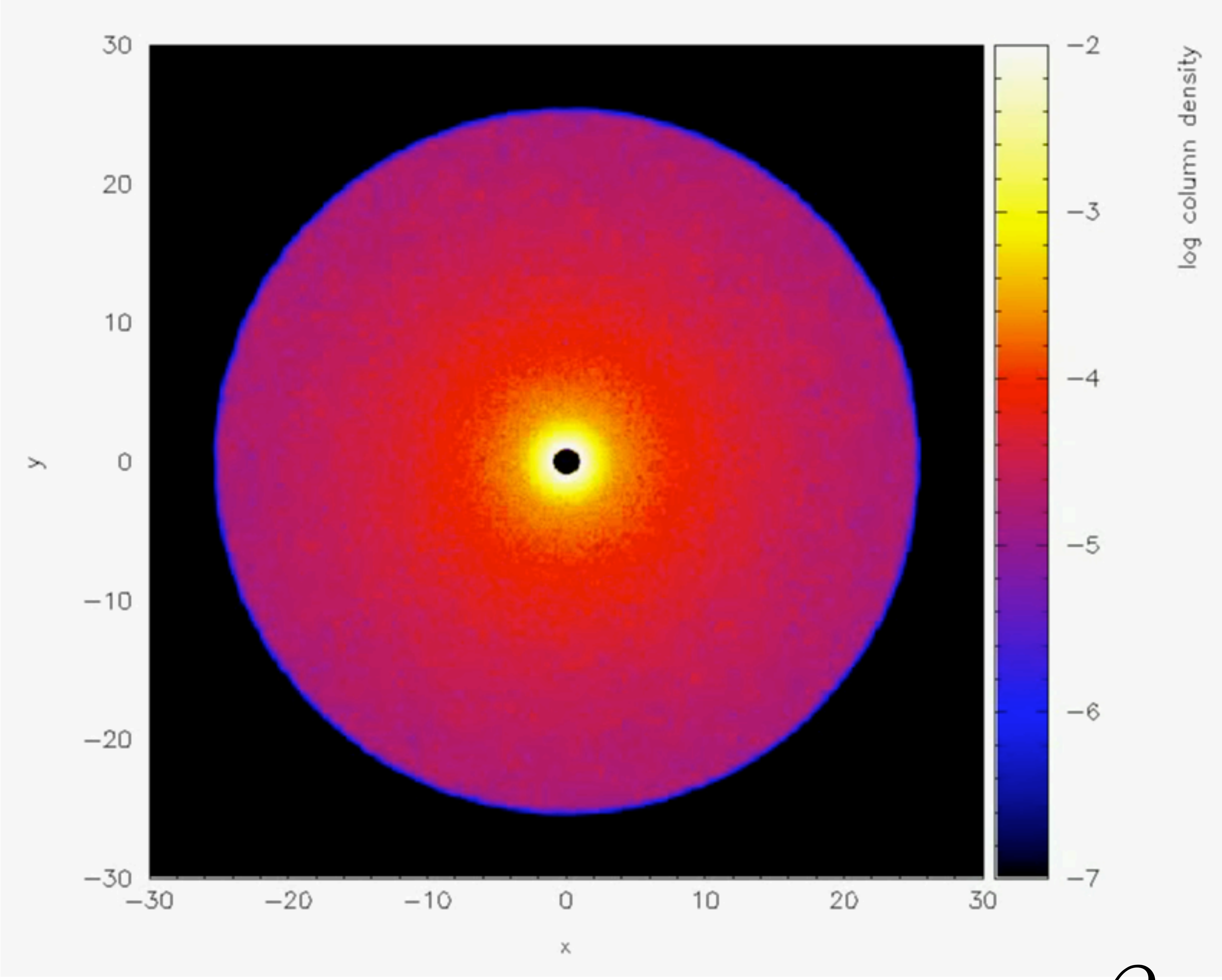

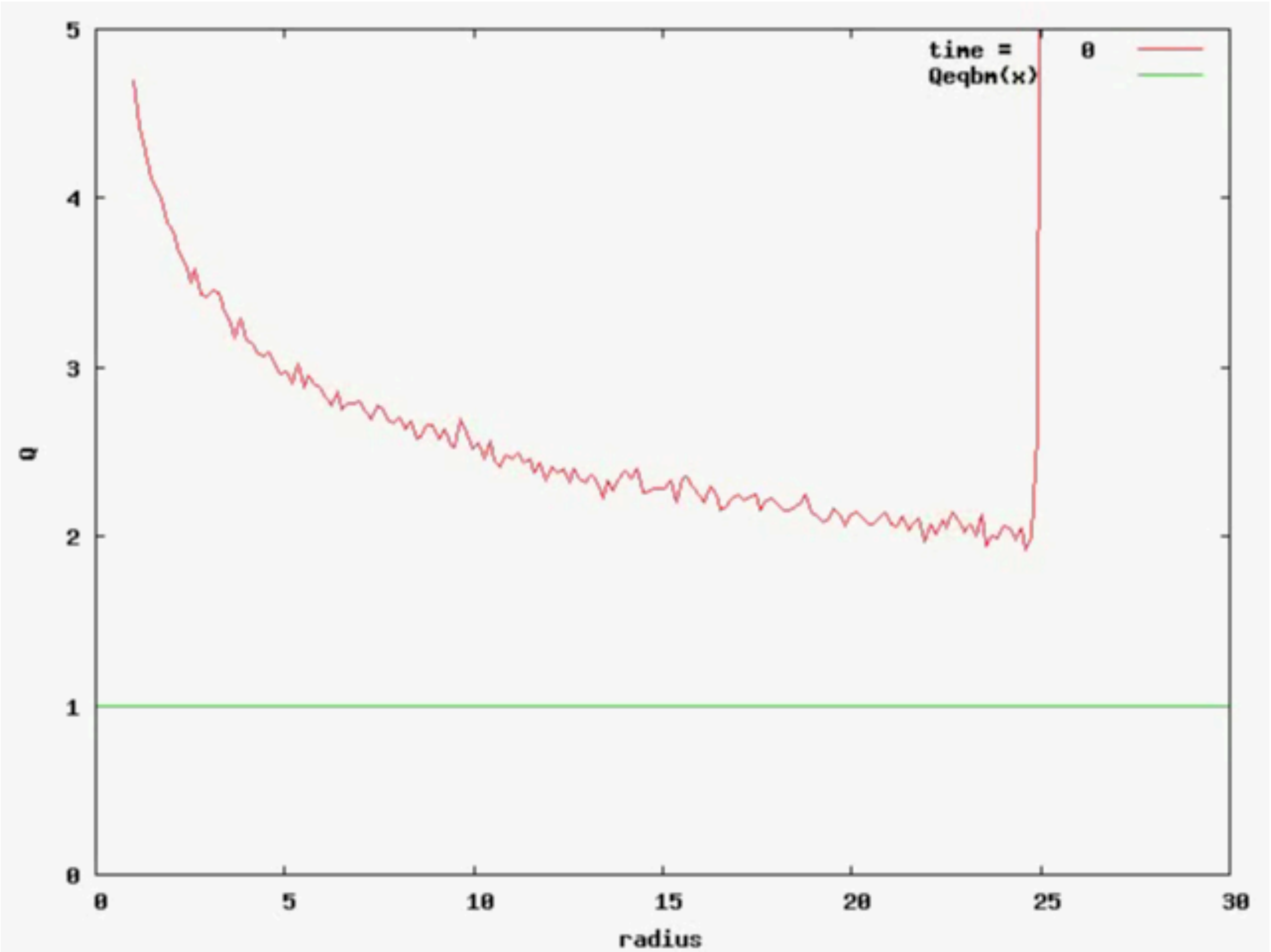

$$\beta = 6$$

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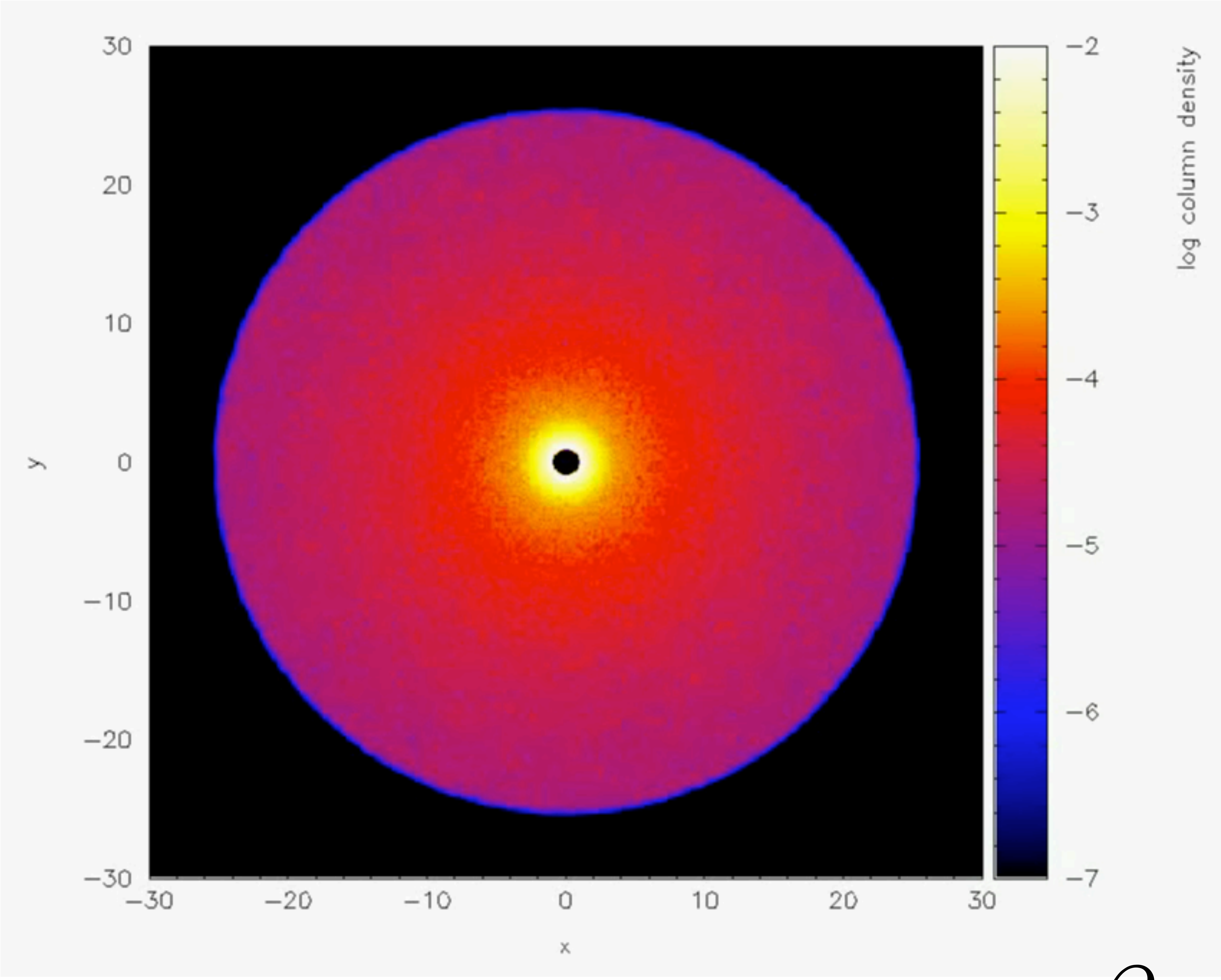

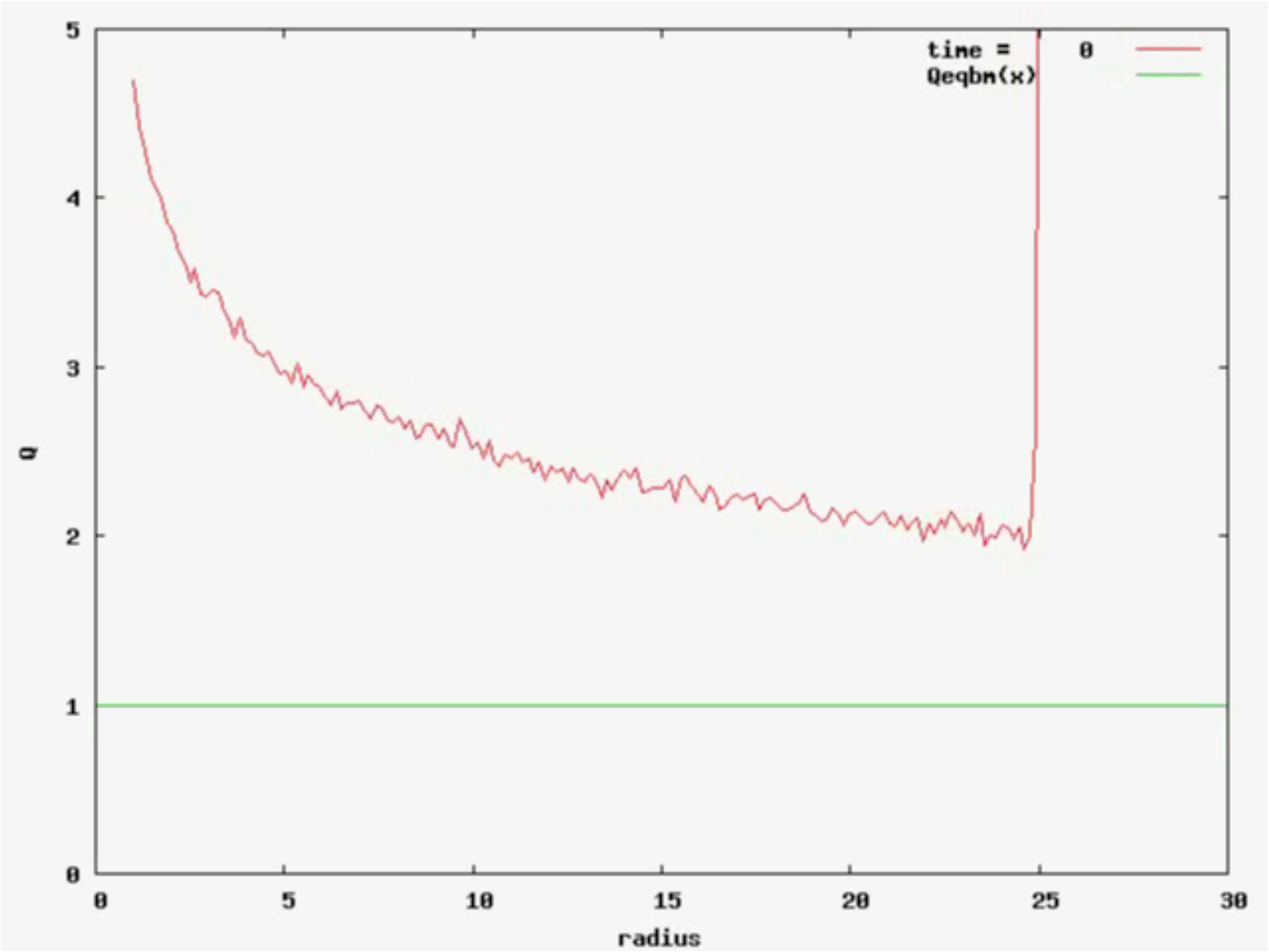

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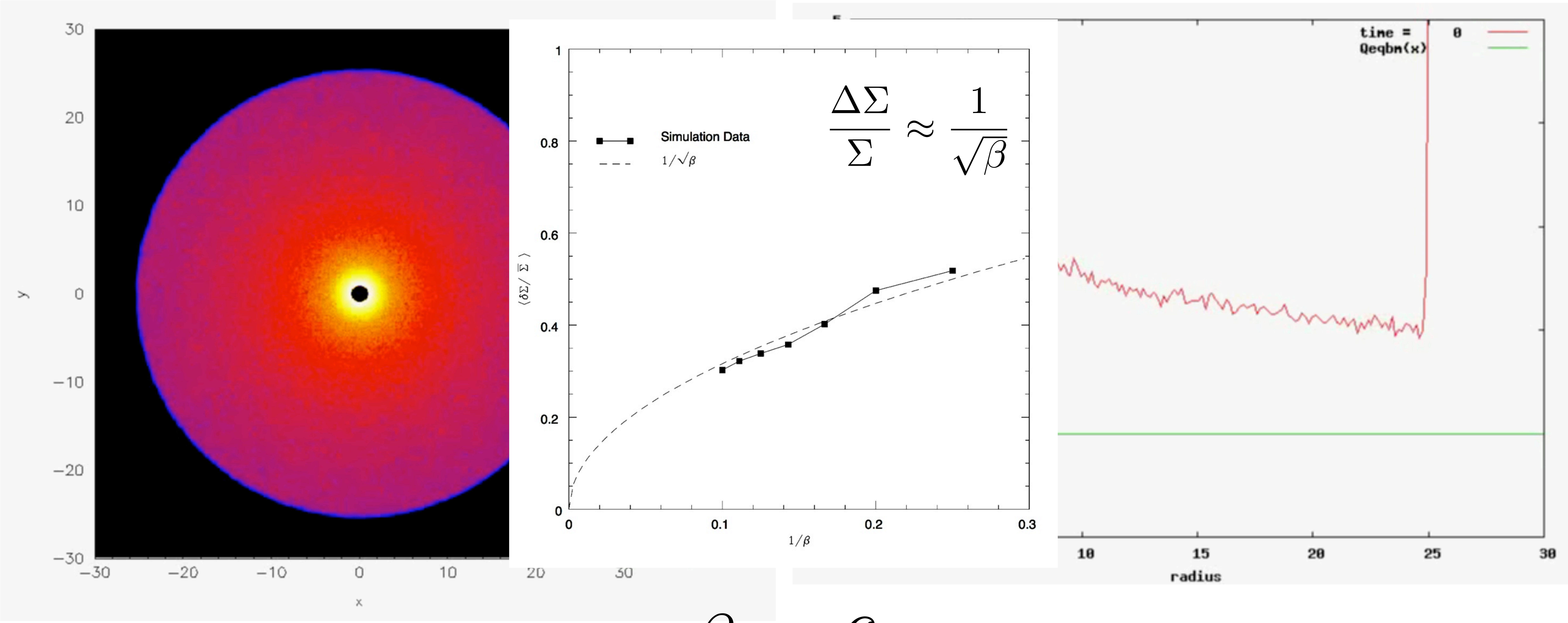

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Effects of disc mass

- **Morphology**
  - As $M_{disc}/M_{\star}$ increases, spiral becomes more open and dominated by fewer modes

$$\frac{M_{\rm disc}}{M_\star} \simeq \frac{\tan i}{m}$$

- **Dynamics**
  - High mass discs are unstable at larger values of Q>1
  - For low disc masses $M_{disc}/M_{\star} < 0.1$, transport is “viscous”-like

$$\alpha = \frac{4}{9\gamma(\gamma-1)}\frac{1}{\beta}$$

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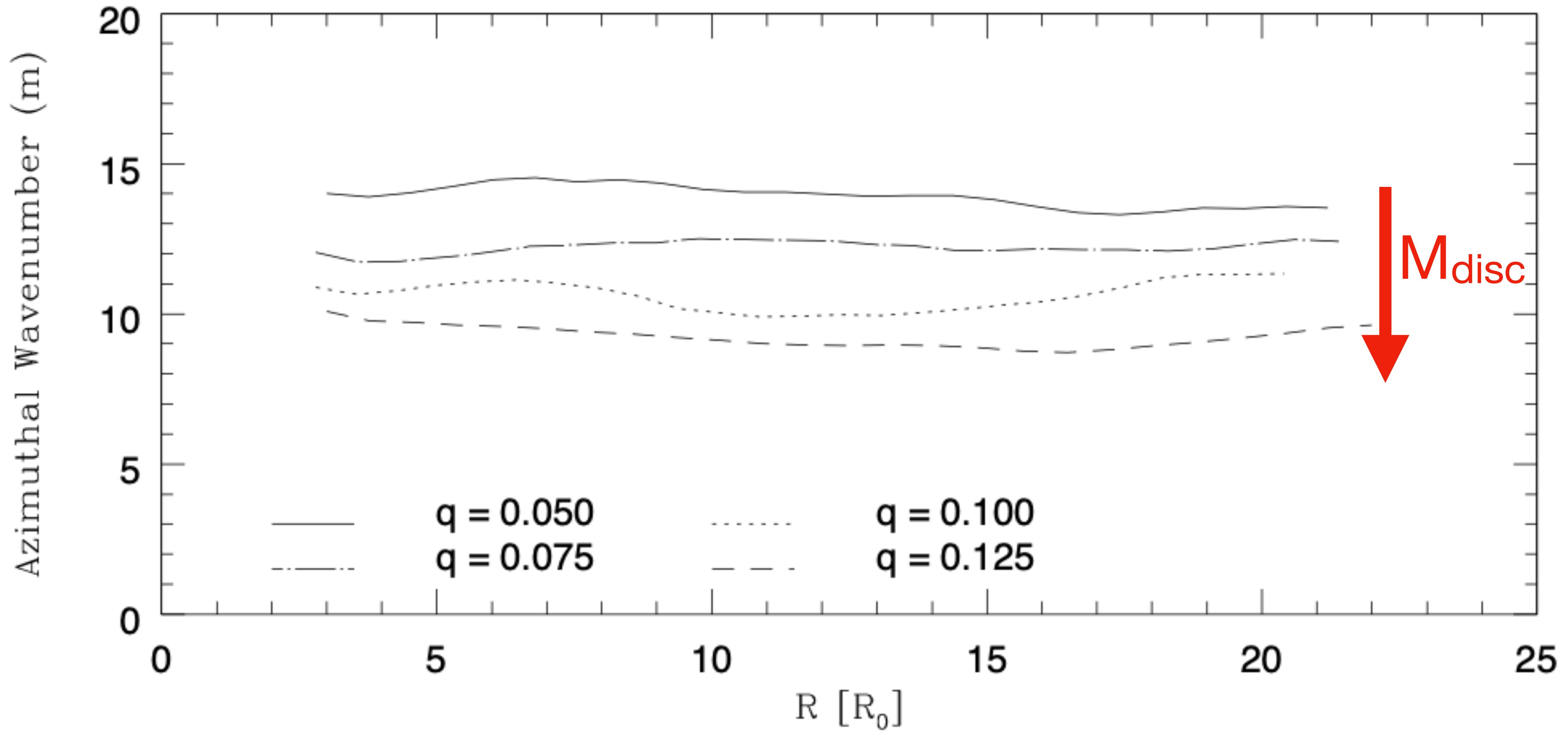

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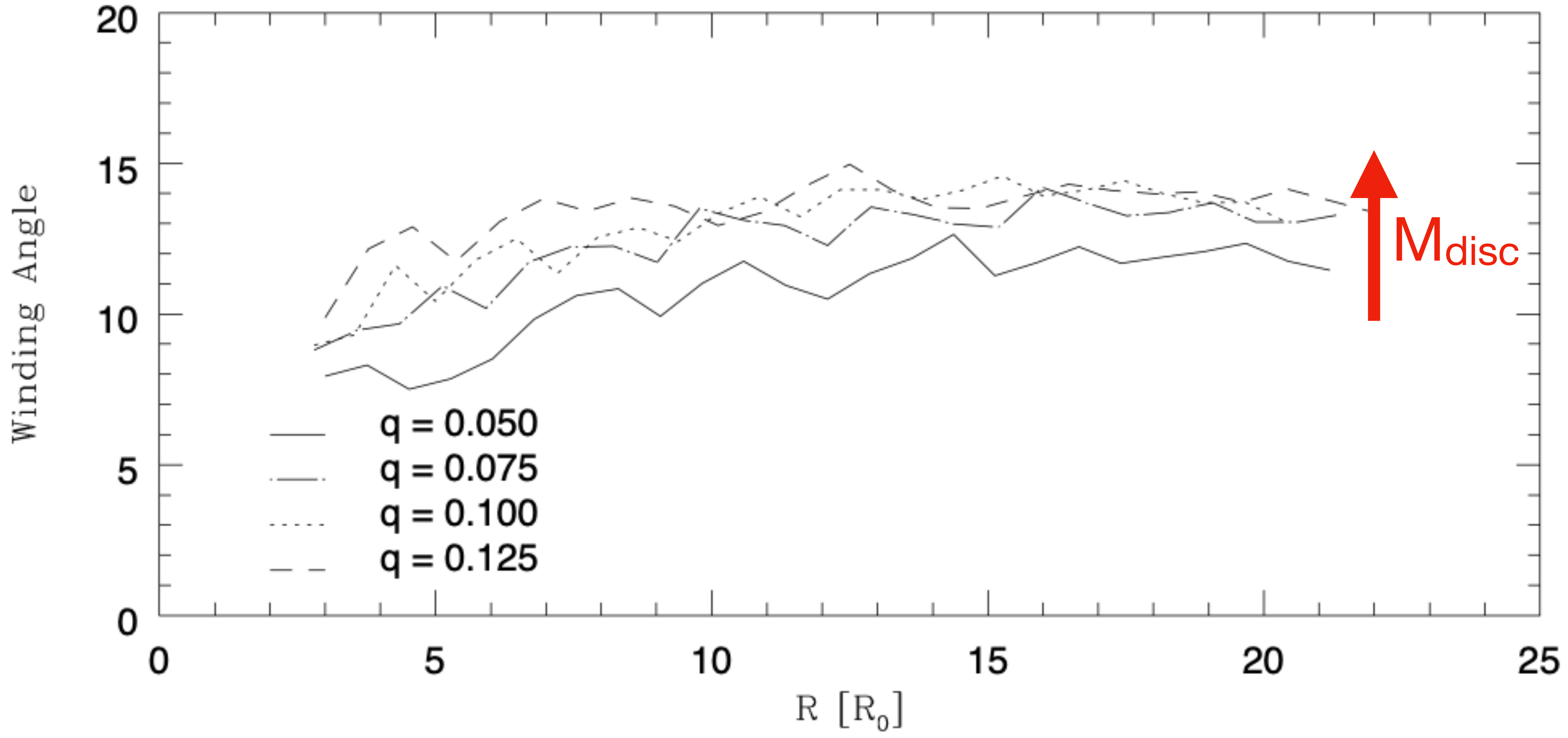

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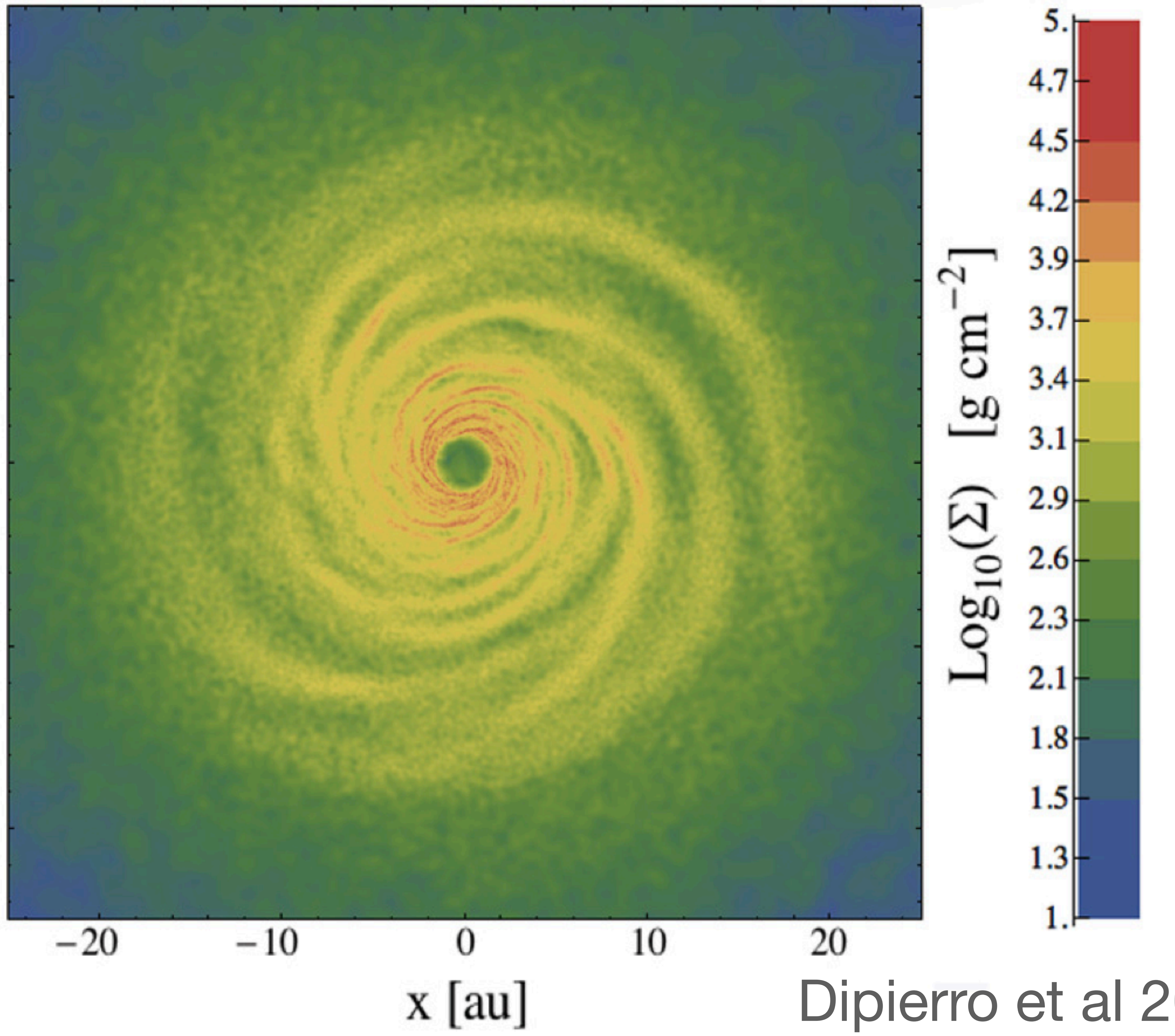
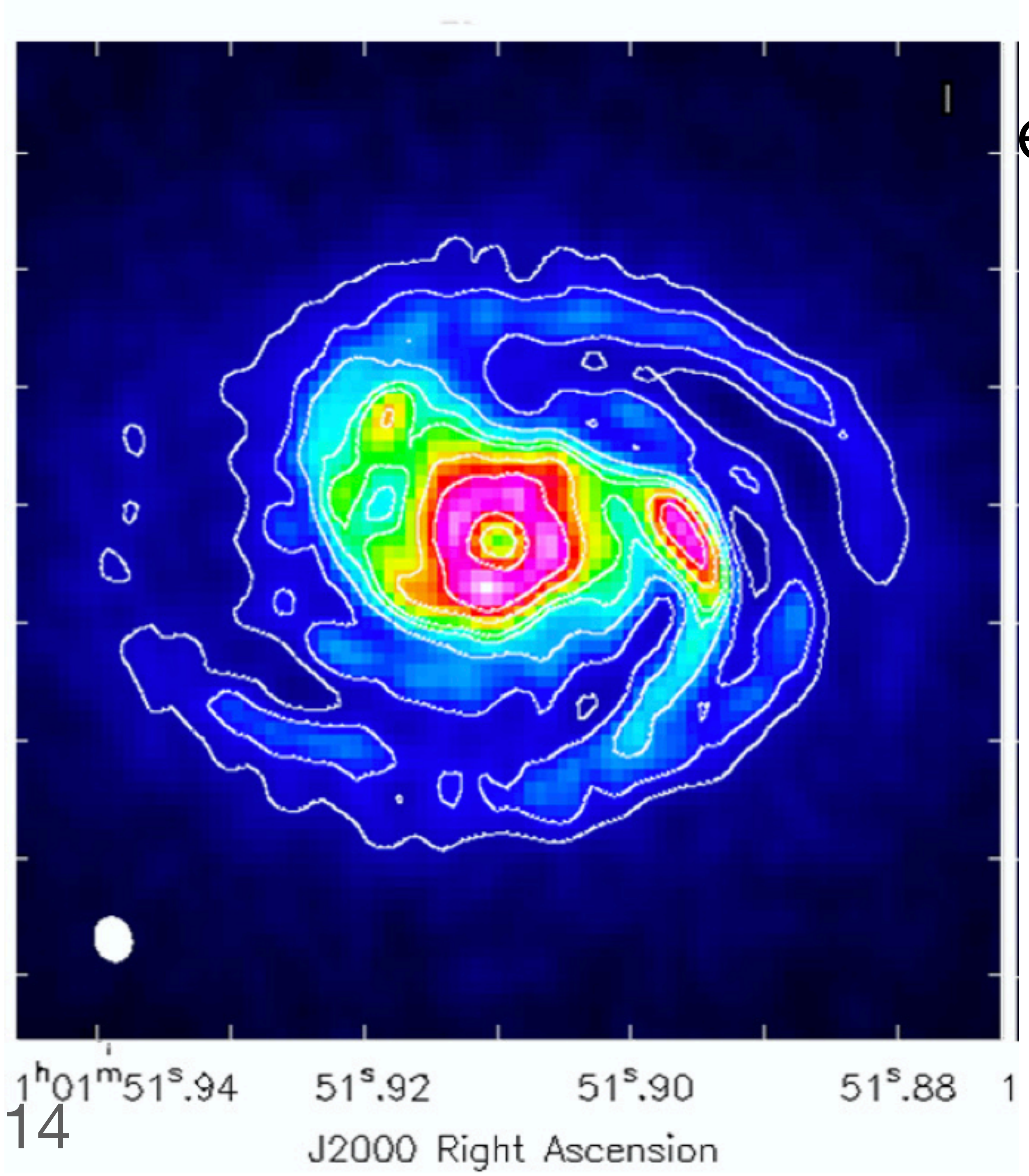


Dipierro et al 2014



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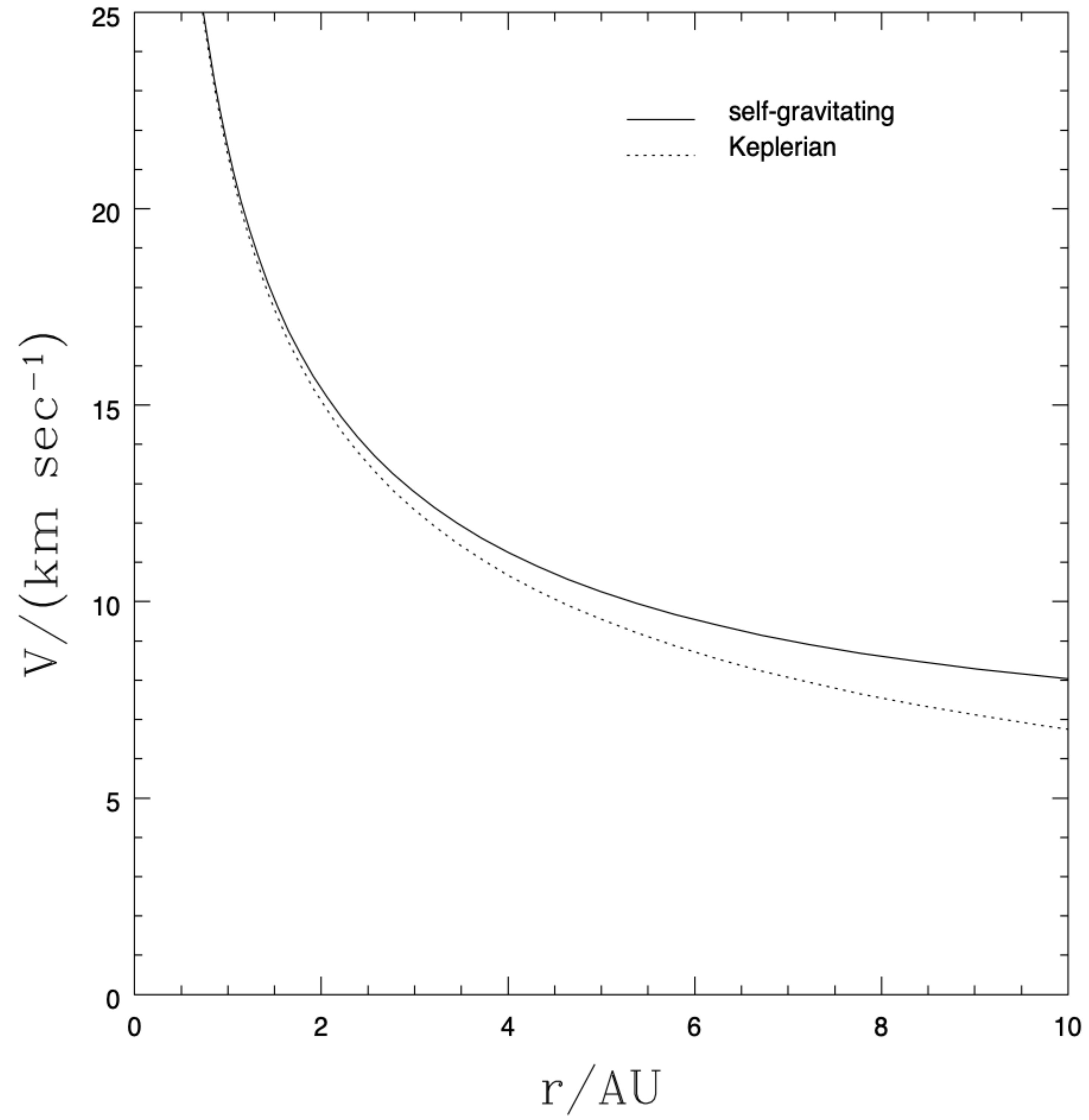
What about kinematics?

- Self-gravity modifies rotation curve

$$\Omega^2 = \frac{1}{R}\frac{\mathrm{d}\Phi_\sigma}{\mathrm{d}R}(R,z) + \frac{\mathcal{G}M_\star}{(R^2+z(R)^2)^{3/2}} + \frac{1}{R}\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}R}$$

super-Keplerian ~ Keplerian sub-Keplerian

Lodato & Bertin 2001



$$\frac{\partial\Phi_\sigma}{\partial R}(R,z) = \frac{\mathcal{G}}{R}\int_0^\infty \left[K(k) - \frac{1}{4}\left(\frac{k^2}{1-k^2}\right)\times\left(\frac{R'}{R}-\frac{R}{R'}+\frac{z^2}{RR'}\right)E(k)\right]\sqrt{\frac{R'}{R}}k\sigma\left(R'\right)dR'$$

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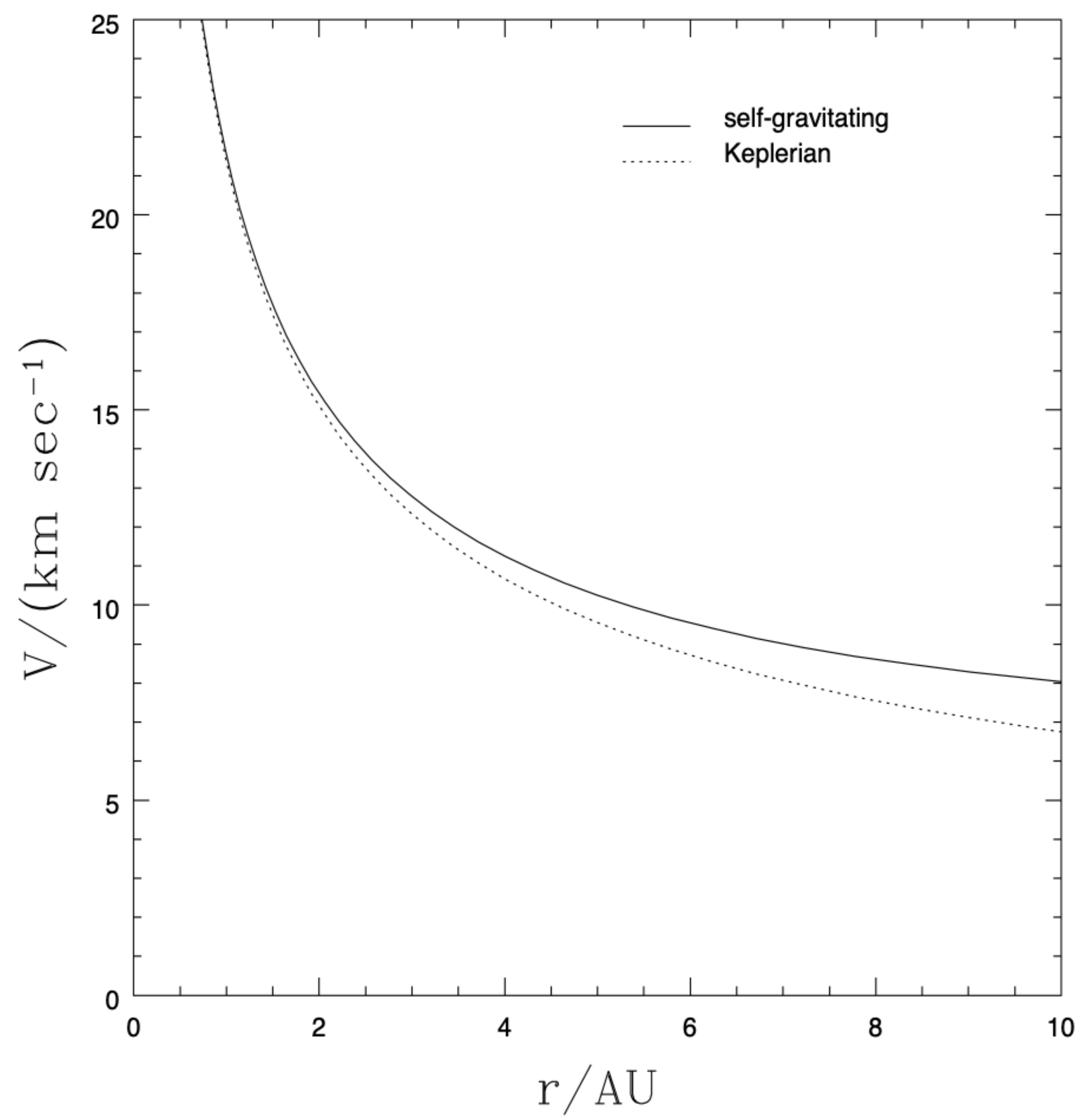

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super-Keplerian → $O\left(\frac{M_{\rm disc}}{M_\star}\right)$

~~~ Keplerian~~ → sub-Keplerian

~~sub-Keplerian~~ → super-Keplerian → $O\left(\frac{H^2}{R^2}\right)$

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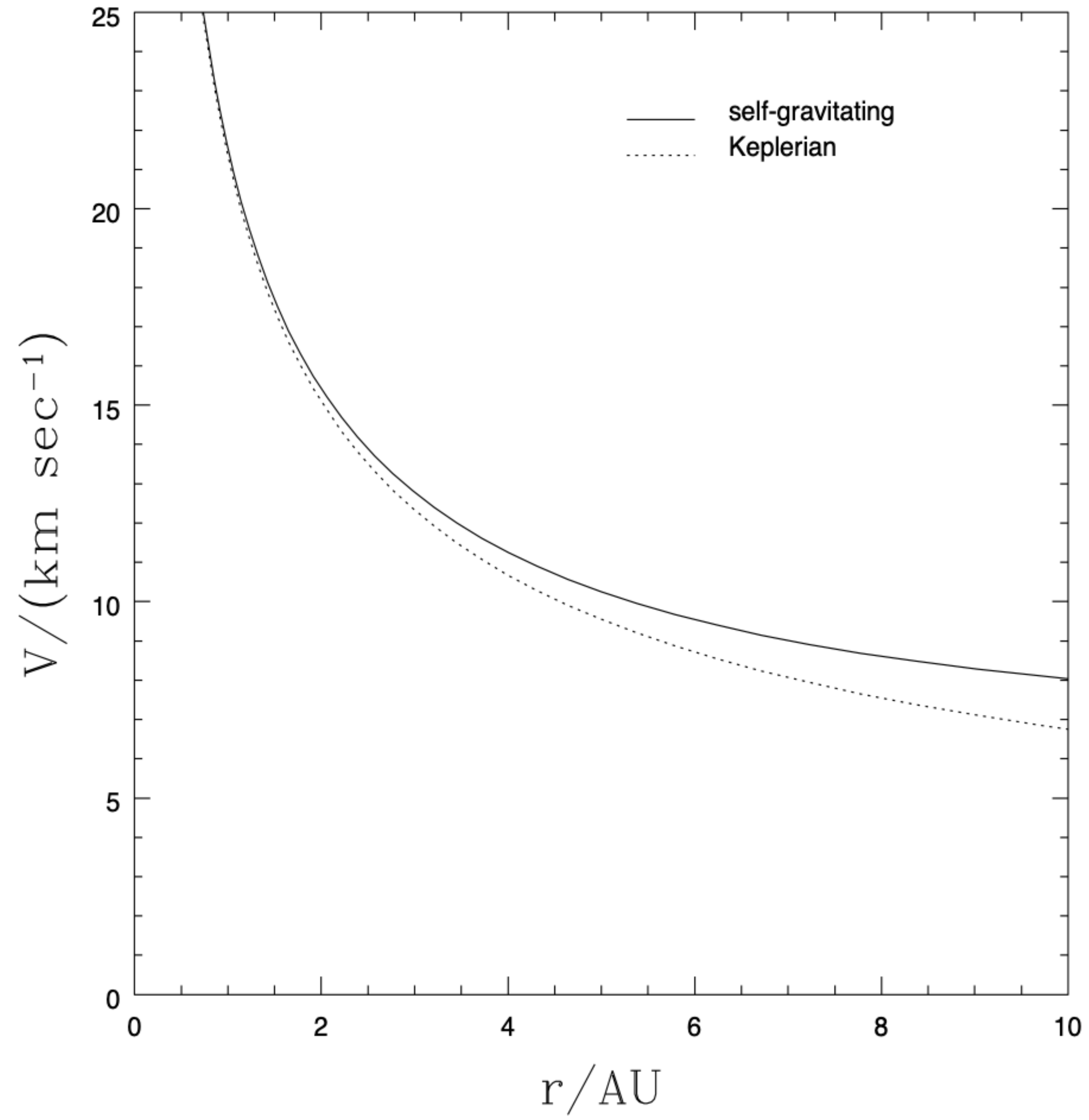

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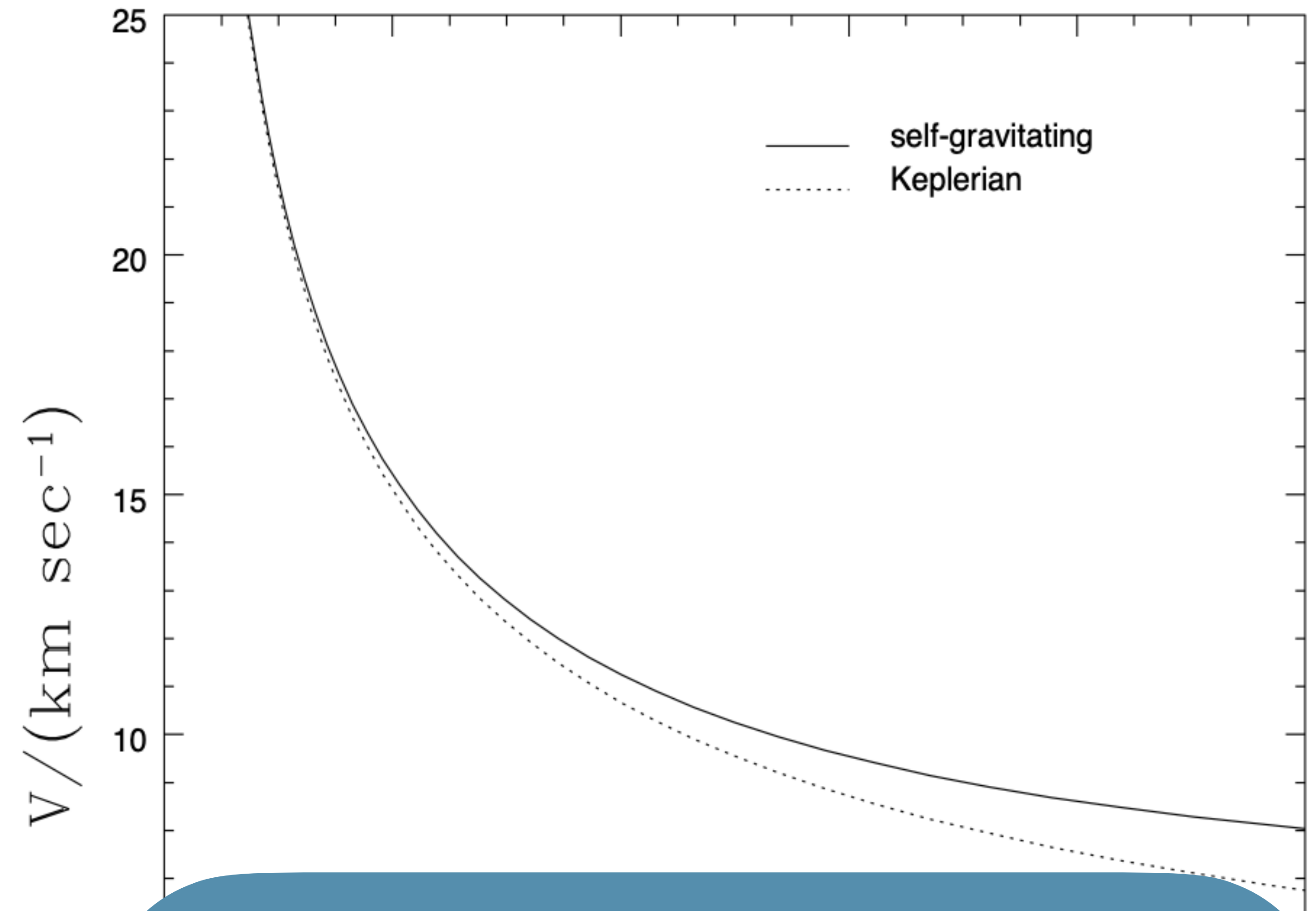

Disc contribution to rotation curve comparable to pressure terms even for discs that are gravitationally stable, $M_{\rm disc}/M_* \sim (H/R)^2$

$$\frac{\partial\Phi_\sigma}{\partial R}(R,z) = \frac{\mathcal{G}}{R}\int_0^\infty \left[K(k) - \frac{1}{4}\left(\frac{k^2}{1-k^2}\right)\times\left(\frac{R'}{R} - \frac{R}{R'} + \frac{z^2}{RR'}\right)E(k)\right]\sqrt{\frac{R'}{R}}k\sigma(R')\,dR'$$
~~~
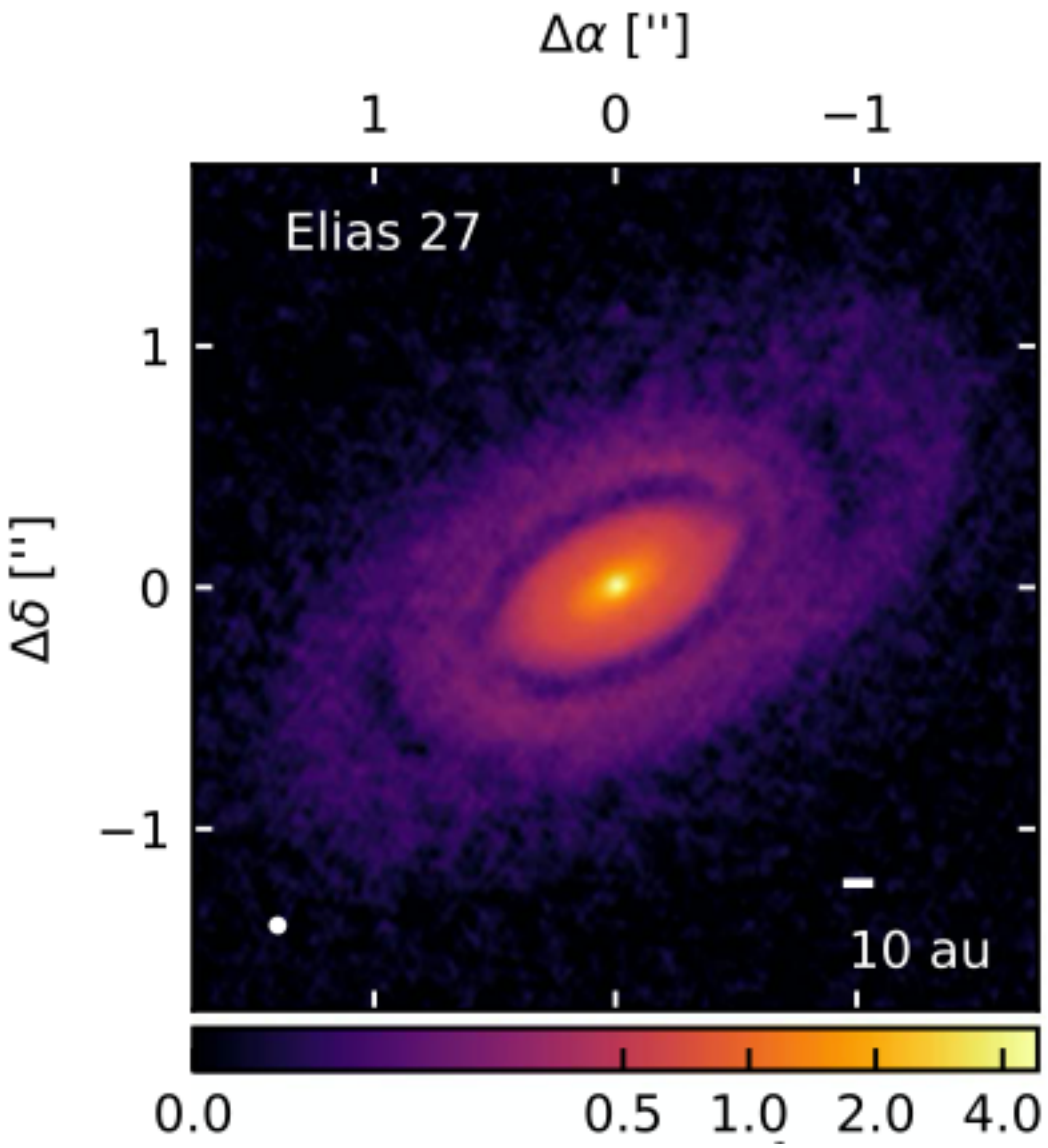
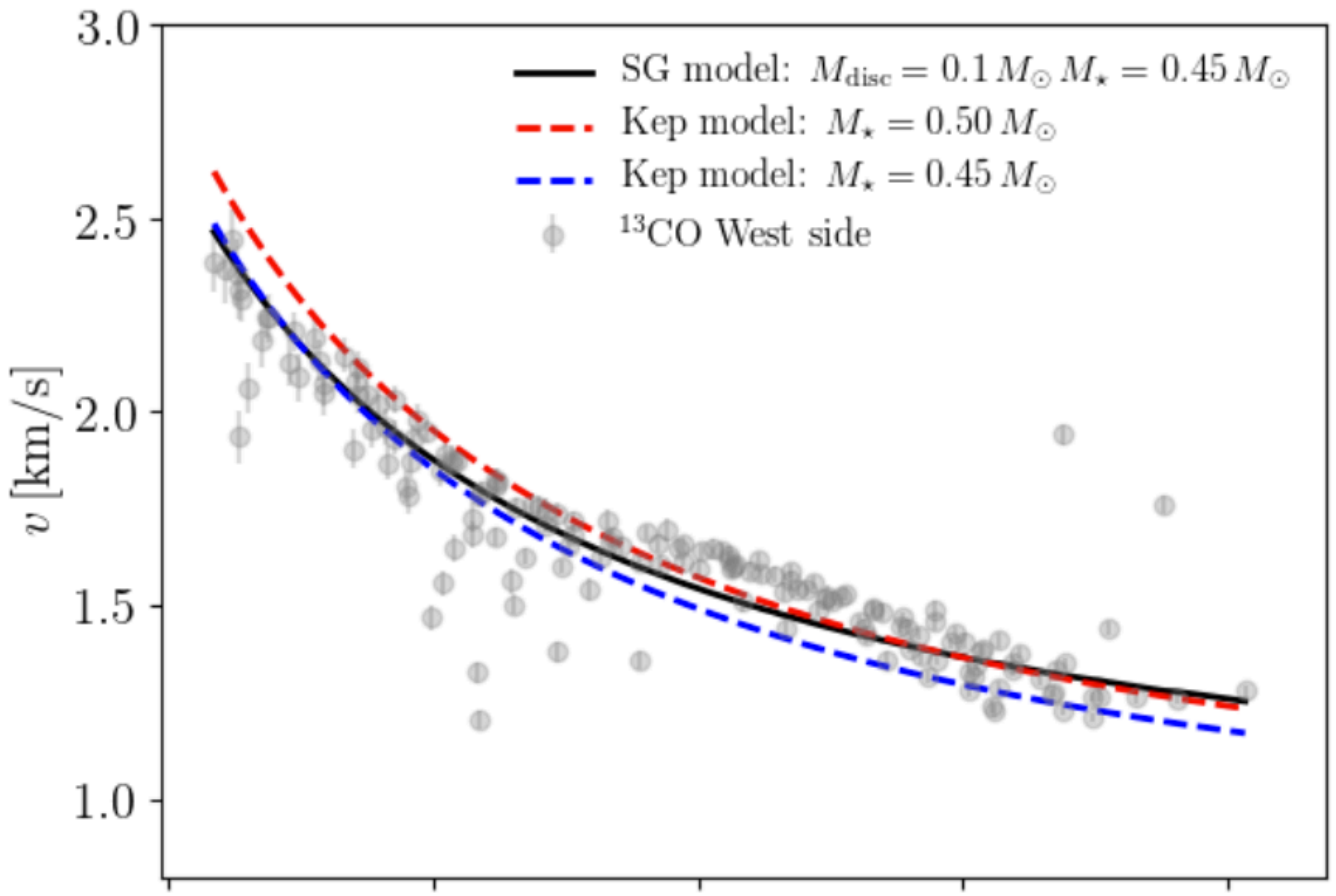

Is it possible to measure such deviations from Keplerianity?

- Yes! See Veronesi et al (2021) for Elias 2-27 (and Benedetta's talk tomorrow!)

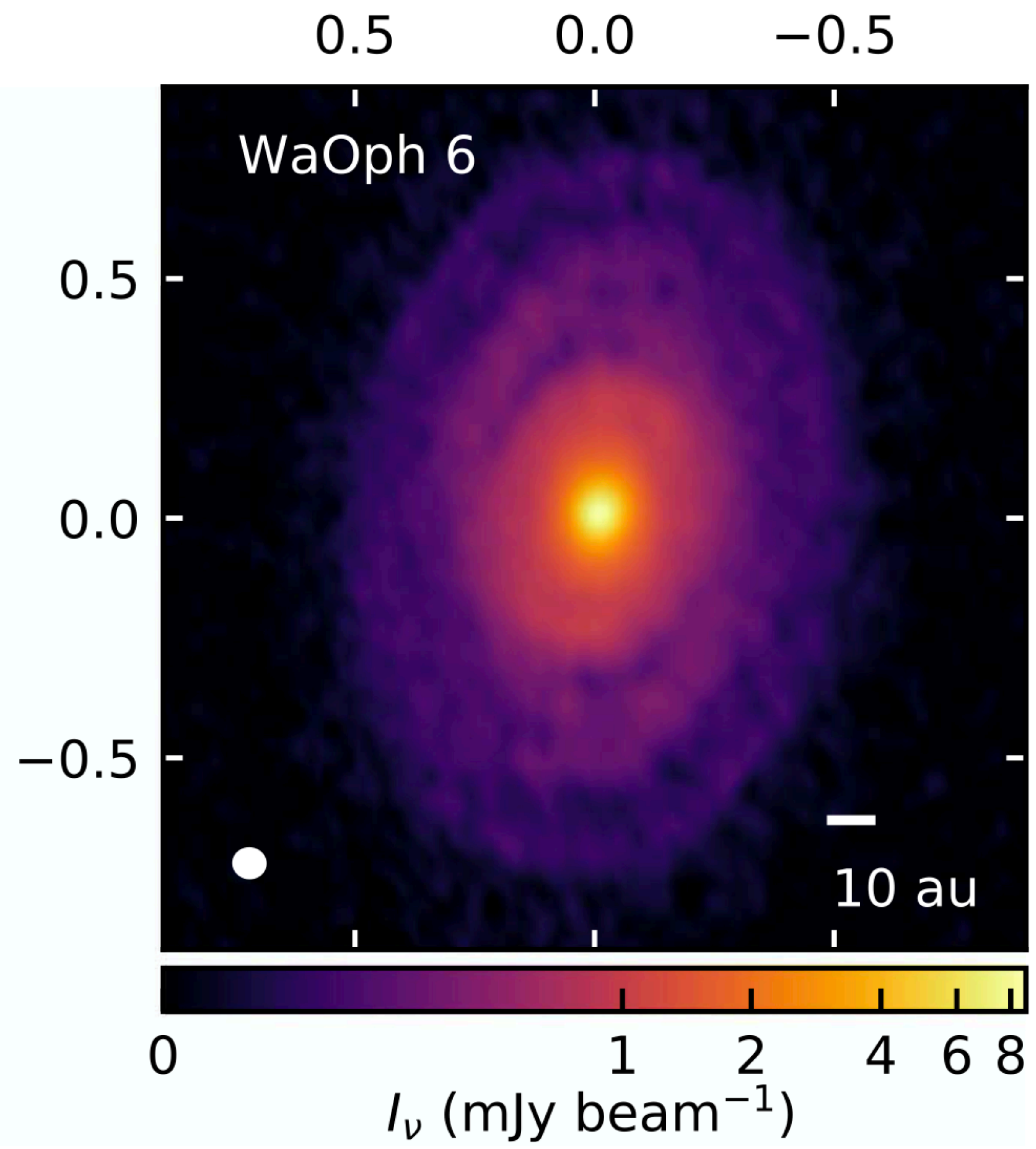




w/ L. Rampinelli, E. Viscardi, B. Veronesi, S. Facchini, L. Testi, A. Miotello, T. Paneque, A. Izquierdo

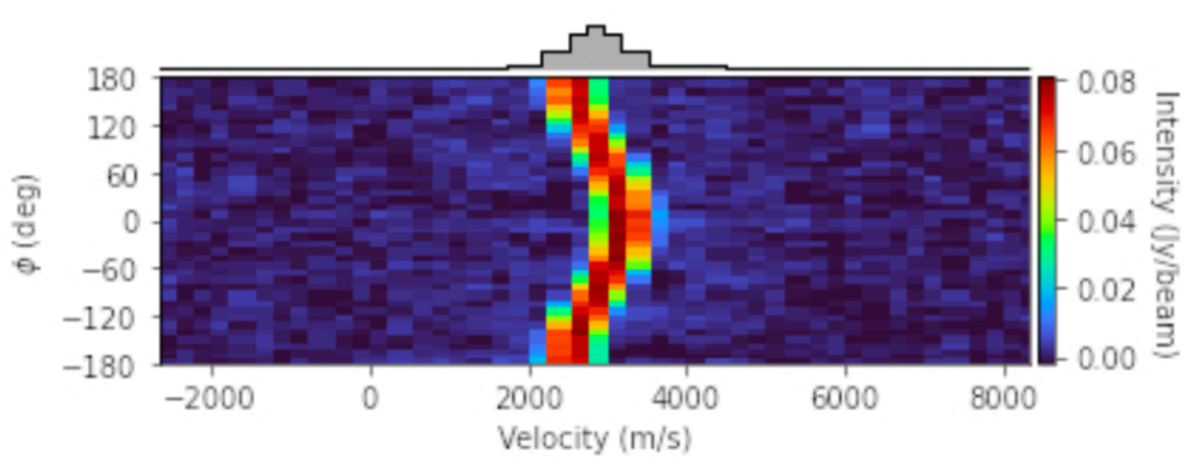
Can we apply the same technique to other sources?

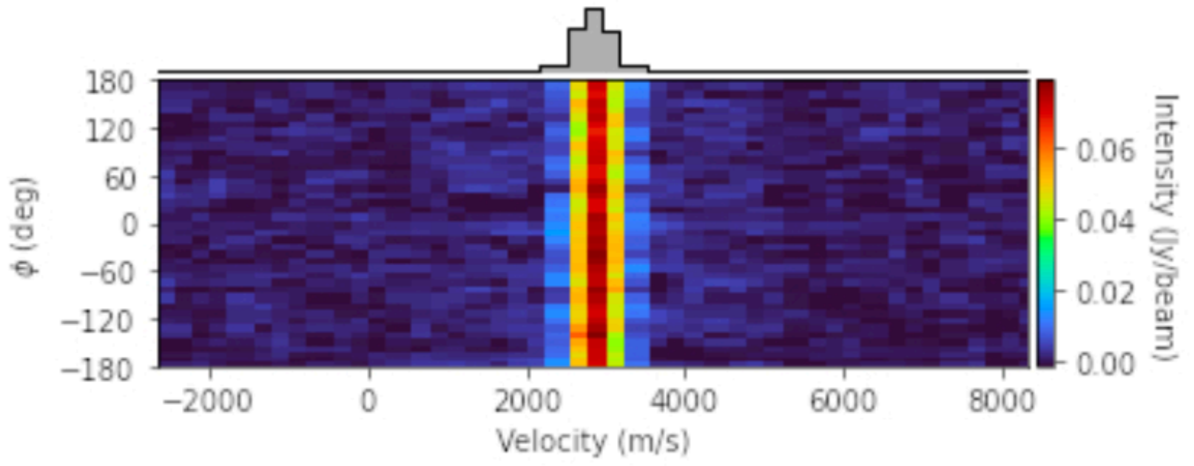
- In DSHARP, also Wa Oph 6 shows evidence of a spiral structure (Huang et al 2019)
- We tested the robustness of the measurement by fitting for the disc mass in different ways:
  - Retrieve the rotation curve using EDDY tools (Teague 2019)
  - Fit for the whole velocity datacube using DISCMINER (Izquierdo et al 2021)
- Data taken from archival Cycle 3 ^{12}CO observations, with 0.3″ angular resolution and 0.25 km/sec spectral resolution

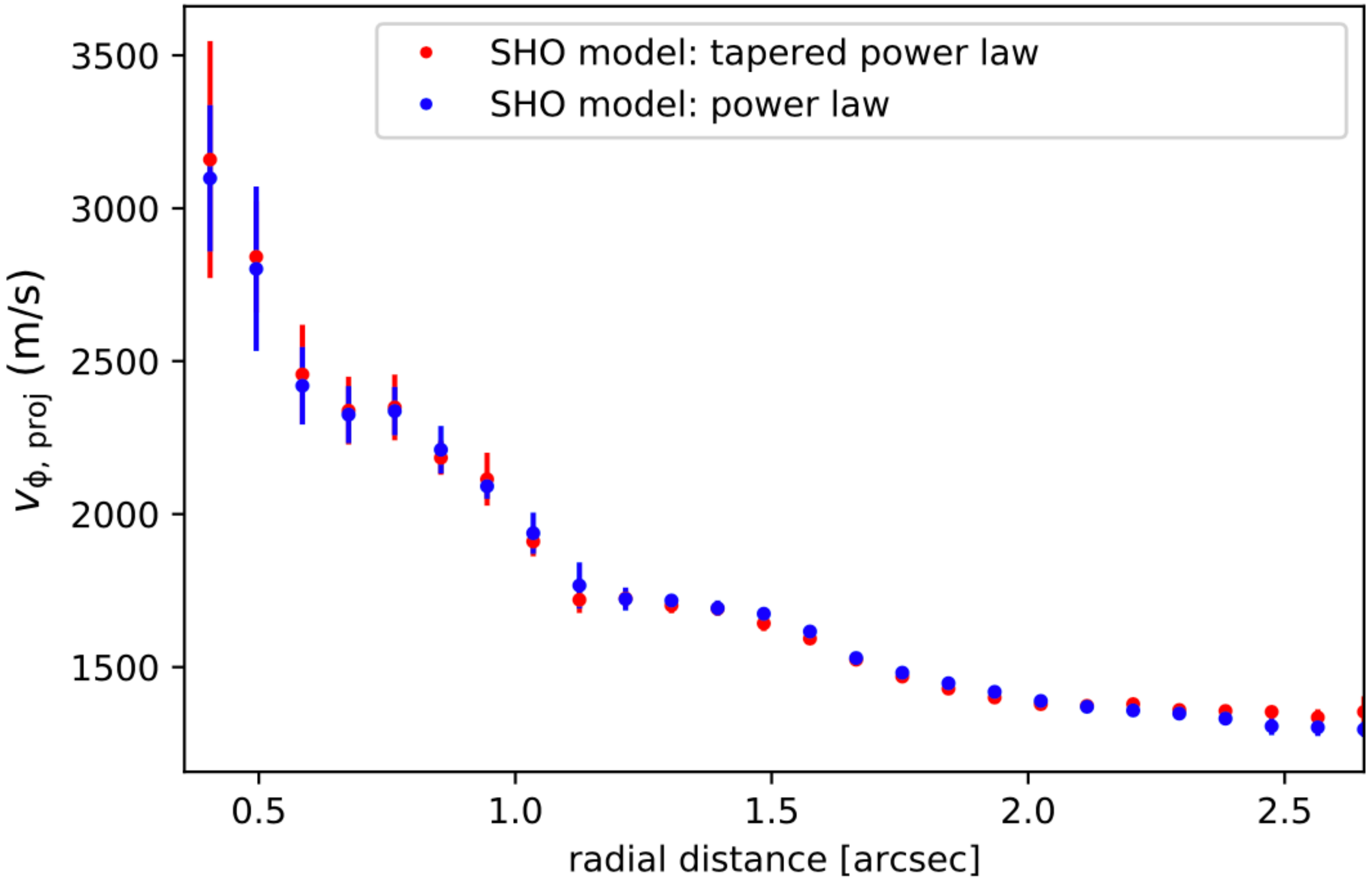


Method 1: Rotation curve from Eddy

- First retrieve the disc geometry and emission layer height (either using Eddy or Pinte et al 2018 method)
- Then use Eddy's SHO model to retrieve rotation curve

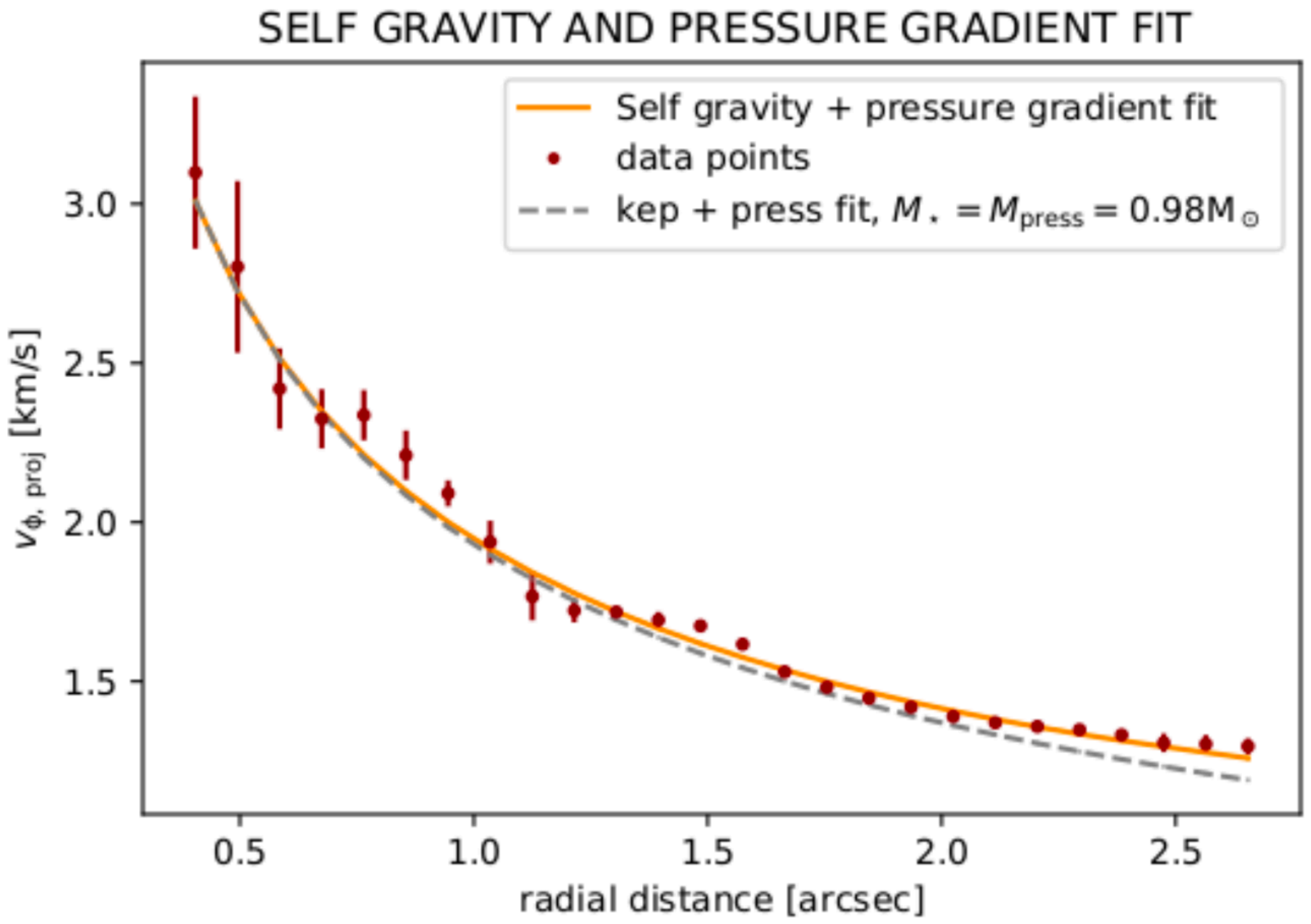


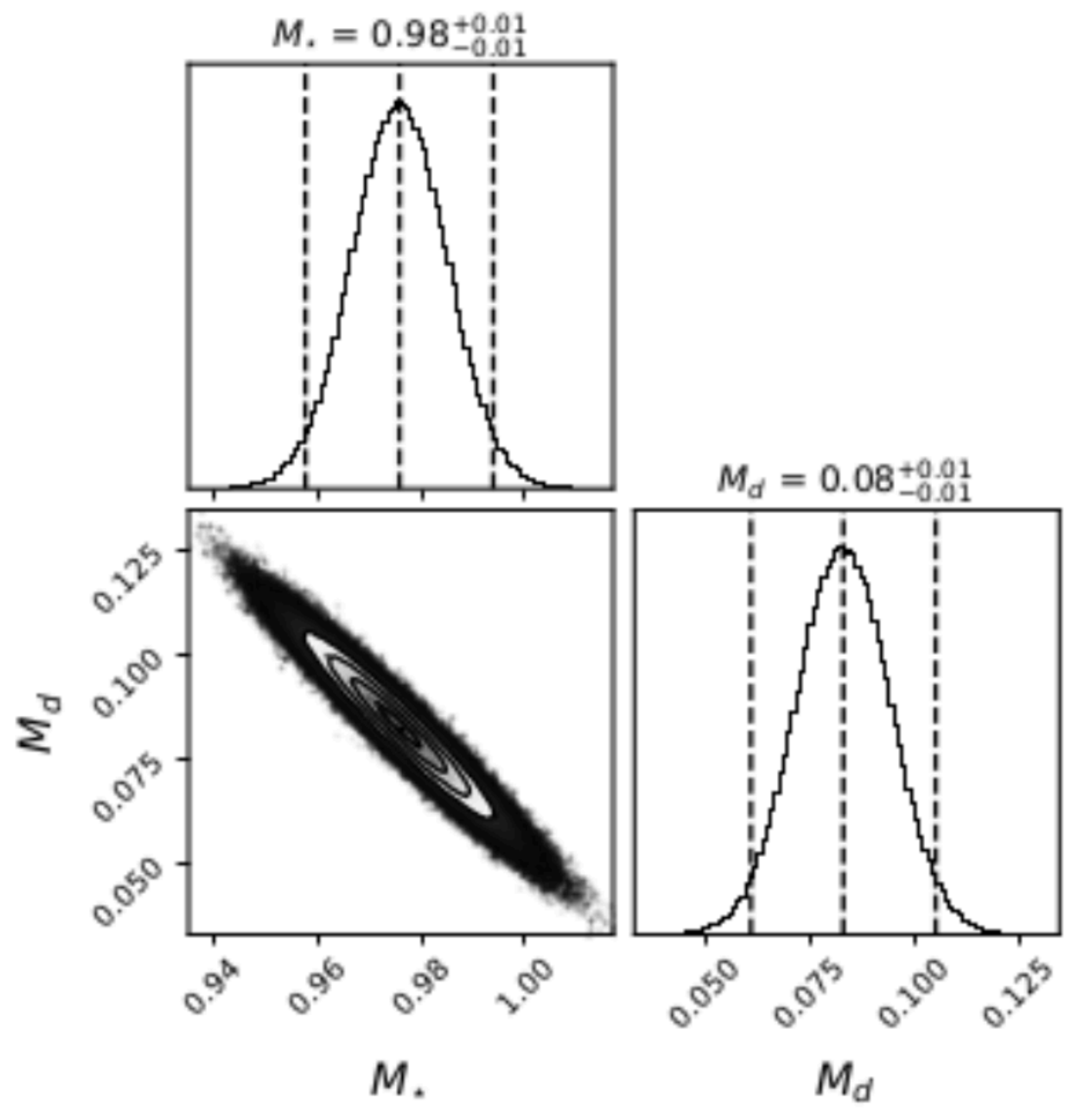




Method 1: Rotation curve from EDDY

- Tried different prescriptions for z(R) and Sigma(R): found differences within ~ 15%
- M_{disc} ~ 0.08 +- 0.01 M_{sun}





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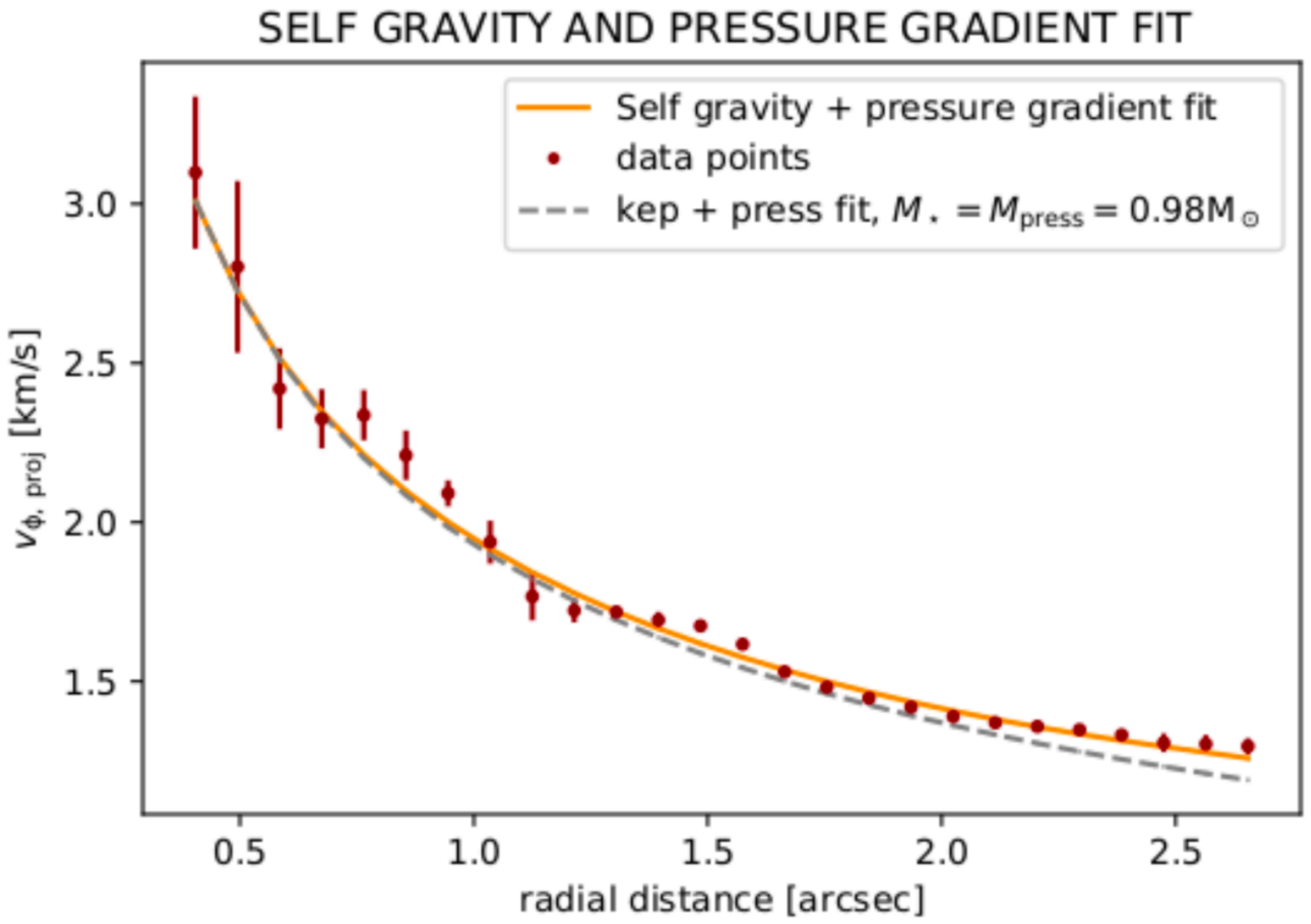

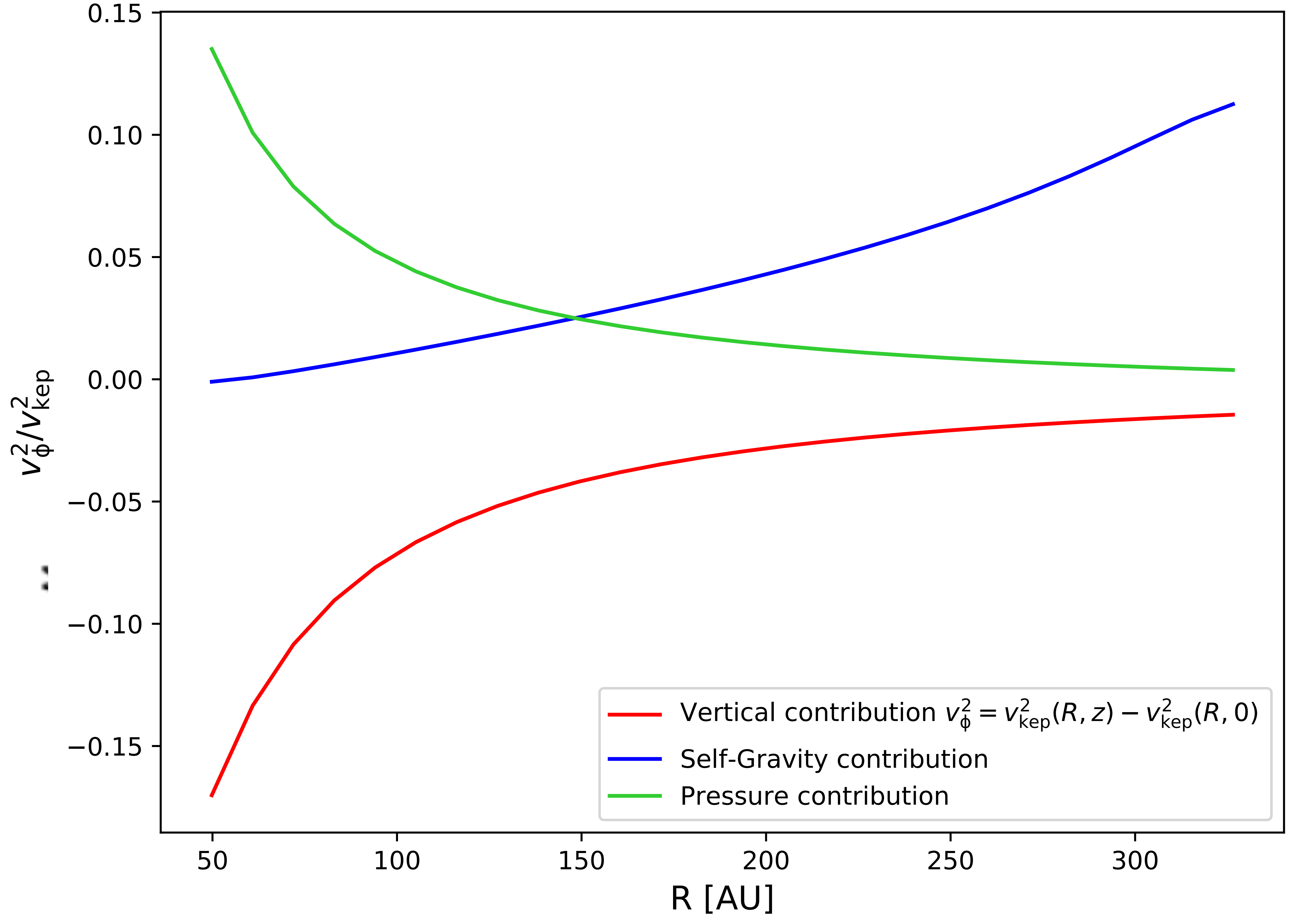

Method 2: Use DiscMiner

- Use a parametric disc model to fit the whole datacube
- Several fit parameters (related to emission profile, height of emitting layer, surface density, geometry,….)
- Tried with several functional forms to test robustness
- M_{disc} ~ 0.06-0.1 M_{sun}
- Andrews et al (2009): M_{dust} ~ 7.7 10^{-4} Msun —> gas/dust ~ 103

	$M_\star$ [$M_\odot$]	M_{disc} [$M_\odot$]
KEP p. law	$1.1270^{+0.0004}_{-0.0003}$	
KEP tp. law	$1.086^{+0.007}_{-0.007}$	
KEP fix	$1.1056^{+0.0008}_{-0.001}$	
SG p. law	$1.0337^{+0.001}_{-0.0009}$	$0.0873^{+0.0002}_{-0.0001}$
SG tp. law 2	$1.0527^{+0.001}_{-0.0008}$	$0.0580^{+0.0002}_{-0.0004}$
SG fix	$0.970^{+0.002}_{-0.002}$	$0.0788^{+0.0002}_{-0.0001}$
Press tp. law	$1.059^{+0.001}_{-0.001}$	$0.098^{+0.003}_{-0.001}$
Press fix	$1.0465^{+0.0006}_{-0.0005}$	$0.150^{+0.002}_{-0.002}$

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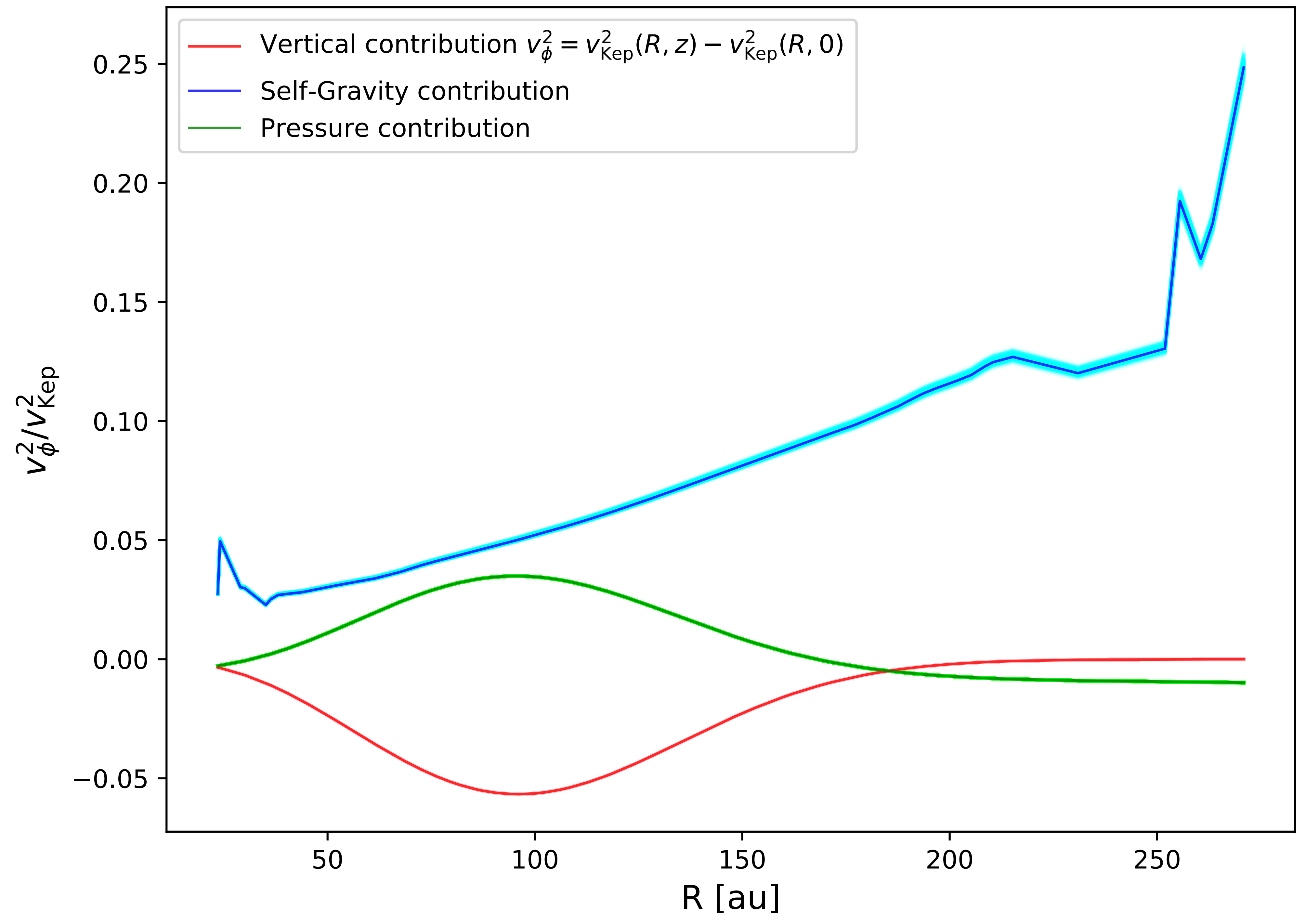

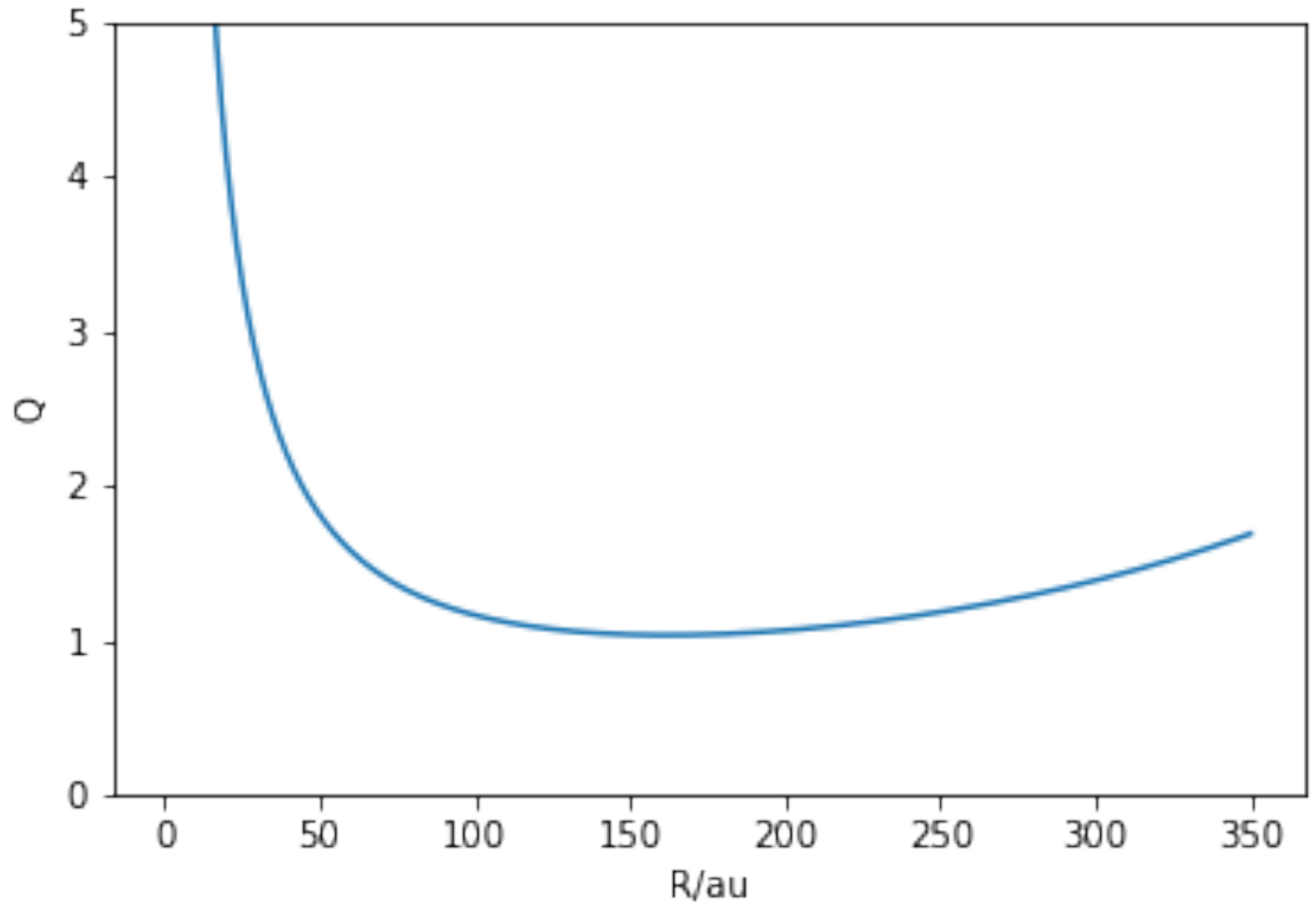

Dynamical measurements of the disc mass

- Disc kinematics has reached the precision to allow a dynamical measurement of the disc mass
- Disc mass contribution to non-Keplerianity comparable to pressure even for gravitationally stable discs!
- For GI unstable discs, disc contribution dominates over pressure

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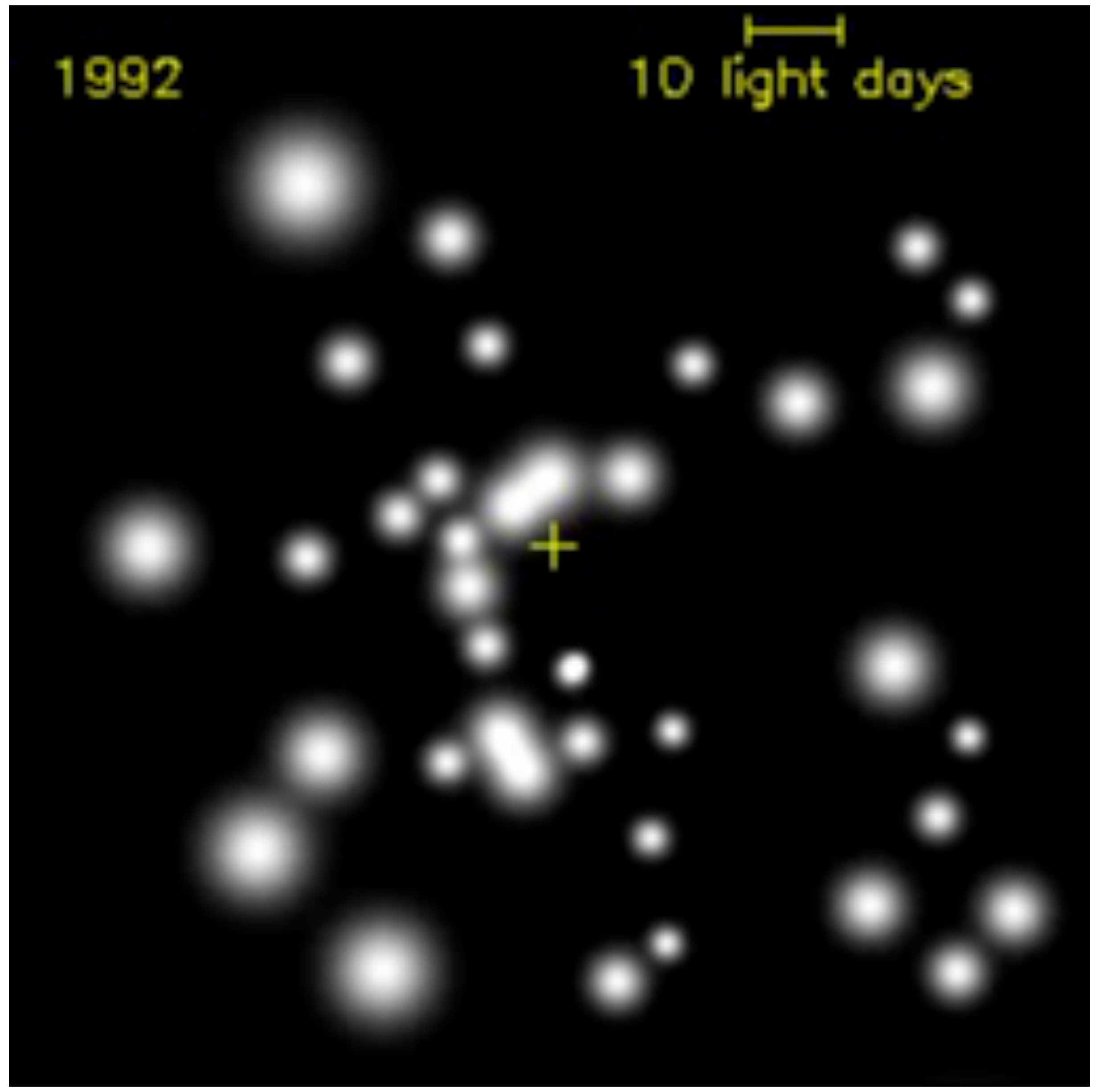

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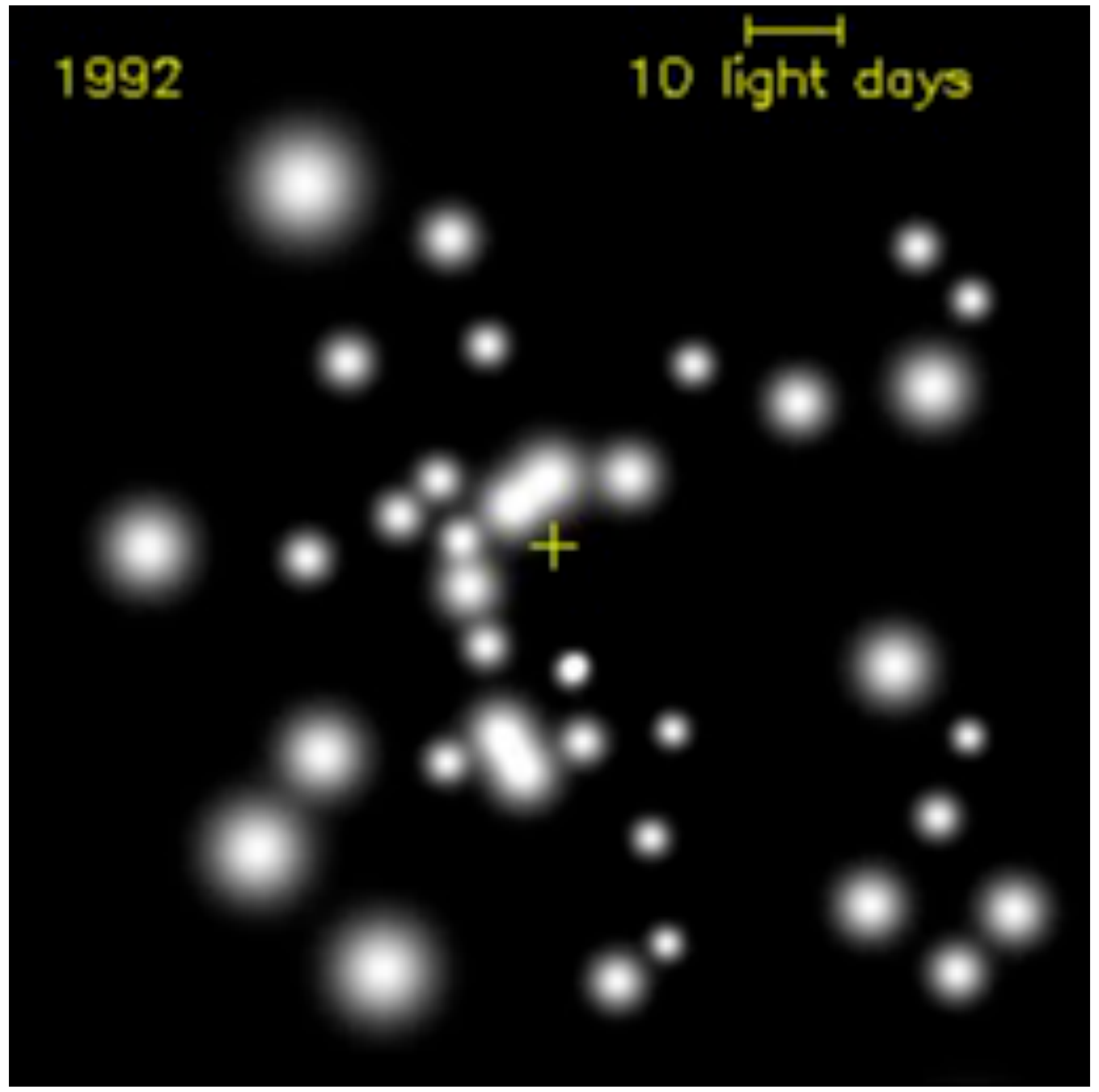

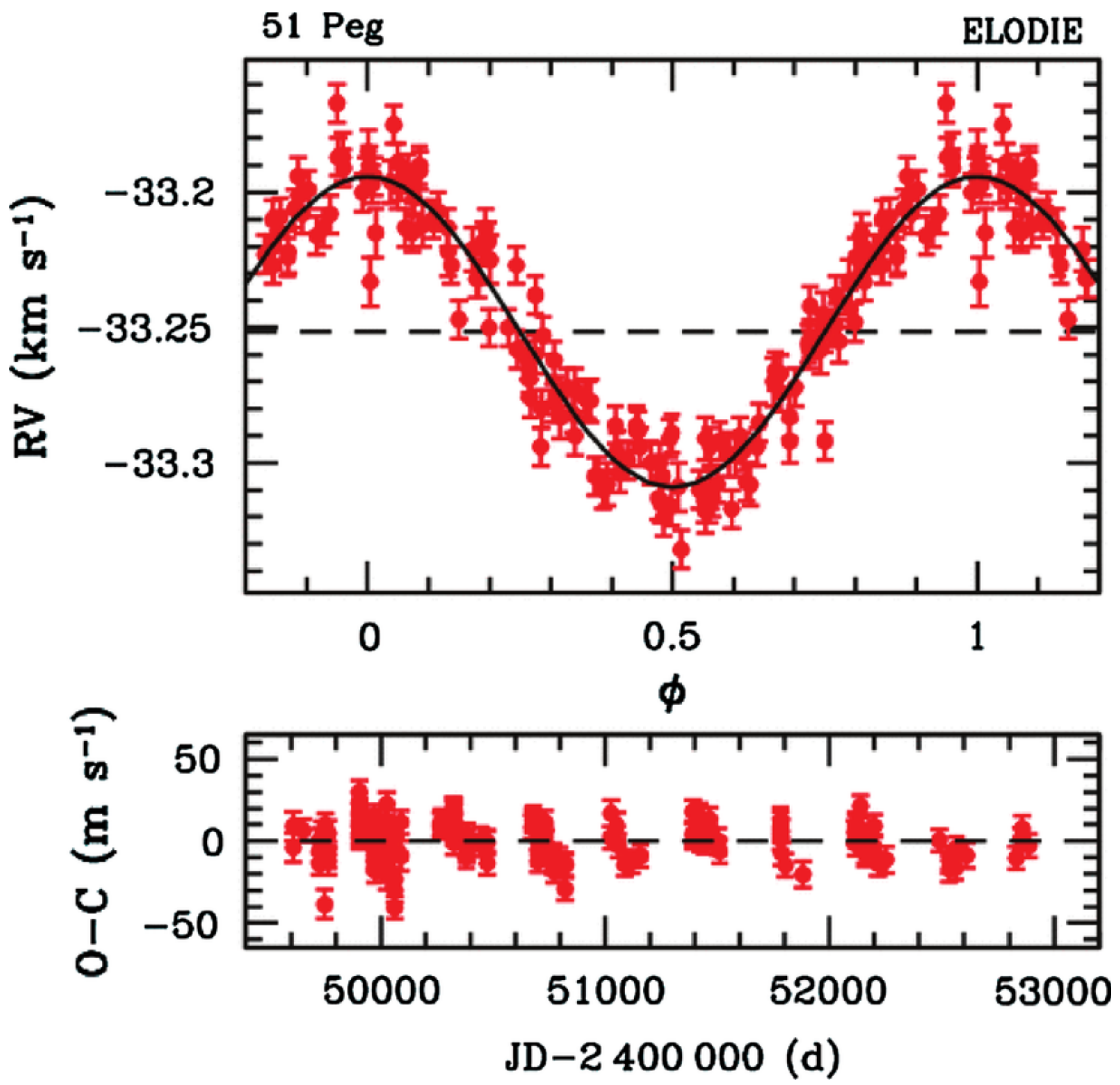

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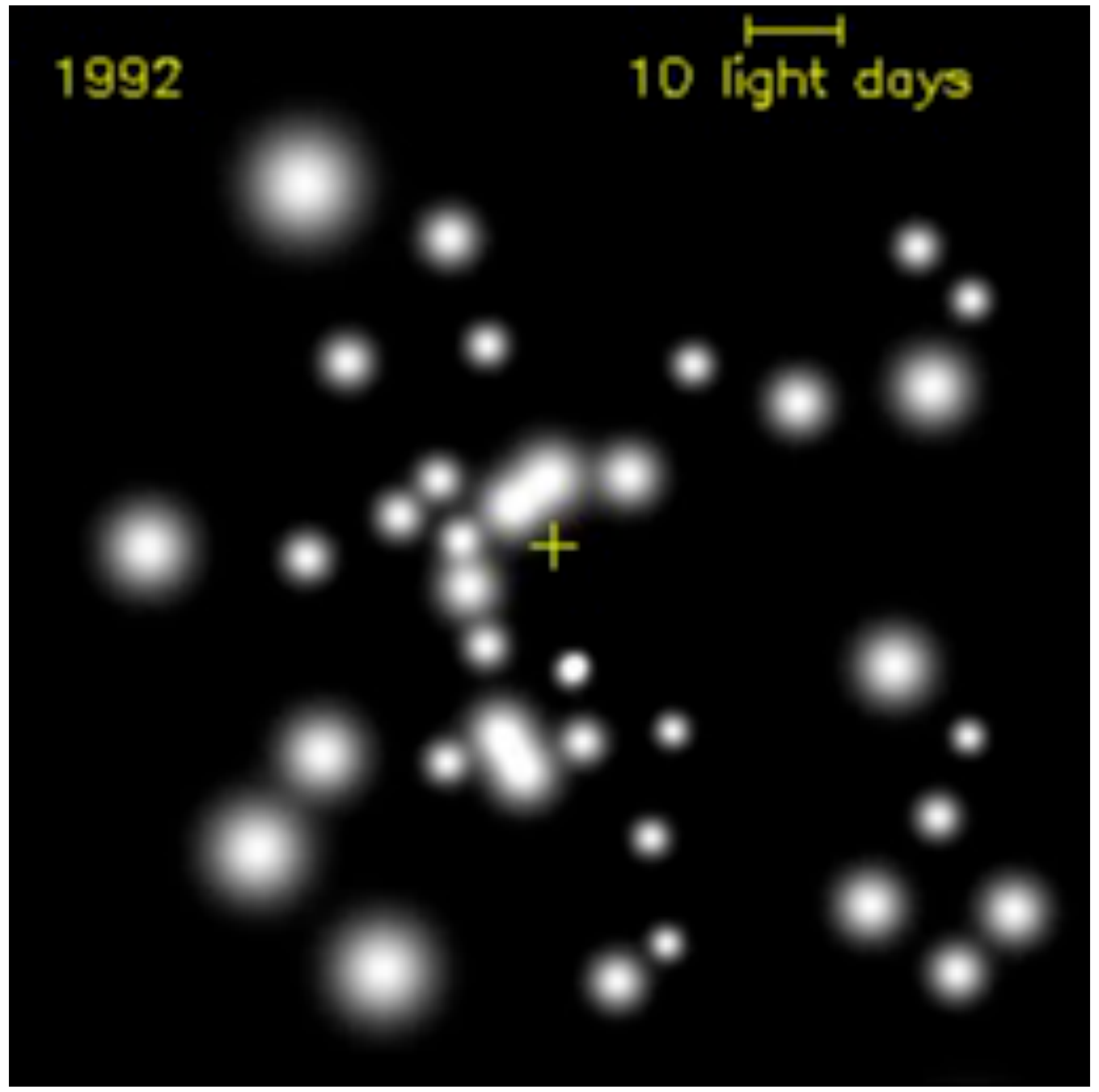

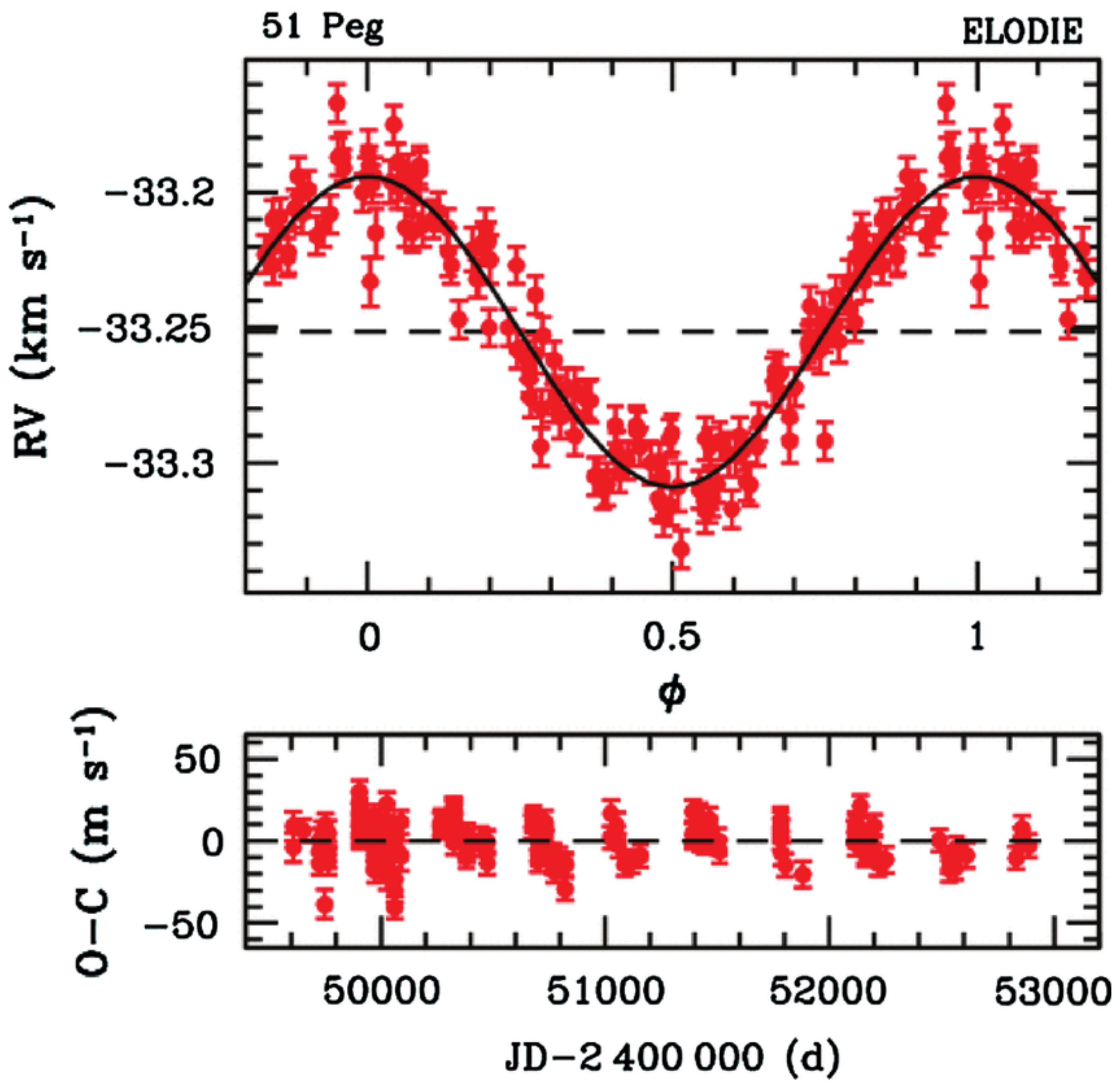

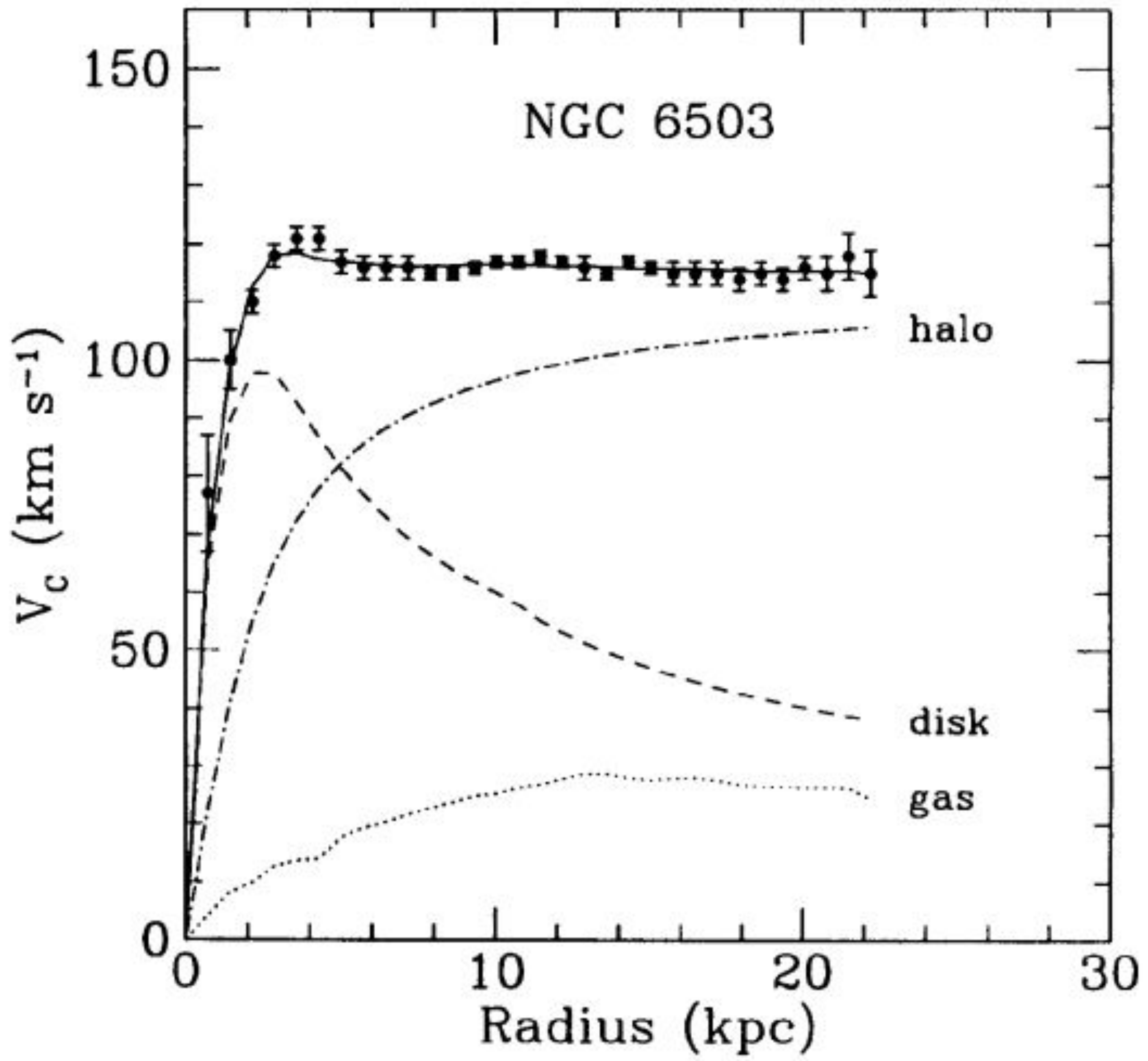

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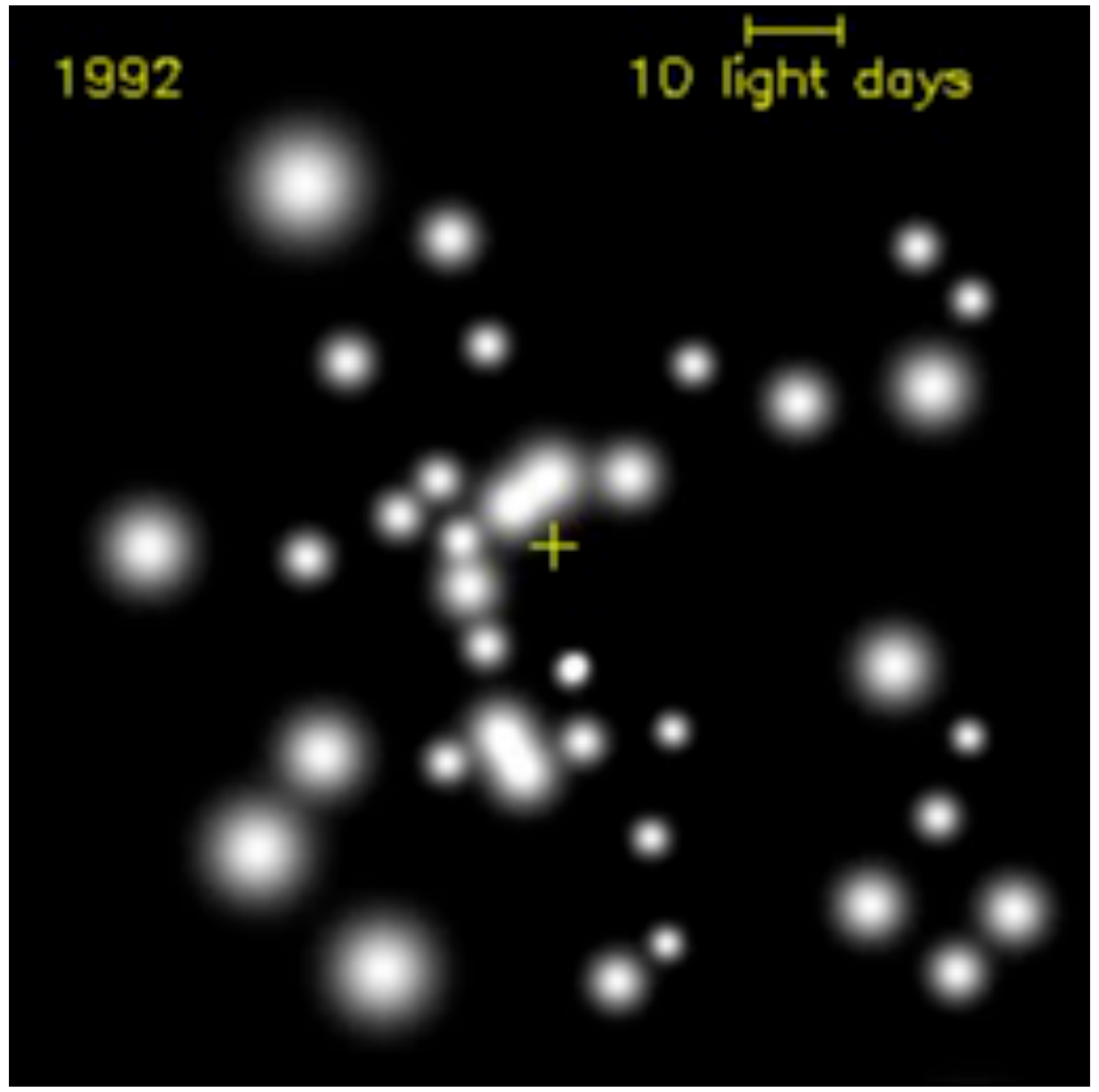
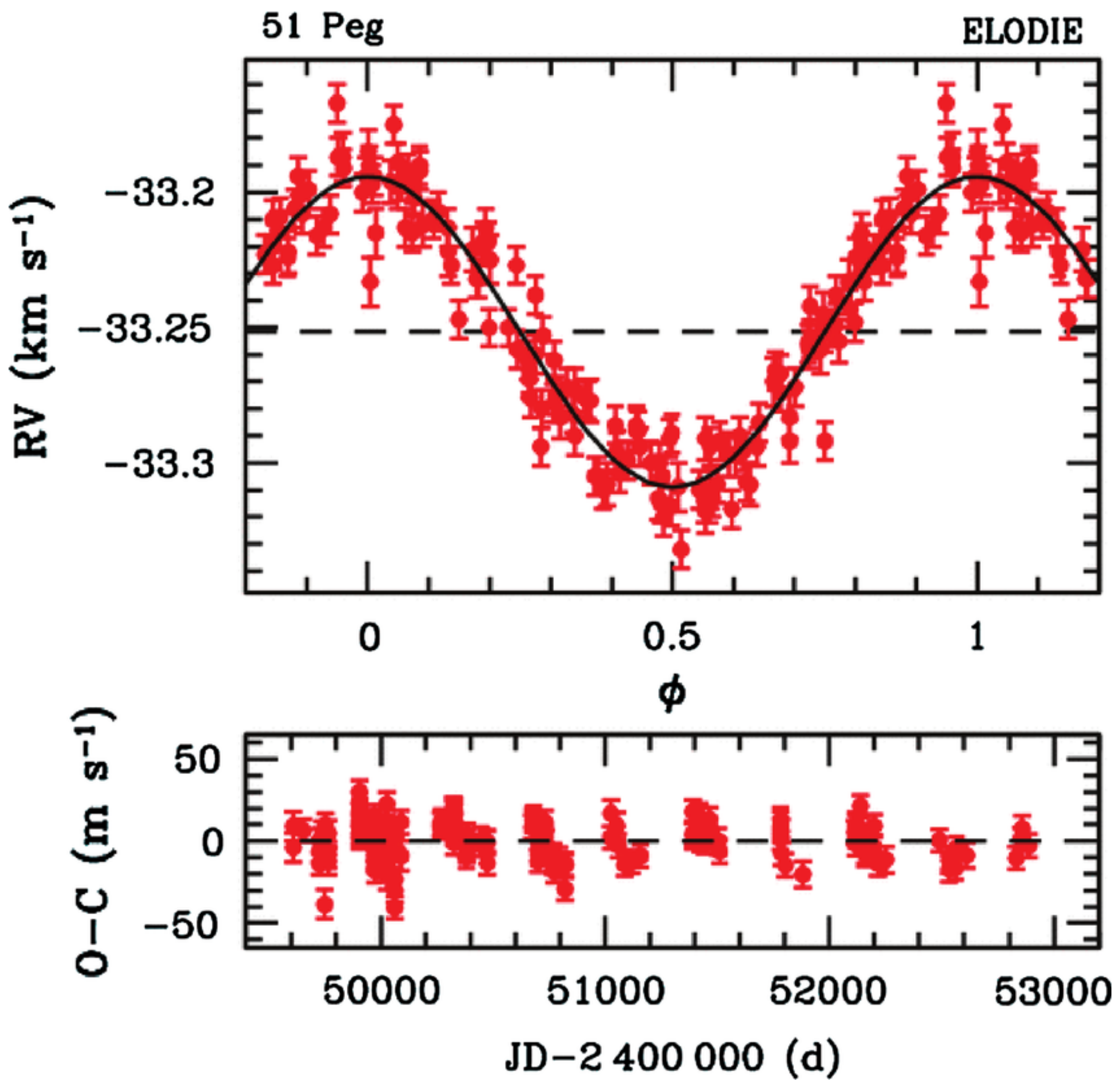
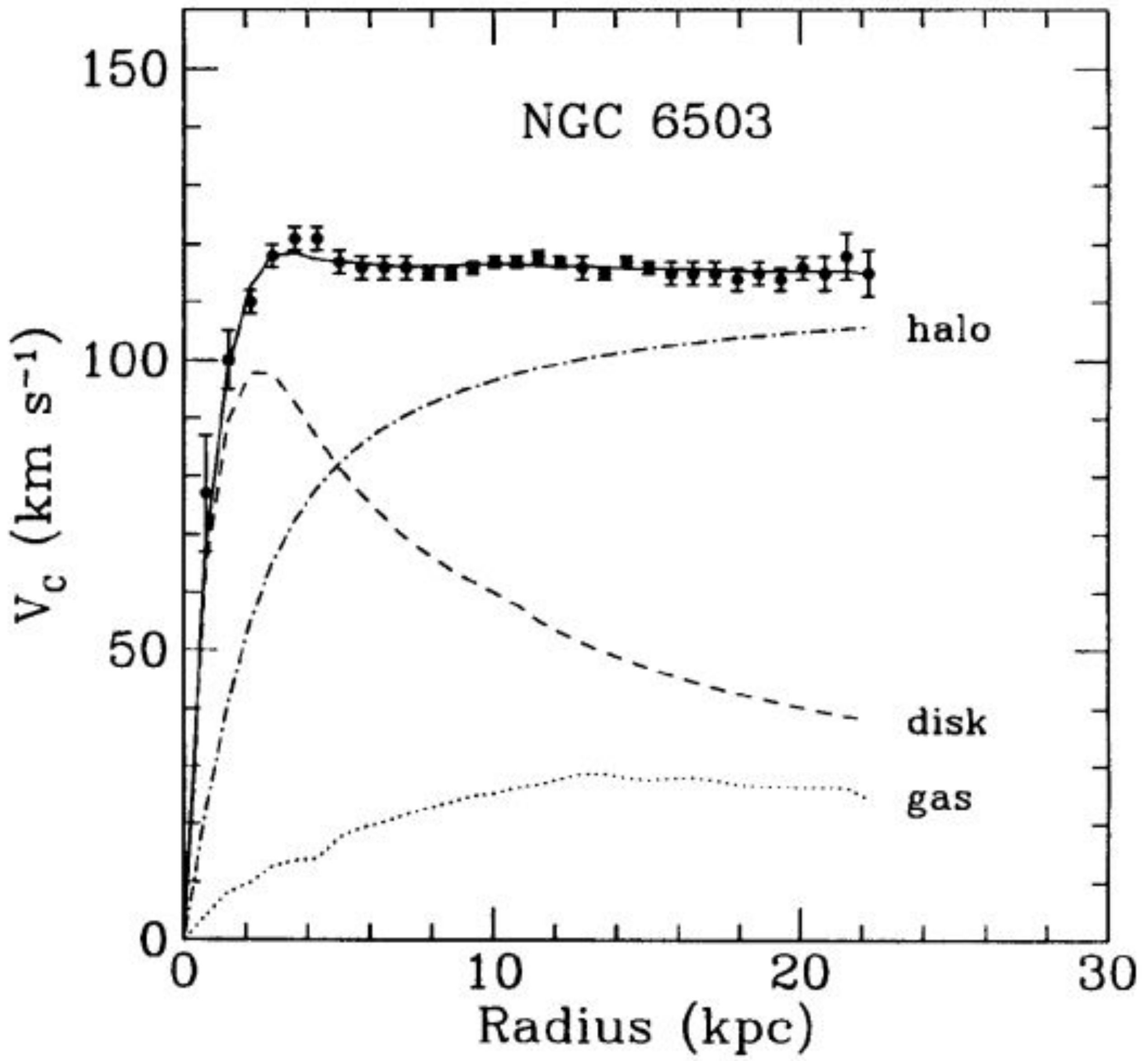


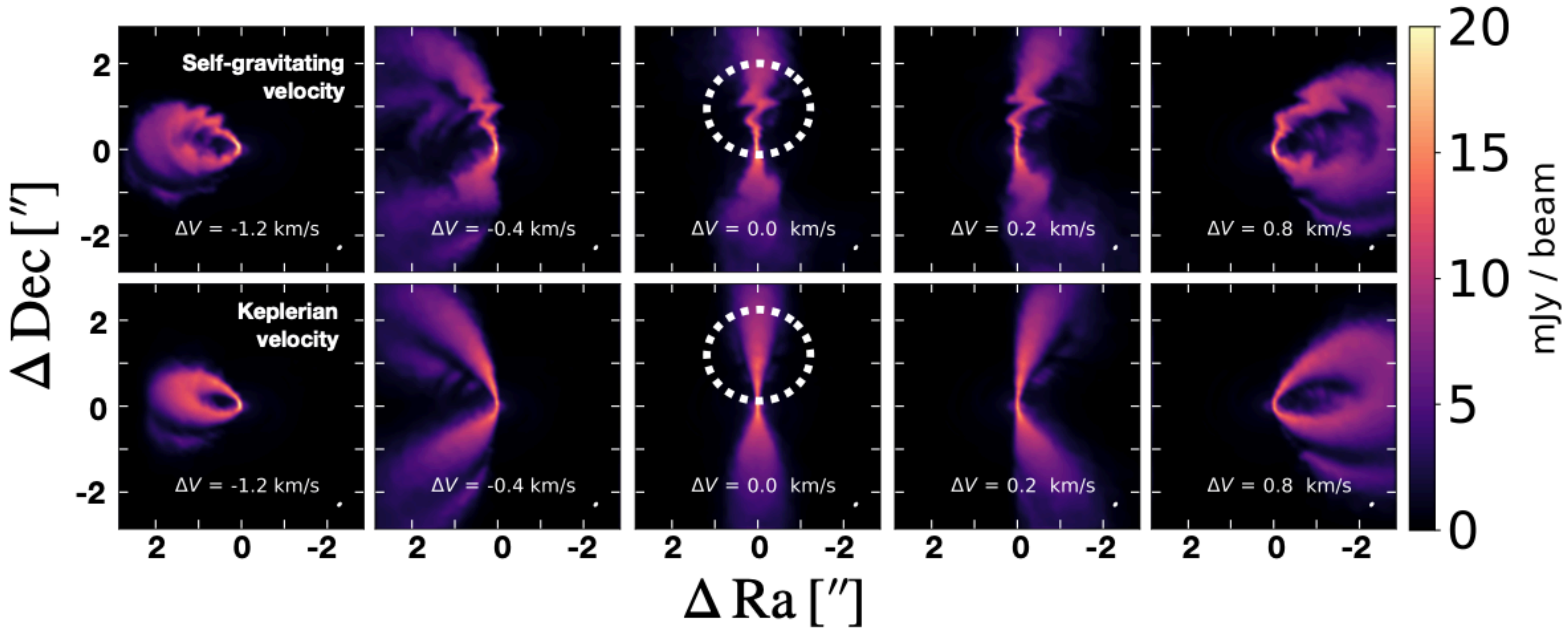


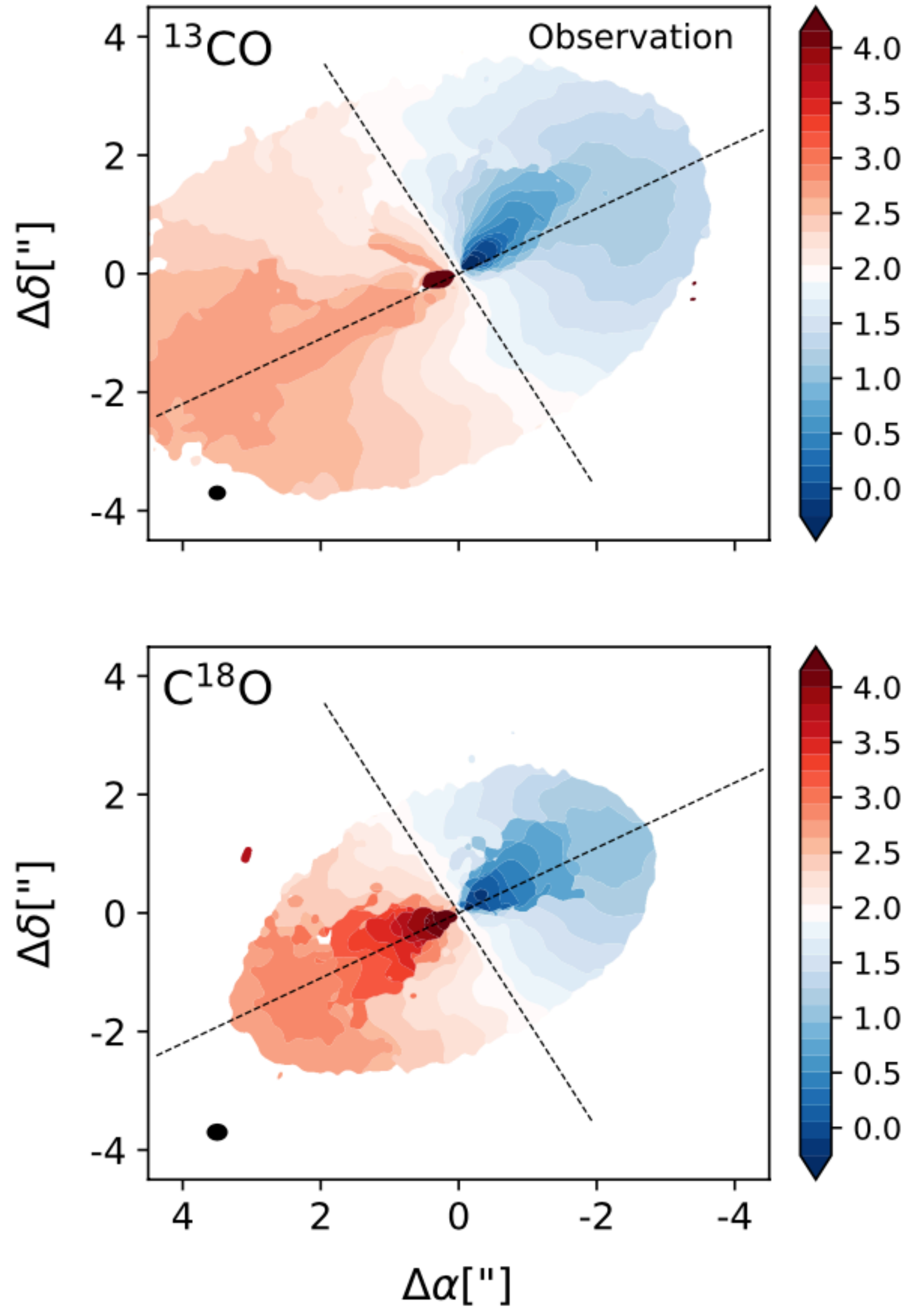


Looking at GI perturbations: the "wiggle"

Longarini, GL et al (2021), ApJL, in press

- Hall et al. (2020) show that the spiral structure induced by GI has characteristic wiggles in the channel maps, as observed in Elias 2-27





Paneque Carreno et al 2021

- Compute analytically the velocity perturbations due to the spiral in a self-gravitating discs (in the same way as can be done for a planet, see Bollati et al, 2021)

Constrain dynamically the cooling rate from the GI-wiggle

Longarini, GL et al (2021), ApJL, in press - see also Terry et al (2021), MNRAS, submitted

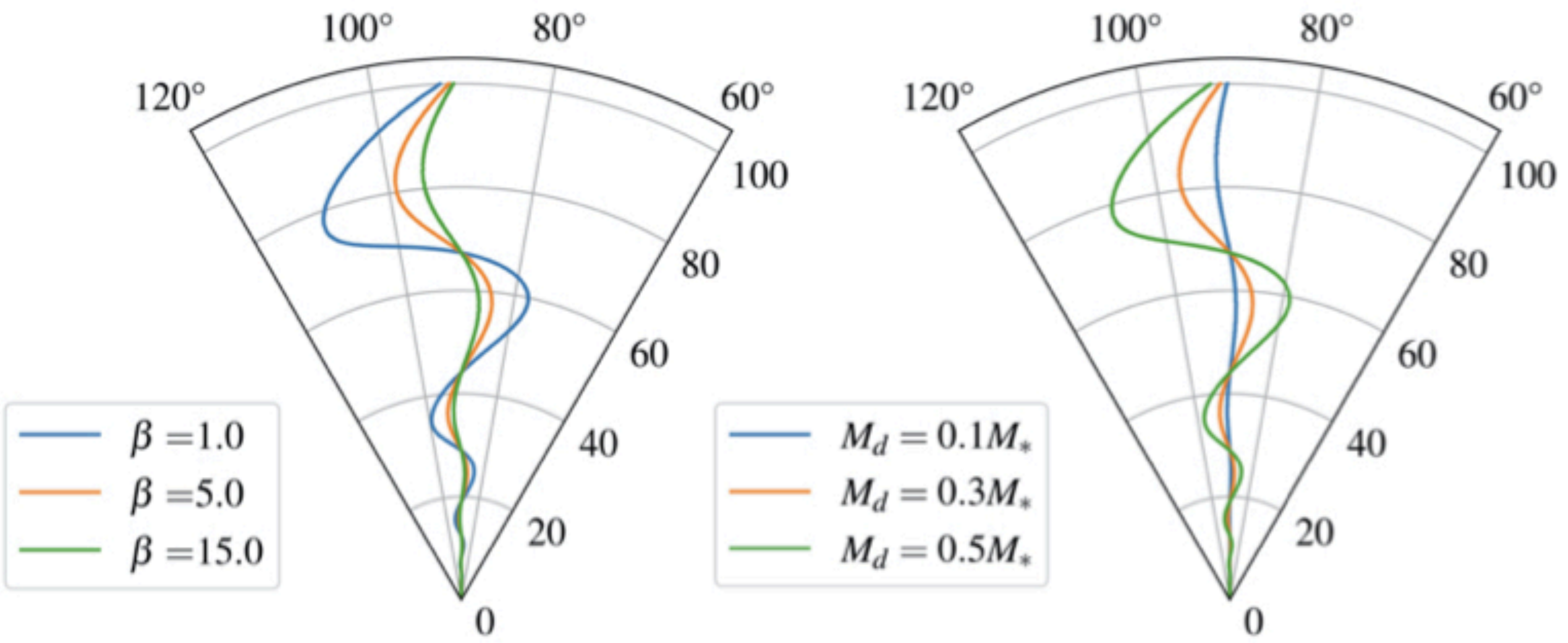
- Use the standard WKB approximation, for nearly Keplerian discs
- Assume that the density perturbation scales with the cooling rate (Cossins et al 2009)

$$\frac{\delta\Sigma}{\Sigma} = \chi\beta^{-1/2}$$

- Obtain velocity perturbations (after some maths!) and wiggle

$$\delta u_r = 2im\chi\beta^{-1/2}\left(\frac{M_d(r)}{M_\star}\right)^2 u_k$$

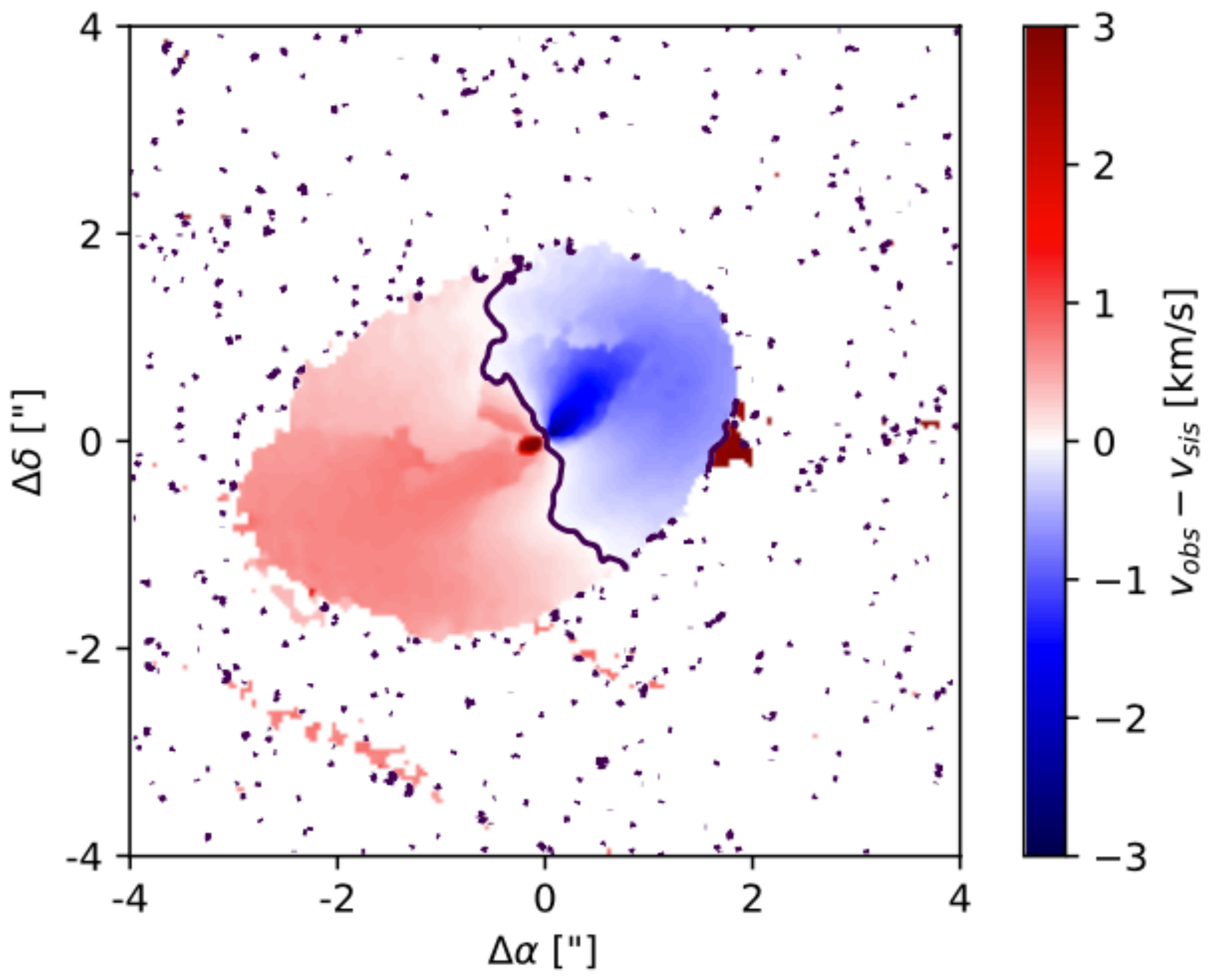
$$\delta u_\phi = -\frac{i\chi\beta^{-1/2}}{2}\left(\frac{M_d(r)}{M_\star}\right) u_k,$$

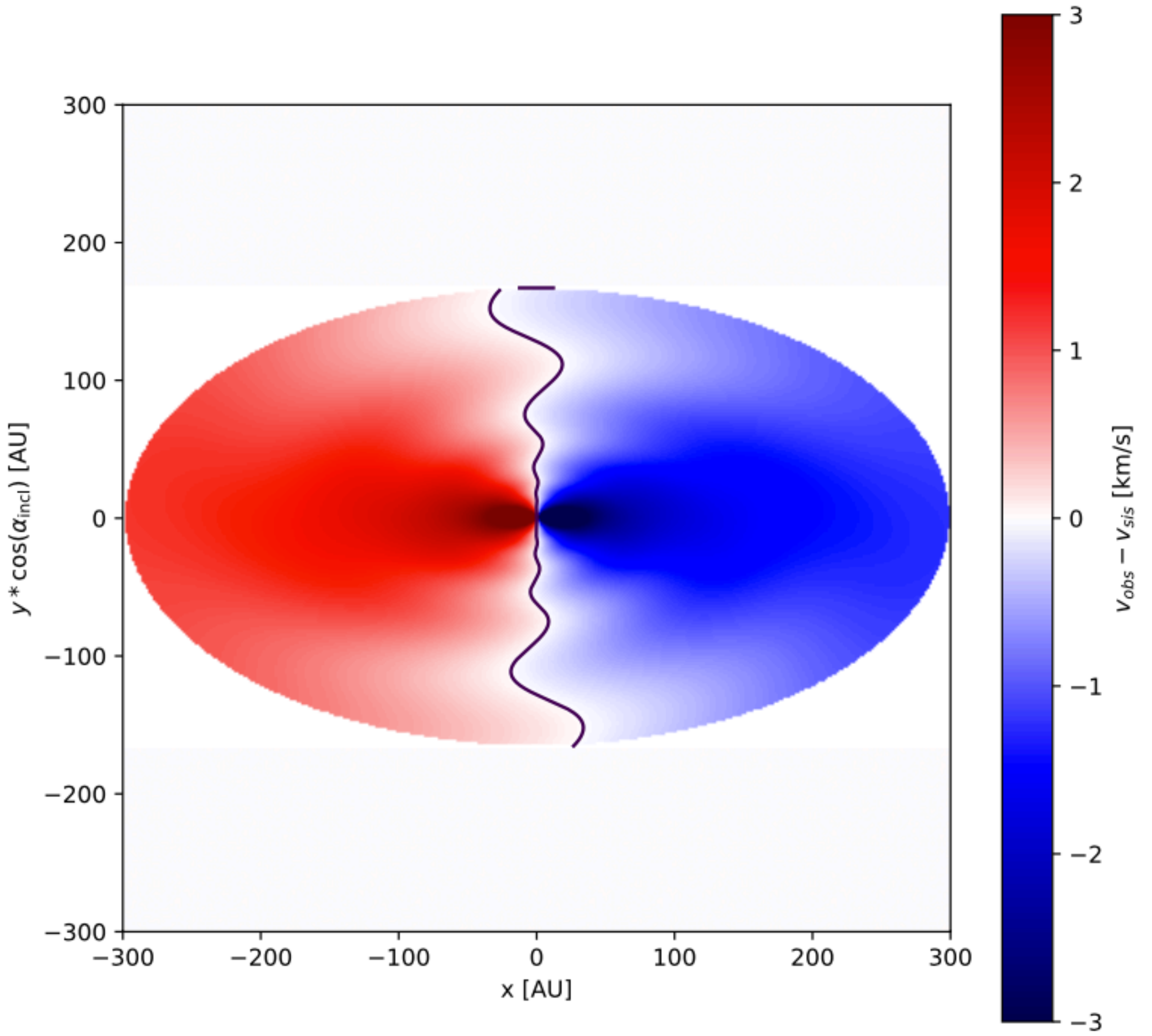


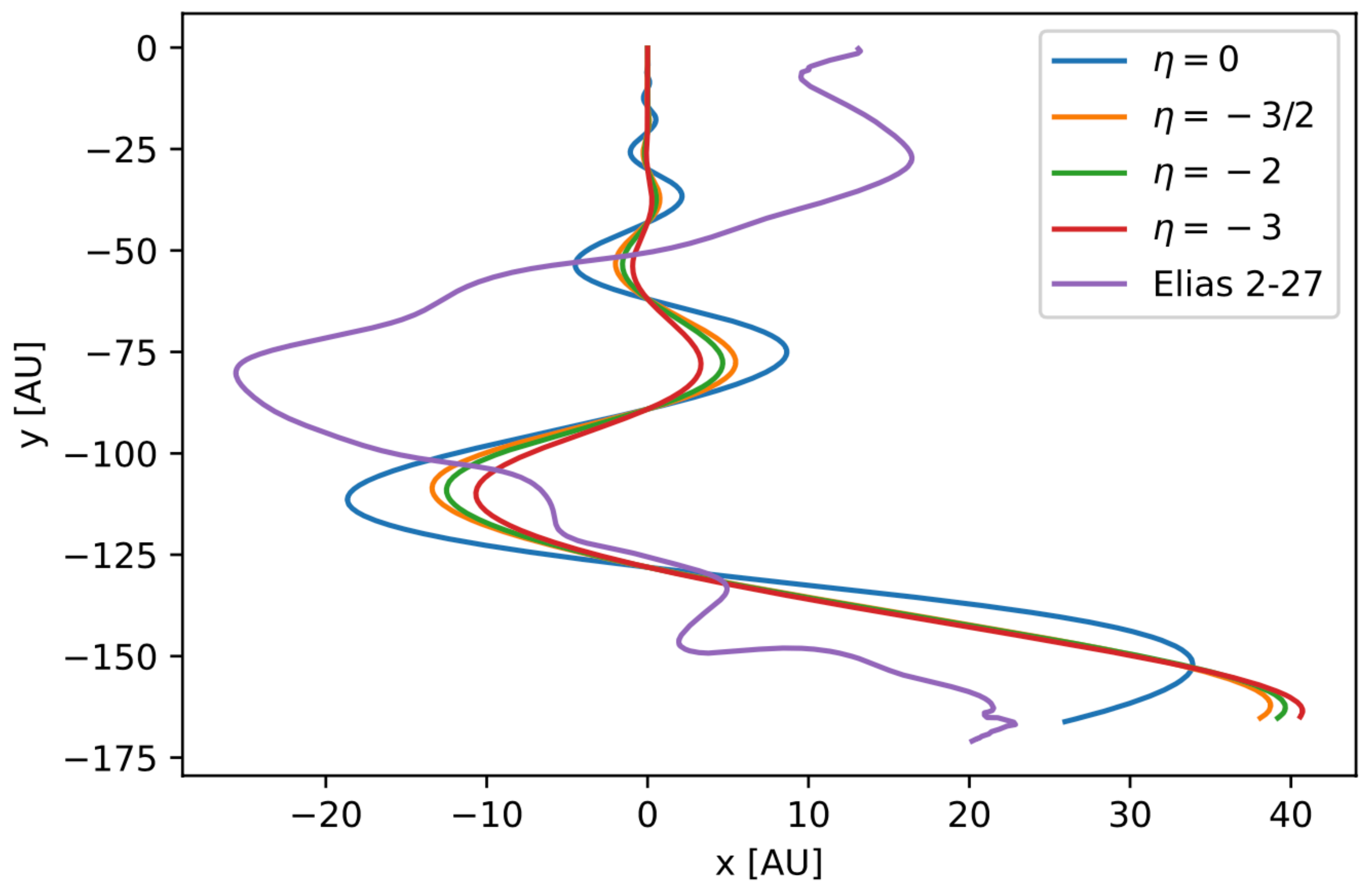
Constraining the outer cooling time in Elias 2-27

Longarini, GL et al, in prep - Bachelor Thesis of E. Arrigoni in Milano

- Apply model to Elias 2-27
- Match wiggle amplitude (std. dev. of zero velocity channel)
- Best match with β ~ 6.7-8.3





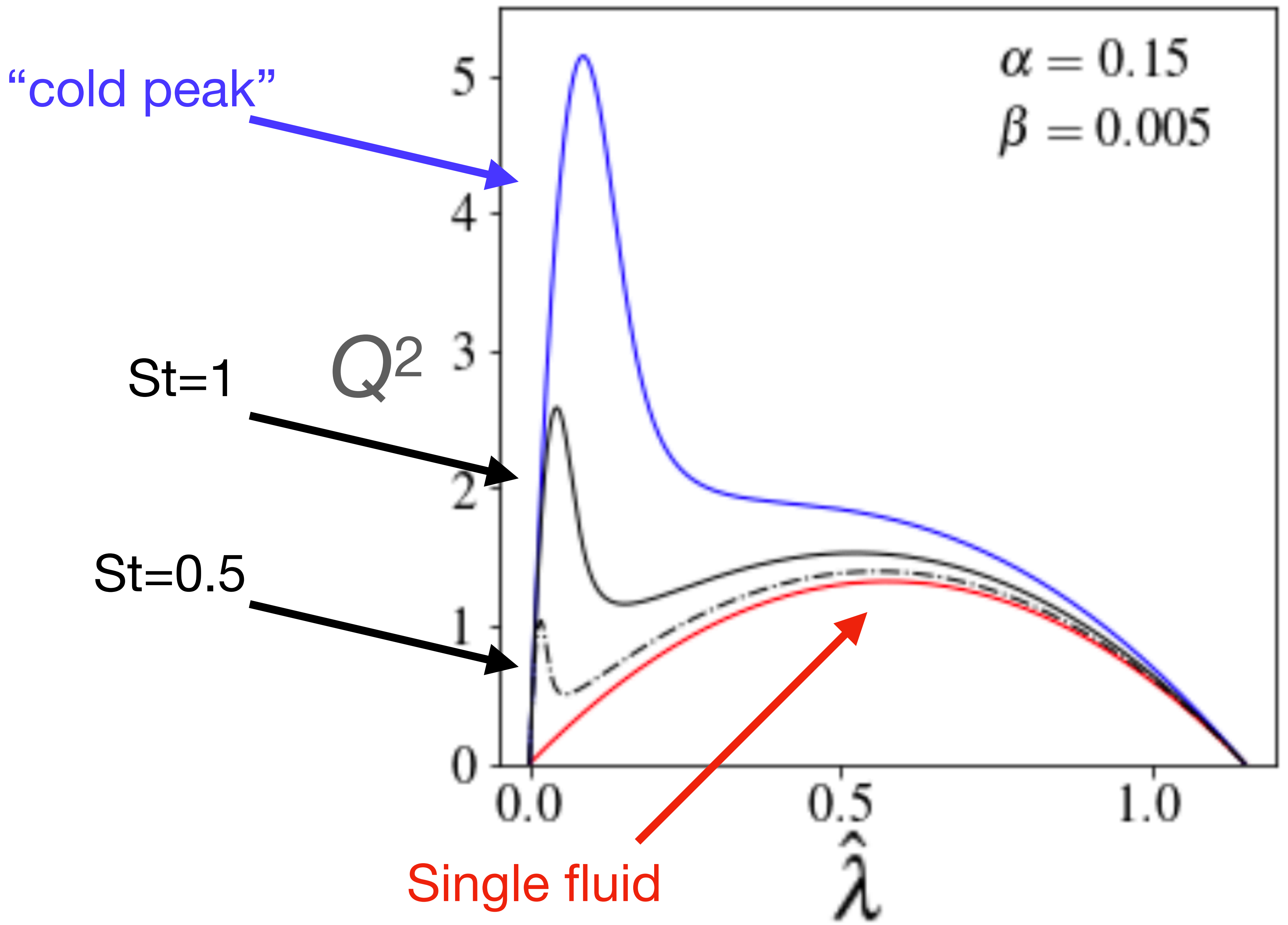


Gravitationally stability of a dust and gas disc

Longarini & GL, in prep

- Problem well studied for galaxies (Jog & Solomon, 1984, Bertin & Romeo, 1988)
- We generalize the stability analysis for two fluids interacting through drag
- Three main parameters:

$$\alpha = \Sigma_{\rm dust}/\Sigma_{\rm gas}$$
$$\beta = (\sigma_{\rm dust}/c_{\rm s})^2$$
$$St = \Omega t_{\rm stop}$$



Gravitationally stability of a dust and gas disc

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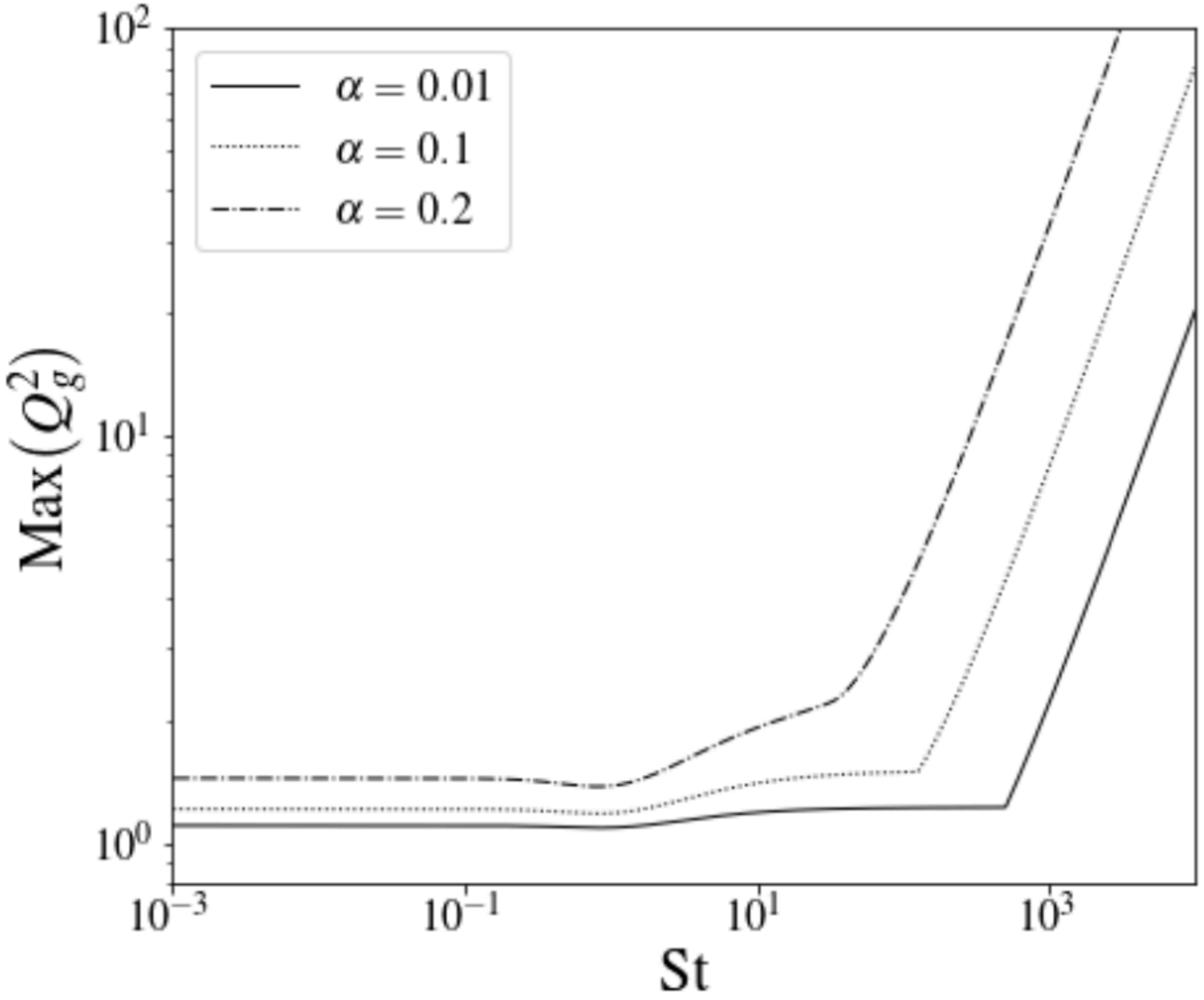
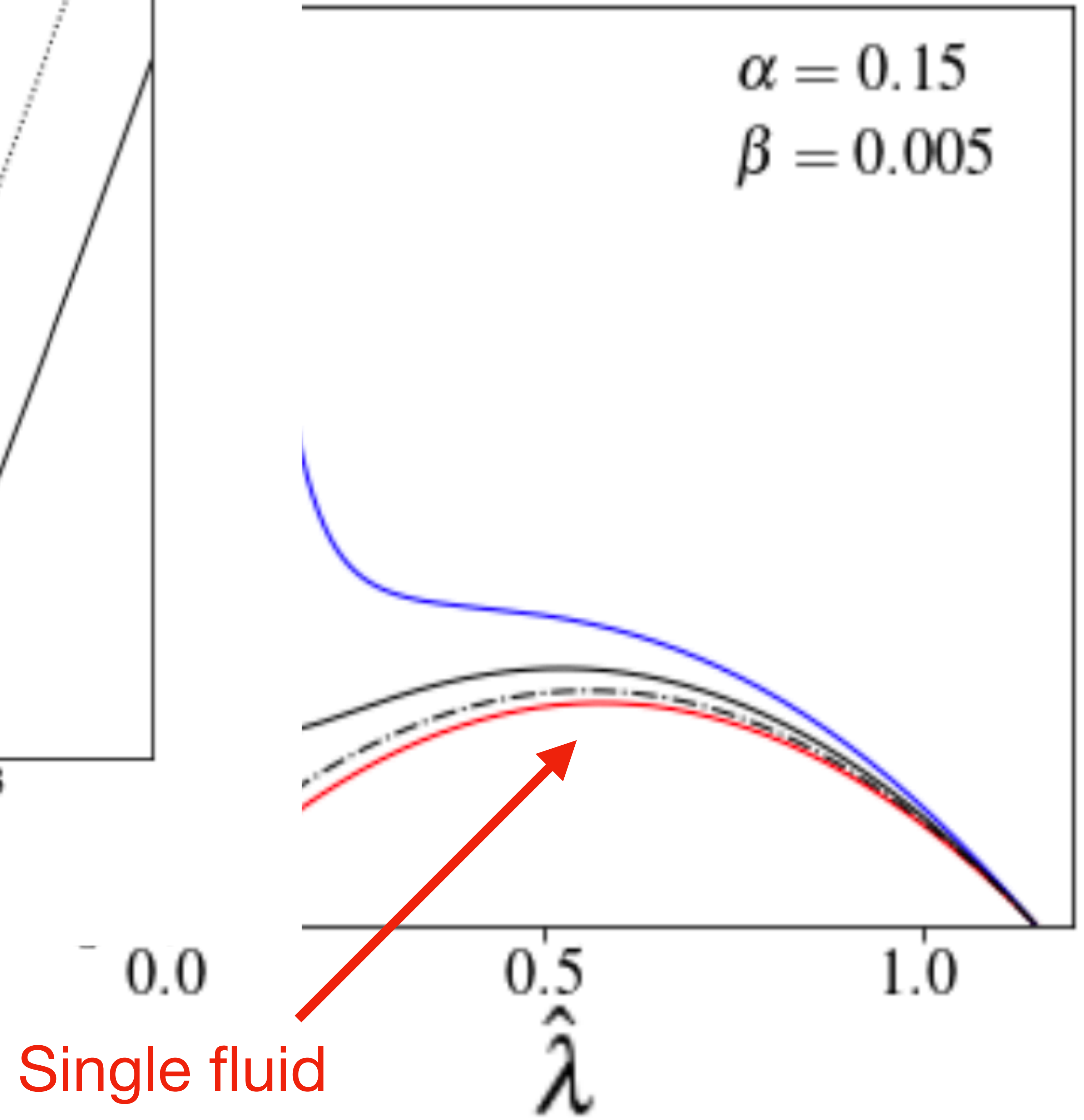
- We generalize the … g through drag
- Three main param…

$$\alpha = \Sigma_{\rm dust}/$$

$$\beta = (\sigma_{\rm dust}$$

$$St = \Omega$$





Gravitationally stability of a dust and gas disc

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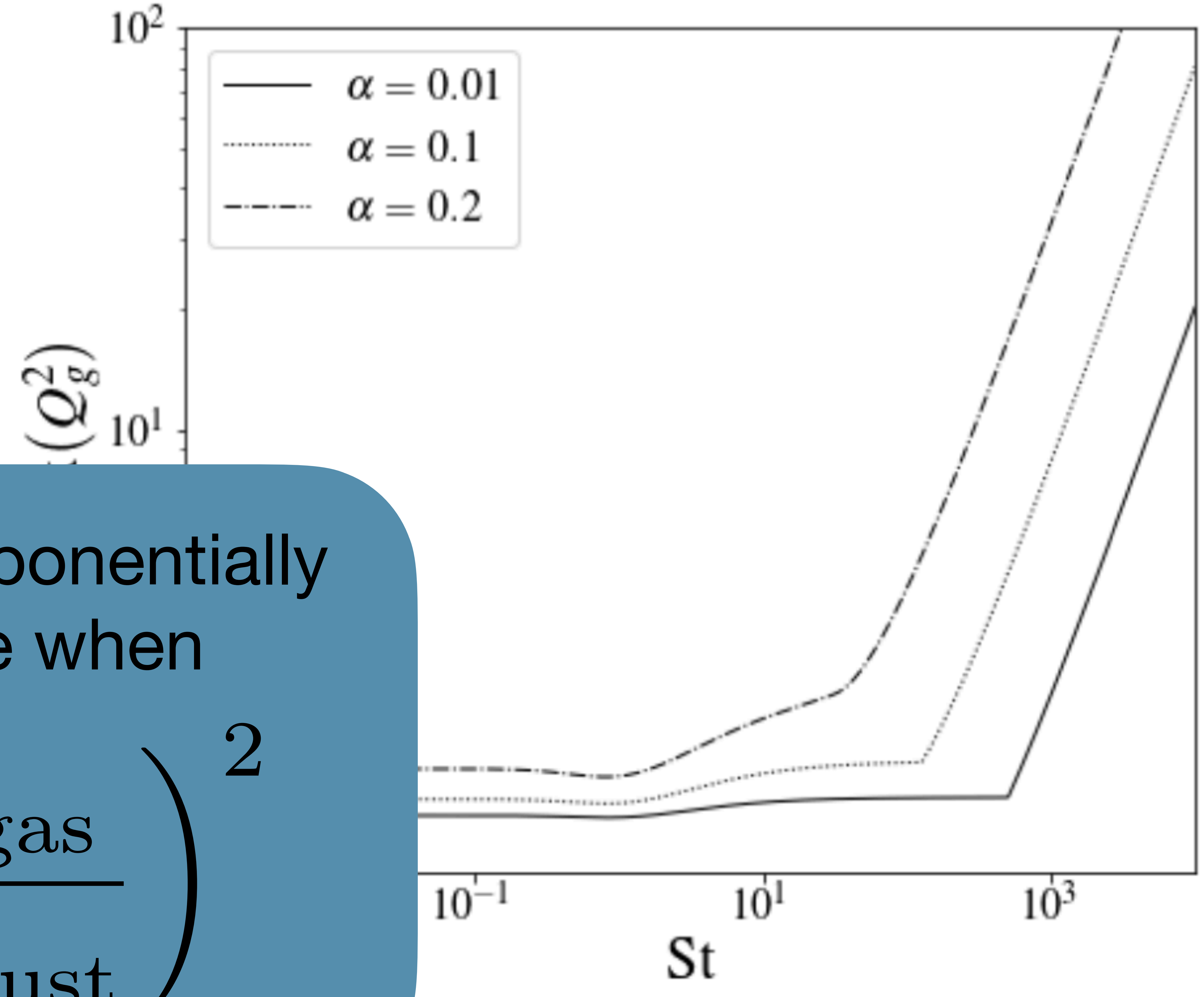
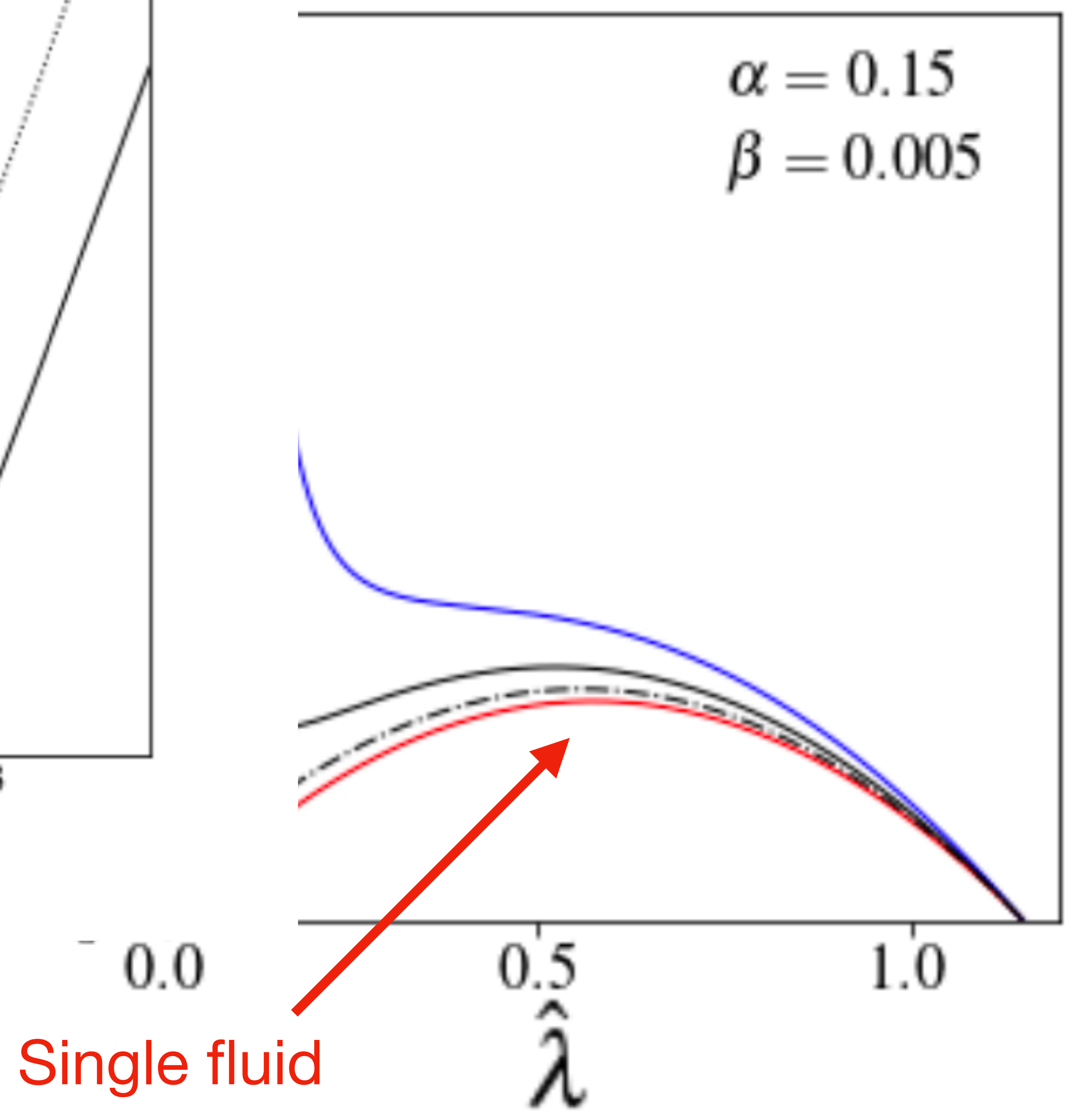


Disc becomes exponentially more unstable when

$$St \gtrsim \left(\frac{\Sigma_{\rm gas}}{\Sigma_{\rm dust}}\right)^2$$

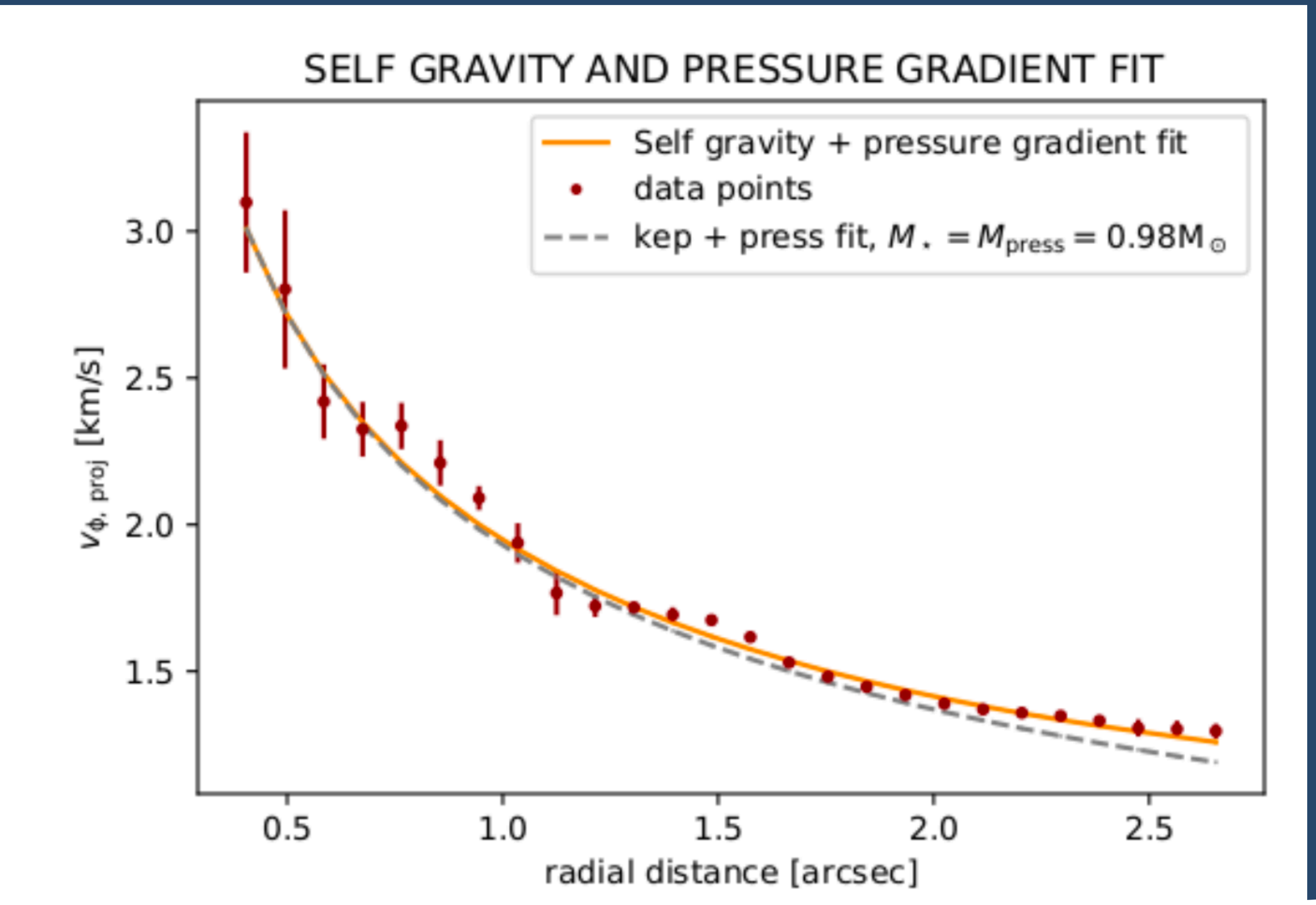


Conclusions

Self-gravity affect the disc in several ways

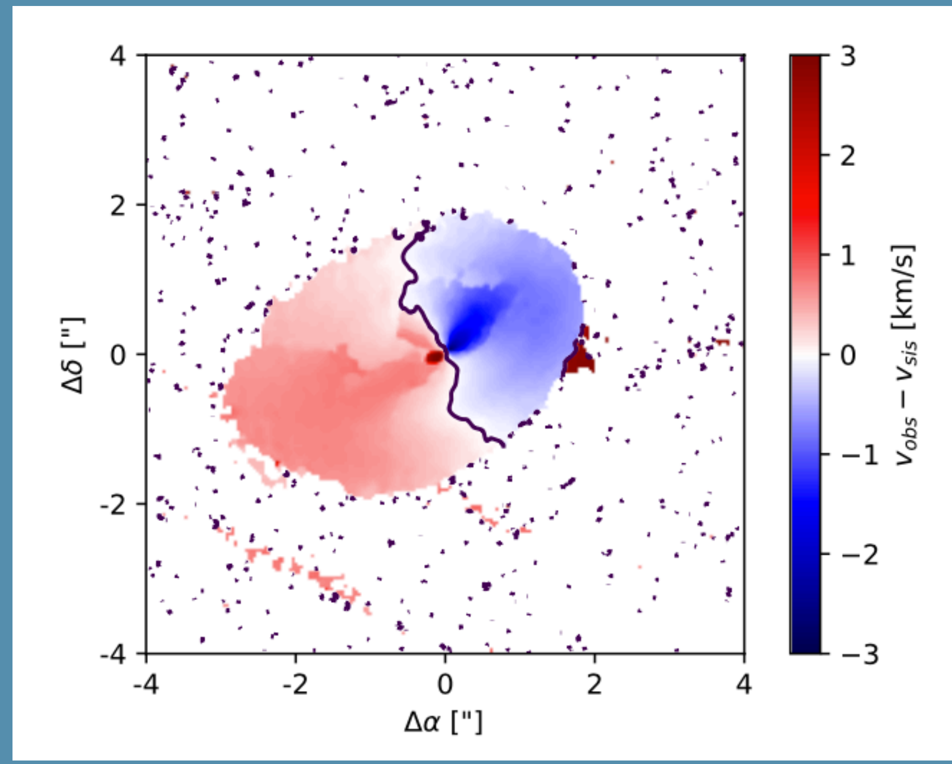
- Dynamics: angular momentun transport, fragmentation
- Morphology: spiral structure
- Kinematics: Overall rotation curve, GI wiggle

- Through rotation curve it is now possible to measure dynamically the disc mass! Done for **Elias 2-27 and WaOph6**
  - In general disc masses consistent with a dust/gas ratio ~ 0.01



Through the GI wiggle possible to measure the cooling rate of the outer disc

- Done for Elias 2-27 —> Close to fragmentation boundary



Analytical models of dust stability with self-gravity

↓

path to planetesimal/core formation?